

Inflation Risk Premium: Evidence from the TIPS Market*

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Abstract

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Abstract

“Inflation-indexed securities would appear to be the most direct source of information about inflation expectations and real interest rates” (Bernanke, 2004). In this paper we study the term structure of real interest rates, expected inflation and inflation risk premia using data on prices of Treasury Inflation Protected securities (TIPS) over the period 2000-2007. The estimates of the 10-year inflation risk premium are between 11 and 22 basis points for 2000-2007 depending on the proxy used for the expected inflation. Furthermore, we find that the inflation risk premium is time varying and, specifically, negative in the first half (which might be due to either concerns of deflation or low liquidity of the TIPS market), but positive in the second half of the sample.

1 Introduction

Treasury Inflation-Protected Securities (TIPS) were first issued by the U.S. Treasury Department in 1997. Since then, the TIPS market has grown substantially to about 10% of the outstanding Treasury debt. One important feature of this indexed debt is that both the principal and coupon payments from TIPS are linked to the value of an official price index - the Consumer Price Index (CPI). Namely, both payments from TIPS are denominated in real rather than nominal terms. As such, TIPS can be considered to be free of inflation risk. The difference between nominal Treasury and TIPS yields of equivalent maturities is known as a breakeven inflation rate¹ and represents the compensation to investors for bearing the inflation risk. This compensation includes both expected inflation and the inflation risk premium due to inflation uncertainty. In a speech before the Investment Analysis Society of Chicago in 2004, Governor Ben Bernanke stressed that estimating the magnitude of the inflation risk premium is important for the purpose of deriving the correct measure of market participants' expected inflation. Indeed, having a good estimate of the inflation risk premium is important for both the demand and the supply sides of the economy. On the demand side, such a measure would allow investors to hedge effectively against inflation risk. On the supply side, the measure would allow the Treasury to tune the supply of TIPS. The former Federal Reserve Chairman Greenspan (1985) also emphasized the importance of the inflation risk premium: *"The real question with respect to whether indexed debt will save taxpayer money really gets down to an evaluation of the size and persistence of the so-called inflation risk premium that is associated with the level of nominal interest rates."*

However, the inflation risk premium is not directly observable and is known to be difficult to estimate accurately. The literature on estimating the magnitude and the volatility of the inflation risk premium is also rather limited. The usual way to estimate the inflation risk premium in the literature is to use the difference between nominal long-term and short-term yields, the so-called term premium. This leads to higher estimates than perceived by the Federal Reserve. In particular, Bernanke (2004) states that *"Estimates of the inflation risk premium for bonds maturing during the next five to ten years are surprisingly large, generally in a range between 35 and 100 basis points, depending on the time period studied."* In this paper, we take a different approach. We estimate the inflation risk premium using the TIPS market prices. Our approach is motivated by the view of Bernanke (2004) who stated that

¹More formally, we define a *breakeven inflation rate* as a difference between nominal and real yields and a *TIPS breakeven rate* as a difference between nominal and TIPS yields of the same maturity. Sometimes, the breakeven inflation rate is also referred to as *the inflation compensation*, see, e.g. Gurkaynak, Sack, and Wright (2008).

“the inflation-indexed securities would appear to be the most direct source of information about inflation expectations and real rates.” In our empirical analysis we use monthly yields on zero-coupon TIPS with five-, seven-, and ten-year maturities from Barclays Capital, and monthly yields on zero-coupon nominal Treasury bonds from the Federal Reserve over the period 2000-2007. As such, unlike those obtained in the existing studies, our estimates of the inflation risk premium are derived directly from the TIPS trading data.

Our estimates of the 10-year inflation risk premium are between 11 and 22 basis points for 2000-2007. In addition, we find that the inflation risk premium is time-varying and more specifically, negative in the first half, but positive in the second half of the sample. The variation in the estimates not only comes from the variation in maturity, but also depends on the proxies for expected inflation. The robust negative inflation risk premium during 2000-2003 appears to be due to liquidity problems in the TIPS market that cause the relatively high real yields. The inflation risk premium is positive in 2004-2007: its five-year estimates vary from 11 to 24 basis points and 10-year estimates vary from 29 to 48 basis points, again depending on different measures of expected inflation.

In a study closely related to ours, D’Amico, Kim, and Wei (2006) estimate a three-factor Gaussian term structure model with nominal Treasury security and TIPS data and conclude that the TIPS yields contain a substantial “liquidity” component. The long-run averages of the inflation risk premium in their study can be also positive or negative. However, the reason is different. The sign of their premia depends not on the maturity or inflation proxy, but on the use of different nominal or TIPS series in the model estimation. Other relevant studies include Jarrow and Yildirim (2003) and Chen, Liu, and Cheng (2005) who use TIPS data to estimate the term structure models but they do not focus on estimating the inflation risk premium. In addition, Hördahl, Tristani, and Vestin (2005) estimate the inflation risk premium in the euro area using information from nominal and index-linked yields.

