**DIFFERENTIATION USING THE CHAIN RULE**

The following problems require the use of the chain rule. The chain rule is a rule for differentiating compositions of functions. In the following discussion and solutions the derivative of a function *h*(*x*) will be denoted by tex2html_wrap_inline53or *h*'(*x*) . Most problems are average. A few are somewhat challenging. The chain rule states formally that

tex2html_wrap_inline57.

However, we rarely use this formal approach when applying the chain rule to specific problems. Instead, we invoke an intuitive approach. For example, it is sometimes easier to think of the functions *f* and *g* as ``layers'' of a problem. Function *f* is the ``outer layer'' and function *g* is the ``inner layer.'' Thus, the chain rule tells us to first differentiate the outer layer, leaving the inner layer unchanged (the term *f*'( *g*(*x*) ) ) , then differentiate the inner layer (the term *g*'(*x*) ) . This process will become clearer as you do the problems. In most cases, final answers are given in the most simplified form.

* + *PROBLEM 1 :* Differentiate tex2html_wrap_inline71.

* + *PROBLEM 2 :* Differentiate tex2html_wrap_inline73.
  + *PROBLEM 3 :* Differentiate tex2html_wrap_inline75.
  + *PROBLEM 4 :* Differentiate tex2html_wrap_inline77.
  + *PROBLEM 5 :* Differentiate tex2html_wrap_inline79.

**SOLUTIONS TO DIFFFERENTIATION OF FUNCTIONS USING THE CHAIN RULE**

*SOLUTION 1 :* Differentiate tex2html_wrap_inline528.

( The outer layer is ``the square'' and the inner layer is (3*x*+1) . Differentiate ``the square'' first, leaving (3*x*+1) unchanged. Then differentiate (3*x*+1). ) Thus,

tex2html_wrap_inline536

= 2 (3*x*+1) (3)

= 6 (3*x*+1) .

*SOLUTION 2 :* Differentiate tex2html_wrap_inline542.

( The outer layer is ``the square root'' and the inner layer is tex2html_wrap_inline544. Differentiate ``the square root'' first, leaving tex2html_wrap_inline544unchanged. Then differentiate tex2html_wrap_inline544. ) Thus,

tex2html_wrap_inline550

tex2html_wrap_inline552

tex2html_wrap_inline554

tex2html_wrap_inline556

tex2html_wrap_inline558

*SOLUTION 3 :* Differentiate tex2html_wrap_inline560.

( The outer layer is ``the 30th power'' and the inner layer is tex2html_wrap_inline562. Differentiate ``the 30th power'' first, leaving tex2html_wrap_inline562unchanged. Then differentiate tex2html_wrap_inline562. ) Thus,

tex2html_wrap_inline568

tex2html_wrap_inline570

tex2html_wrap_inline572

*SOLUTION 4 :* Differentiate tex2html_wrap_inline574.

( The outer layer is ``the one-third power'' and the inner layer is tex2html_wrap_inline576. Differentiate ``the one-third power'' first, leaving tex2html_wrap_inline576unchanged. Then differentiate tex2html_wrap_inline576. ) Thus,

tex2html_wrap_inline582

tex2html_wrap_inline584

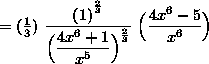
(At this point, we will continue to simplify the expression, leaving the final answer with no negative exponents.)

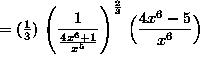
tex2html_wrap_inline586

tex2html_wrap_inline588

tex2html_wrap_inline590

tex2html_wrap_inline592





tex2html_wrap_inline598

tex2html_wrap_inline600

tex2html_wrap_inline602

tex2html_wrap_inline604

tex2html_wrap_inline606

tex2html_wrap_inline608.

*SOLUTION 5 :* Differentiate tex2html_wrap_inline610.

( First, begin by simplifying the expression before we differentiate it. ) Thus,

tex2html_wrap_inline612tex2html_wrap_inline614

( The outer layer is ``the negative four-fifths power'' and the inner layer is tex2html_wrap_inline616. Differentiate ``the negative four-fifths power'' first, leaving tex2html_wrap_inline616unchanged. Then differentiate tex2html_wrap_inline616. )

tex2html_wrap_inline622

tex2html_wrap_inline624

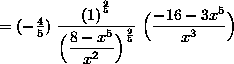
(At this point, we will continue to simplify the expression, leaving the final answer with no negative exponents.)

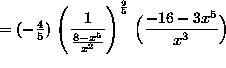
tex2html_wrap_inline626

tex2html_wrap_inline628

tex2html_wrap_inline630

tex2html_wrap_inline632





tex2html_wrap_inline638

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tex2html_wrap_inline644

tex2html_wrap_inline646.