**Chapter 19 Solutions**

1. A call option in Black-Scholes formula needs following five variables:
2. Current price of the firm’s common stock (*S*);
3. Exercise price (or strike price) of the option (*X*);
4. Term to maturity in years (*T*);
5. Variance of the stock’s price (); and
6. Risk-free rate of interest (*r*).

and the accumulative normal distribution functions are also used in Black-Scholes call option as follows:

,

Where *C* = Price of the call option, , , *N(d)* is the value of the cumulative standard normal distribution.

On the other hand, the binomial option pricing model can be written as



and *C=0* if *m>T*,

where n is the number of periods per year of term to maturity (T), *m* = minimum number of upward movements in stock price that is necessary for the option to terminate “in the money,” and , *X* = option exercise price (or strike price), *R = 1 + r = 1* + risk-free rate of return, *u = 1 +* percentage of price increase, *d = 1 +* percentage of price decrease,, and.

By a form of the central limit theorem, when , the cumulative binomial density function can be approximated by the cumulative normal density function as



where



It can be shown that



Then Equation can be rewritten as



Use the definition of m and property of lognormal distribution, it can be shown that Z1=d1 and Z2=d2



1. The stock price follows a log-normal distribution under Black-Scholes model framework. That is,  is a random variable with a log-normal distribution, where  is the stock price in jth period,  is the rate of return in jth period and is a random variable with normal distribution. Let *Kt* have the expected value  and variance  for each *j*. Then  is a normal random variable with expected value  and variance . Thus, we can define the expected value (mean) of  as

 (19.52)

Under the assumption of a risk-neutral investor, the expected return  is assumed to be  (where *r* is the riskless rate of interest). Therefore, the relationship between the expected stock return and risk-free rate under the risk-neutral assumption is 

1. The stock price follows a log-normal distribution under Black-Scholes model framework.
2. 1. P(X > 1.65) = 0.0495
   2. P(X < -2.38) = 0.0087
   3. P(X > -1.37) = 0.9147
   4. P(1.72>X > -2.43) = 0.9498
3. As the assumption in question 4, find z
   1. P(X < z) = 0.99 , by the inverse function of standard normal cumulative distribution, z= 2.33
   2. P(X > z) = 0.05, P(X < z)=1- P(X > z)=1-0.05=0.95

by the inverse function of standard normal cumulative distribution, z= 1.64

* 1. P (z <X<0)=0.0130, P (z <X<0)=P (X<0)- P (X<z)=0.5- P (X<z) =0.0130

P (X<z)=0.5-0.013=0.487, by the inverse function of standard normal cumulative distribution, z= -0.0326

* 1. P (-z <X<z)=0.5408, P (-z <X<z)=1- P(X > z)- P(X < -z)=1-2 P(X < -z),

Therefore, P(X < -z)= [1- P (-z <X<z)]/2= (1-0.5408)/2=0.2296

by the inverse function of standard normal cumulative distribution, -z = -0.74016,

z=0.74016.

1. Y ~N(5, 2) , then standard normal z=(Y-5)/2
   1. P(5<Y<9)= P((5-5)/2 < (Y-5)/2 < (9-5)/2 )= P(0<Z<2) =0.4772
   2. P(-1<Y<3)= P((-1-5)/2 < (Y-5)/2 < (3-5)/2 )= P(-3<Z<-1)=0.15
   3. P(Y>6)=P( (Y-5)/2 > (6-5)/2 )=P(Z>0.5)=0.3085
   4. P(Y <10) = P( (Y-5)/2 < (10-5)/2) =P(Z<2.5) =0.9938
2. Let x= the amount withdrawn, xp= the amount of cash prepared

P(x> xp)=0.001

Therefore, the standard normal value (xp-5) /1=3.08 by using the inverse function of standard normal cumulative distribution. xp=$8.08 (millions).



If we define y as the number of bottles that contain less than 20 ounces of milk, then y can be approximated by a normal distribution.

Y~N( np, np(1-p))=N(400(), 400 () (1-))=N(63.48, 7.3082)

1. By Equation (19.61), call option value is as follows

1. When X=90, use Equation (19.61), call option value is as follows