A few related studies obtain the estimates of the inflation risk premium without using the information from the TIPS market. For example, Campbell and Shiller (1996) estimate the inflation risk premium based on the nominal term premium and find that it is between 50 and 100 basis points. Buraschi and Jiltsov (2005) analyze both nominal and real risk premia of the U.S. term structure of interest rates based on the structural monetary version of a real business cycle model. They find that the inflation risk premium is time-varying, ranging from 20 to 140 basis points. Their estimate of the average 10-year inflation risk premium is roughly 70 basis points over a 40-year period. Ang, Bekaert, and Wei (2008) consider a term structure model with regime switches and find that the unconditional five-year inflation risk

premium is around 1.15%, but its estimates vary with regimes. Haubrich, Pennacchi, and Ritchken (2008) estimate a term structure model of real and nominal interest rates using data on nominal Treasury yields, survey forecasts of inflation, and inflation swap rates. Their estimated 10-year inflation risk premium is between 38 and 60 basis points during the 1982-2008 sample period, with the average of 51 basis points. Interestingly, Evans (2003) models U.K term structure of indexed bonds using Markov regime switching technique. He documents the large negative inflation risk premium, which varies between negative 1% and 3.5%.

Our paper is different from the above studies in a sense that we use explicit information from the TIPS market. The estimates of the inflation risk premium that we obtain using the TIPS market prices are substantially lower than the ones obtained by Campbell and Shiller (1996), Buraschi and Jiltsov (2005), Ang, Bekaert, and Wei (2008), and Haubrich, Pennacchi, and Ritchken (2008). However, the results in these studies may not be directly comparable to ours due to differences in sample periods, estimation methods, and data sets used.

The estimation methodology we use has two building blocks. First, we follow Evans (1998), who employs the equilibrium first-order conditions to link the nominal and real term structures. Evans (1998) uses the equilibrium conditions in order to test the Fisher hypothesis on U.K. data. He strongly rejects the Fisher hypothesis but he does not provide any estimates of the inflation risk premium in the U.K. Second, we use several proxies of expected inflation: (1) naive forecasts of inflation as the average of past inflation, (2) VAR methodology to derive expected inflation forecasts, and (3) the Survey of Professional Forecasters and the Survey of the Blue Chip Economic Indicators. Recently, there has been a considerable interest in employing surveys as proxies for expected inflation. For example, Ang, Bekaert, and Wei (2007) study several methods for forecasting inflation and find that surveys outperform other forecasting methods both in-sample and out-of-sample. On the other hand, Chernov and Mueller (2008) find systematic biases in survey forecasts. Nevertheless, they find that surveys along with private sector inflation expectations produce realistic inflation forecasts. We report the forecasting errors of the three approaches that we entertain to estimate expected inflation. In our sample, inflation forecasting errors are the lowest in the case of historical-based forecasts.

The rest of the paper is organized as follows. Section 2 provides an overview of the TIPS market and describes the data used in our empirical analysis. Section 3 describes the methodology. Section 4 presents estimation results for real yields, expected inflation, and

the inflation risk premium. Finally, Section 5 concludes.

2 Data

In this section we provide a brief review of the TIPS market and then describe the TIPS and inflation data sets used in our analysis.

A An Overview of the TIPS Market

TIPS were first introduced in 1997. This indexed debt was initially called Treasury Inflation Protected Securities (TIPS). Later on its official name was changed to Treasury Inflation Indexed Securities. Nevertheless, market participants keep calling these instruments TIPS, so we retain this abbreviation for our study. The TIPS market has been growing significantly since its inception. The first inflation-indexed debt issue had a maturity of 10 years. Since 1997, the Treasury has been issuing regularly additional 10-year debt, and 5-, 20-, and 30-year debt irregularly. For instance, in September 2007, \$450 billion TIPS were outstanding, representing roughly 10% of nominal Treasury debt, with \$70 billion in new issuance every year, and over \$8 billion in average turnover daily.² The main advantage of TIPS over nominal Treasuries is that TIPS investors are almost fully hedged against inflation risk. The TIPS coupon rate is fixed in real terms, and the principal amount grows with inflation over the life of indexed debt. The real return (purchasing power) of TIPS does not vary with inflation. However, the real return of nominal Treasury declines as inflation increases. Therefore, nominal Treasury debt holders are not protected against inflation risk.

The following example illustrates how TIPS investors are protected against inflation fluctuations, but nominal debt holders are not. For this example, we assume that the real yields are constant and ignore the indexation lag. Suppose in January 2006 the Treasury auctioned 5-year TIPS with a 2% coupon rate and 5-year nominal Treasury debt with a 4.5% coupon rate. If an investor buys the January TIPS and holds it to maturity, he will receive a real return on his investment equal to 2%. If an investor buys nominal Treasury with 4.5% coupon rate, his real return will depend on the level of the actual Consumer Price Index (CPI) inflation rate. If inflation rate turns out to be 2.5%, then the real return on nominal debt will be 2%. If instead, the inflation rate turns out to be 3%, then the real return will be only 1.5%. Part of the purchasing power will be eroded by higher inflation against which nominal debt holders are not protected. The real rate of return, or real yield,

²See, for example, US Treasury press release on <http://www.ustreas.gov/press/releases/hp570.htm>.

is the only relevant measure for investors because it measures the result of the investment in terms of the purchasing power.

B Dataset description

Our sample period extends from January 2000 to December 2007 due to the low liquidity in the TIPS market prior to 2000. Liquidity problems that plagued this market have been documented in, e.g. D’Amico, Kim, and Wei (2006) and Shen (2006).³ For TIPS data, we use monthly yields on *zero-coupon* TIPS of 5-, 7-, and 10-year maturities from Barclays Capital Bank. We have also considered zero-coupon data estimated by Gurkaynak, Sack, and Wright (2008) and available on www.federalreserve.gov/econresdata/researchdata.htm. The descriptive statistics is presented in Table 1. As one can see from Panels A and B, two data sets are essentially identical, and therefore, yield essentially identical results. For nominal data, we use monthly yields on zero-coupon nominal Treasury bonds from Federal Reserve. Panels A and B of Figure 1 plot monthly term structure of nominal Treasury bonds and TIPS, respectively. These graphs show that both nominal and TIPS yields decrease steadily since the beginning of our sample period until about the middle of 2003. The period of low long-term rates between 2004 and 2006 is often called “conundrum,” referring to the fact that the increase in the short-term rates did not lead to a consequent increase in the long-term rates.⁴ The mild increase of the long-term rates (both nominal and real) in 2006 and 2007 was associated with the lower volatility of interest rates than in the first half of our sample. Panel C of Figure 1 presents the breakeven rate defined as the difference between 10-year nominal and TIPS zero-coupon yields. Clearly, the breakeven rate is relatively low and volatile in the first half of the sample and relatively high and less volatile in the second half.

To calculate the inflation risk premium, we need to estimate expected inflation. In our first approach we construct expected inflation using information in past inflation rates, so we use two measures of realized inflation. One is the seasonally-unadjusted Consumer Price Index (CPI) (to which TIPS are linked) and the other is the Core CPI (Consumer Price Index Less Food and Energy).

For our second approach, we entertain a Vector Autoregression (VAR) model to estimate expected inflation, following Ang and Piazzesi (2003) and Chernov and Mueller (2008). We

³Even in May 2001, the Treasury Advisory Committee of the Bond Market Association recommended that the TIPS program to be discontinued.

⁴The Federal Open Market Committee increased the federal funds rates 17 times from one percent up to 5.25 percent between June 2004 and June 2006.

include in the VAR real activity and inflation variables. Real activity variables include *HELP* - the Index of Help Wanted Advertising in the Newspapers, *EMPLOY* - the growth rate of employment, *IP* - the growth rate of the industrial production index IP, and *UE* - the unemployment rate. This is the standard list of variables used in monthly VARs in macroeconomic literature (see, e.g. Ang and Piazzesi (2003)). Inflation variables used in our VAR include the combination of realized and forecasted inflation variables, available on monthly basis. In particular, we include the seasonally-unadjusted consumer price index *CPI*, the Core *CPI*, the production price index inflation rate *PPI*, and the Blue Chip Forecasters' inflation rate *BC* in our VAR regression.

In our third approach we use surveys as proxies for expected inflation measures. This is motivated by Ang, Bekaert, and Wei's finding that survey measures forecast inflation the best. One proxy we use is from the Survey of Professional Forecasters (SPF) conducted by the Federal Reserve Bank of Philadelphia every quarter and the other is the Blue Chip Forecasts provided by Aspen Publishers.⁵ SPF forecasts of one-year ahead and 10-year ahead inflation are available on a quarterly basis while one-year ahead Blue Chip forecasts are available monthly. Before presenting the summary statistics, we would like to describe the Blue Chip inflation forecasts in a little more detail because they are not reported in a conventional way.

Blue Chip Forecasts of Financial and Economic Indicators represent consensus forecasts of about 50 professional economists in the leading financial and economic advisory firms, and investment banks each month. The survey contains the forecasts of key financial and macroeconomic indicators, including the CPI inflation (inflation hereafter).

In particular, Blue Chip Economic Indicators provide monthly estimates of one-year inflation forecast for both the current year and the next year. For instance, in January 1999, Blue Chip provides the expected inflation for both 1999 and 2000. In February 1999, they also provide inflation forecasts for 1999 and 2000, but the forecast horizon is actually 11 months. In December 1999, analysts again provide forecasts of the 1999 and 2000 year inflation albeit with a forecast horizon of one month only. This feature of the survey results in a time-varying forecast horizon for any variable in question. In our empirical analysis we need monthly forecasts for a fixed horizon. For instance, in February 1999 we need a one-year ahead inflation forecast, but the Blue Chip Survey has only 11-month and 23-month ahead forecasts available. Following Chun (2007), we obtain monthly fixed horizon forecast

⁵We do not use the Livingston Survey because it is conducted only twice a year and, unlike other surveys, participants of the Livingston survey are asked to provide their forecasts of non-seasonally-adjusted CPI levels six and twelve months in the future.

by doing linear interpolations.

We now proceed to the summary statistics of the data used in our empirical analysis. Panel A of Table 1 reports the summary statistics on zero-coupon TIPS yields. We observe that on average TIPS yields in our sample are between 2.21% and 2.57% for 5- and 10-year index-linked bonds, respectively.

Panel B reports sample statistics for nominal zero-coupon Treasuries. Nominal Treasuries yields are between 4.24% (5-year yields) and 4.89% (10-year yields) on average during our sample period. This indicates that the spread between nominal and inflation-linked bonds, the breakeven rate, is between 2.03% and 2.32% depending on the maturity of the bonds. As we mentioned earlier, the breakeven rate is also quite volatile (see Panel C of Figure 1).

Panel C of Table 1 reports the statistics of various realized and expected inflation measures. The average realized CPI-based inflation is 2.74% with a 0.82% volatility during our sample period, while the average realized Core inflation is 2.18%, naturally, with a much lower volatility of only 0.43%, because this index excludes very volatile energy and food prices. In addition, we report the statistics for Producer Price Index inflation, the variable that we use in the VAR estimation of expected inflation. On average, PPI inflation is 2.81% with 2.22% volatility. Next, we report the descriptive statistics for the Survey of Professional Forecasters (SPF) and Blue Chip Economic Indicators (BCF). SPF includes one- and 10-year forecasts of seasonally-unadjusted CPI and BCF produces one-year ahead CPI forecasts. The SPF forecasts are available quarterly, while BCF forecasts are available monthly. The SPF 10-year CPI inflation rate forecast mean is 2.48% per year with a standard deviation of 0.04%. This allows us to proxy the SPF 10-year expected inflation by a single number, 2.5%. The average BCF one-year ahead forecast of the CPI-based inflation is 2.45% with .40% volatility. Note that this one-year forecast is higher than the forecast reported by SPF. Overall, the summary statistics in Panel C indicates the measures of realized and expected inflation far exceed the breakeven inflation. It follows then that either nominal rates were too low or real yields were too high. We take the latter view, linking high real yields during our sample period to the liquidity problems that the TIPS market has experienced.

Panel D reports sample summary statistics of the real activity variables that we use in our VAR(1) model to estimate expected inflation. All growth rates that we compute in Table 1 are the log difference of the index levels at time t and $t - 12$.

3 Methodology

First, we need to construct a yield curve of real interest rates because TIPS rates do not equal real rates since TIPS coupon and principal payments are linked to the three-month lagged inflation index, rather than the current level of the index. This presents some additional difficulty in computing the real rates, because there is a risk of inflation shock during those three months that investors are not protected against. Therefore, we have to establish a relationship between three term structures: the term structure of nominal rates, the TIPS term structure, and the term structure of real rates. We follow Evans (1998) in constructing the real yield curve. As mentioned earlier, this method is based on the first-order equilibrium conditions, and, therefore, is arbitrage-free.

Second, we compute the inflation risk premium as the difference between the nominal-real yield spread and expected inflation. Below is the detailed description of our methodology.

A Notation

Before proceeding with the description, we define some notation as follows:

Nominal Bonds. Let $Q_t(h)$ denote the time- t nominal price of a zero-coupon bond paying \$1 at period $t + h$. Then define the continuously compounded yield on a bond of maturity h as

$$y_t(h) \equiv -\frac{1}{h} \ln Q_t(h). \quad (1)$$

Define k -period nominal forward rate h periods forward as

$$F_t(h, k) \equiv \left[\frac{Q_t(h)}{Q_t(h+k)} \right]^{1/k} - 1. \quad (2)$$

Real Bonds. Let $Q_t^*(h)$ denote the nominal price of a zero-coupon bond at period t paying $\$ \frac{P_{t+h}}{P_t}$ at period $t + h$, where P_t is the (known) price level at t . $Q_t^*(h)$ also defines the real price of one consumption bundle at $t + h$. By definition, such a bond completely indexes against future movements in price levels h periods ahead. Then define the continuously compounded real yield on a bond of maturity h and forward rates as

$$y_t^*(h) \equiv -\frac{1}{h} \ln Q_t^*(h) \quad \text{and} \quad F_t^*(h, k) \equiv \left[\frac{Q_t^*(h)}{Q_t^*(h+k)} \right]^{1/k} - 1. \quad (3)$$

Bonds with incomplete indexation. Let $Q_t^+(h)$ denote the nominal price of an index-linked (IL) zero-coupon bond at period t paying $\$ \frac{P_{t+h-l}}{P_t}$ at period $t+h$, where $l > 0$ is the indexation lag. When $h = l$, then such a bond pays out \$1 at maturity. Therefore, we have $Q_t(l) = Q_t^+(l)$ in the absence of arbitrage. The yields and forward rates of IL bonds are defined as:

$$y_t^+(h) \equiv -\frac{1}{h} \ln Q_t^+(h) \quad \text{and} \quad F_t^+(h, k) \equiv \left[\frac{Q_t^+(h)}{Q_t^+(h+k)} \right]^{1/k} - 1. \quad (4)$$

The TIPS indexation lag is three months, so $l = 3$ in our case.

B Nominal, Real, and Index-Linked Term Structure

In this section we establish the relationship between nominal, IL, and real prices: Q_t , Q_t^+ , and Q_t^* . The expressions below are given in terms of the stochastic discount factor M_t . We assume that the price index P_t for the month t is known at the end of the period t . This seems to be a reasonable approximation of the US data since the index is published with a two-week delay only. Let M_{t+1} be a random variable that prices one-period state-contingent claims. In the absence of arbitrage opportunities, the one-period nominal returns for all traded assets, $i = 1, \dots, N$, are given by

$$E_t[M_{t+1}R_{t+1}^i] = 1, \quad (5)$$

where R_{t+1}^i is the gross return on asset i between t and $t+1$, and E_t is the expectation operator conditioned on the information set at time t . We use equation (5) to find the prices of nominal, real, and IL bonds. In the case of nominal bonds, for $h > 0$ it becomes:

$$Q_t(h) = E_t[M_{t+1}Q_{t+1}(h-1)]. \quad (6)$$

In the case of real bonds, the nominal return at $t+1$ of the claim of $\$(P_{t+h}/P_t)$ at $t+h$ is given by:

$$\frac{Q_{t+1}^*(h-1) \frac{P_{t+1}}{P_t}}{Q_t^*(h)}. \quad (7)$$

Therefore, (5) becomes:

$$Q_t^*(h) = E_t[M_{t+1}^* Q_{t+1}^*(h-1)], \quad (8)$$

where $M_{t+1}^* = M_{t+1} \times \frac{P_{t+1}}{P_t}$ is the real stochastic discount factor. In a similar way, we find the price of IL claims. We know that $Q_t(l) = Q_t^+(l)$, so we need to find prices only for IL claims with maturities $h > l$. In the same way as for real bonds, we can show that

$$Q_t^+(h) = E_t[M_{t+1}^* Q_{t+1}^+(h-1)]. \quad (9)$$

Let lowercase letters stand for the natural logarithms of their uppercase counterparts, e.g. $q_t(h) = \ln Q_t(h)$ etc. Log-linearizing equations and applying $Q_t(0) = Q_t^*(0) = 1$ we have:

$$q_t(h) = E_t \sum_{i=1}^h m_{t+i} + \frac{1}{2} V_t \left(\sum_{i=1}^h m_{t+i} \right), \quad (10)$$

$$q_t^*(h) = E_t \sum_{i=1}^h m_{t+i}^* + \frac{1}{2} V_t \left(\sum_{i=1}^h m_{t+i}^* \right), \quad (11)$$

$$q_t^+(h) = E_t \sum_{i=1}^{\tau} m_{t+i}^* + E_t q_{t+\tau}(l) + CV_t(\sum_{i=1}^{\tau} m_{t+i}^*, q_{t+\tau}(l)) + \frac{1}{2} \left[V_t \left(\sum_{i=1}^h m_{t+i}^* \right) + V_t(q_{t+\tau}(l)) \right], \quad (12)$$

where $\tau \equiv h - l$, $V_t(\cdot)$ and $CV_t(\cdot, \cdot)$ represent the conditional variance and covariance given period t information. The equations are approximations in general, but hold exactly if the joint distribution for $\{M_{t+j}, P_{t+i+1}/P_{t+i}\}_{j>0, i>0}$ conditional on the period t information is log normal.

C Term Structure of Real Interest Rates

Let $\Delta^\tau p_{t+\tau} = \log(P_{t+\tau}/P_t)$, and, in particular, $\Delta p_{t+1} = \log(P_{t+1}/P_t)$. Using (10), (11), (12), and the definition of $m_{t+1}^* \equiv m_{t+1} + \Delta p_{t+1}$, we link the prices of nominal, real, and IL bonds by the following formula:

$$q_t^+(h) = q_t^*(\tau) + [q_t(h) - q_t(\tau)] + \gamma_t(\tau), \quad (13)$$

where $\tau = h - l$ and

$$\gamma_t(\tau) \equiv CV_t(q_{t+\tau}(l), \Delta^\tau p_{t+\tau}). \quad (14)$$

Equation (13) shows that the log price of real bonds is not only a function of nominal prices and IL prices, but also depends on $\gamma_t(\tau)$, the conditional covariance between the τ -period

future inflation and future nominal prices. $\gamma_t(\tau)$ represents the compensation for the risk of high inflation. By no-arbitrage condition, the IL bond prices depend on future nominal bond prices $q_{t+\tau}(l)$, and this affects the choice between real and IL bonds. In the periods of high expected inflation, future nominal prices drop, causing negative $\gamma_t(\tau)$. Therefore, IL bonds will sell at a discount (compared to real bonds) to compensate for this risk.

Equation (13) can be rewritten in terms of the yields in order to derive the estimates of the real term structure. Let $y_t(h)$, $y_t^*(h)$, and $y_t^+(h)$ be the continuously compounded yields for nominal, real, and IL bonds, respectively. Therefore, we can rewrite equation (13):

$$y_t^*(\tau) = \frac{h}{\tau} y_t^+(h) - \frac{l}{\tau} f_t(\tau, l) + \frac{1}{\tau} \gamma_t(\tau). \quad (15)$$

Using the above equation, we can estimate real yields y_t^* because we observe IL yields y_t^+ , and log nominal forward rates f_t . The only variable that has to be estimated is $\gamma_t(\tau)$. To estimate $\gamma_t(\tau)$, we follow the VAR methodology proposed by Evans (1998). We consider first-order vector autoregression:

$$z_{t+1} = Az_t + e_{t+1}, \quad (16)$$

where $z_t' \equiv [\Delta p_t, q_t(l), x_t]$, where x_t is a vector of conditioning variables that can potentially include relevant macro-variables which would affect the covariance between inflation and nominal bond prices. For now, we just use a unit vector, so $x_t = \mathbf{1}$. As a result of estimated (16), $\gamma_t(\tau)$ is given by:

$$\gamma_t(\tau) = i_1' \left[\sum_{i=1}^{\tau} A^{\tau-i} \left(\sum_{j=1}^i A^{i-j} V(e_{t+j}|z_t) A^{i-j'} \right) \right] i_2, \quad (17)$$

where i_k , $k = 1, 2$ is the selection vector such that $\Delta p_t = i_1' z_t$ and $q_t(l) = i_2' z_t$. Equation (17) shows how the covariance between $\Delta p_{t+\tau}$ and $q_{t+\tau}(l)$ conditioned on z_t is defined through the coefficient matrix A and the innovation variances $V(e_{t+j}|z_t)$.⁶ The VAR(1) results are presented in Table 2. We discuss the properties and the magnitude of $\gamma_t(\tau)$ in more detail in Section 4.

⁶It seems that there is a typo in the derivation of $\gamma_t(\tau)$ in Evans (1998) and we present the corrected formula.

D Expected Inflation Proxies

In this section we discuss three methods to obtain the estimates of inflation expectations.

D.1 Expected Inflation Based on Historical Average

First, we compute expected inflation as the average of past inflation rates at a particular month t . Formally, we define τ -period inflation rate at $t + \tau$ as

$$\pi_{t+\tau}(\tau) = \frac{1}{\tau} \log \left(\frac{P_{t+\tau}}{P_t} \right) = \frac{1}{\tau} \Delta^\tau p_{t+\tau}. \quad (18)$$

We define the τ -period expected inflation based on the T -year historical average as

$$E_t \pi_{t+\tau}(\tau) = \frac{1}{T} \sum_{k=1}^T \frac{1}{\tau} (p_{t-k} - p_{t-k-\tau}). \quad (19)$$

In our empirical estimation, we vary both the estimation horizon T and the inflation horizon τ .

D.2 Expected Inflation Based on the VAR

For our second approach we assume that the state vector of the economy is governed by the vector $z_t = (r'_t, i'_t)'$, where r_t is the vector of real activity variables, and i_t is the vector of inflation variables. We include real activity variables to the VAR using the motivation from Phillips curve, according to which real activity measures should be important in predicting inflation. In particular, $r_t = (HELP_t, UE_t, EMPLOY_t, IP_t)$, where $HELP_t$ is the Index of Help Wanted Advertising in the Newspapers, $EMPLOY_t$ is the growth rate of employment, IP_t is the growth rate of the industrial production index IP, and UE_t is the unemployment rate. Inflation variables include CPI_t , $Core_t$, PPI_t , and the Blue Chip Forecasters' inflation rate BC_t . So, the vector of inflation variables $i_t = (CPI_t, Core_t, PPI_t, BC_t)$. We assume that the state vector z_t follows VAR(1) process:

$$z_t = \mu + \Phi_1 z_{t-1} + \epsilon_t, \quad (20)$$

where $\epsilon_t \sim N(0, \Sigma)$. Therefore, the τ -period ahead conditional expectation of z_t is given by:

$$E_t(z_{t+\tau}) = \Psi^\tau \mu + \Phi^\tau z_t, \quad (21)$$

where

$$\Psi^\tau = \sum_{l=1}^{\tau-1} \Phi^l = (I - \Phi)^{-1}(I - \Phi^\tau). \quad (22)$$

D.3 Surveys' Inflation Forecasts

Last, but not least, we use three forecasts of inflation from the Survey of Professional Forecasters and the Blue Chip Economic Indicators. Quarterly Surveys of Professional Forecasters produce one-year ahead and 10-year ahead forecasts. Monthly Blue Chip Economic Indicators data are used to construct one-year ahead forecasts. As stated above, Ang, Bekaert, and Wei (2007) conclude that surveys outperform other forecasting models and methods. However, they do not include Blue Chip Forecasts in their study. In the related study, Chernov and Mueller (2008) include Blue Chip Forecasts among others and conclude that their model, which combines yields and inflation, produces dominating out-of-sample forecasts of both inflation and yields. We discuss the estimates of expected inflation in Section 4.

E Inflation Risk Premium

Consider the following equation that defines the inflation risk premium $IRP_t(\tau)$:

$$y_t(\tau) = y_t^*(\tau) + E_t\pi_{t+\tau}(\tau) + IRP_t(\tau). \quad (23)$$

This equation can be also derived from log-linear pricing equations (10), (11), (12), and (15). Equation (23) presents the variation of the Fisher equation that equates the τ -period nominal yield with the τ -period real yield plus τ -step ahead expected inflation and the inflation risk premium $IRP_t(\tau)$. One way to estimate the inflation risk premium is to specify the real pricing kernel in a model with a representative agent. Examples of this approach include Fisher (1975), Benninga and Protopapadakis (1983), Evans and Wachtel (1992), and Buraschi and Jiltsov (2005). Evans (1998) notes that in general, the inflation risk premium can be positive or negative depending on how the real pricing kernel covaries with inflation. He proposes an alternative approach that does not require any specification of the real pricing kernel. We use his method to estimate the inflation risk premium and report the empirical results in Section 4.

4 Empirical Results

In this section we present the estimates of the real yields, expected inflation, and the inflation risk premium. In addition, we control the estimates of the inflation risk premium for liquidity. The TIPS market experienced liquidity problems in the early years following its inception and especially in the first years of our sample. Our estimates of the inflation risk premium can be affected by a sound liquidity component, so we investigate this opportunity too.

A Estimated Real Yields

In order to compute the real yields, we first estimate the covariance $\gamma_t(\tau)$ between future inflation $\Delta^\tau p_{t+\tau}$ and future nominal bond prices $q_{t+\tau}(l)$. So, we estimate the first-order VAR (16) and compute $\gamma_t(\tau)$ given by (17) and present the results in Table 2. In order to assess how $\gamma_t(\tau)$ affects the IL yields, we annualize these estimates by multiplying them by $-1200/\tau$. Although $\gamma_t(\tau)$ seems to manifest in a uniformly upward sloping term structure, its impact, even for 10-year yields, is not large, resulting in .2075 per cent to the annualized yields.

The estimated real yields are reported in Table 3. The real yields (and TIPS yields) are quite high during the first half of our sample, 2000-2003. For example, implied real yields on average vary between 2.55% and 2.88% depending on the maturity and manifest the upward sloping term structure. The volatilities of the real yields vary between 1.06% and .75%. In the second half of our sample (2004-2007), the real yields drop on average and vary from 1.49% to 1.75%, based on their maturity. They again display the upward sloping term structure, like in the first half of the sample. The volatility of the real yields drops significantly ranging from 1.49% to 1.75%. Overall, during our sample period the implied real yields display the upward sloping term structure, from 1.97% to 2.29% for five- and ten-year maturity bonds, respectively. Note that implied real yields are on average 20 basis points lower than TIPS yields, because of the indexation lag risk $\gamma_t(\tau)$ that we accounted for when we converted TIPS yields into real yields.

Figure 2 further illustrates how the dynamics of the real yields, in particular the relatively high real yields in 2000-2003, contribute to the widening of the 10-year breakeven inflation rate. It also provides the comparison between the breakeven rate and the 10-year inflation forecast from the Survey of Professional Forecasters. By definition, the inflation risk premium is the difference between the breakeven inflation rate and the expected in-

flation. In general, we do not expect these series to coincide, because we have rejected zero inflation risk premium albeit using the historical average as a proxy for expected inflation. Panel A of Figure 2 reveals an interesting pattern. It shows that the inflation risk premium is negative in the first half, but positive in the second half of the sample period. We interpret it as the change in the TIPS market liquidity over the sample period. Due to the illiquid nature of the TIPS market, the liquidity premium, it seems, comprised a substantial component in the TIPS yields. Therefore, high TIPS yields lead to the lower breakeven inflation rate than the SPF inflation forecast in the first half of our sample. This interpretation is consistent with D’Amico, Kim, and Wei (2006) who find that TIPS yields contained a significant “liquidity premium” of about 1% and may not have been priced fully efficiently in its early years.

B Expected Inflation Estimates

Table 4 reports two measures of expected inflation based on the historical inflation averages of seasonally-unadjusted CPI and Core CPI indices in Panels A and B, respectively. We use equation (19) to compute these forecasts. There are two distinguished features worth noting. First, like the real yields, $E_t\pi_{t+\tau}(\tau)$ displays the upward-sloping term structure. For example, for a one-year estimation period ($T = 1$ year), expected inflation increases from 2.47% to 2.54% when maturity of the implied yields τ changes from 57 months (roughly five years) to 117 months (roughly 10 years).⁷ When $T = 5$ years, these estimates vary from 2.45% to 2.73% depending on the horizon. Other researchers obtain similar estimates. In particular, Carlstrom and Fuerst (2004) find that 10-year ahead expected inflation varies between 2.5% and 2.6%. Second, for longer estimation periods our estimates are biased upward and closer to 3% per year but this reflects the fact that they encompass periods of high inflation of early and mid 90s.

The estimates of expected inflation based on Core CPI are reported in Panel B. As in Panel A, we observe the upward sloping term structure for each estimation period, and the upward bias of the estimates based on a longer T . At the same time, estimates here are slightly lower than those based on the standard CPI index. On average, the estimates of expected inflation based on the estimation period of $T = 1, 3,$ and 5 years are around 2.5%.

Next, we report forecasting errors for historical-based, VAR-based and surveys’ forecasts

⁷In Table 4 and in the subsequent tables, we display our results by maturities in months, $\tau = 57, 81,$ and 117 rather than by years. Because of the three-month indexation lag, TIPS yields’ and implied real yields’ corresponding maturity differs by three months.

of inflation. We use seasonally-unadjusted CPI and Core inflation indices as benchmarks and compute root mean square errors for historical-based and VAR-based inflation. We use only the CPI benchmark in the case of surveys. In our sample, forecasting errors are the lowest for the historical-based expected inflation given by (19). In particular, forecasting errors for the CPI inflation are between eight and 14 basis points for historical-based forecasts, while they range between 12 and 21 basis points for VAR-based forecasts. Both sets of errors decrease with the horizon of expected inflation. Surveys' forecasting errors are higher than either historical-based or VAR-based forecasting errors. They are between 27 and 42 basis points depending on the survey series. This result, however, does not necessarily invalidate the result of Ang, Bekaert, and Wei (2007) because surveys' forecasting errors are not directly comparable with the forecasting errors of other methods we consider. First, the sample periods in our study and their's are different. Second, the horizons of historical- and VAR-based forecasts and surveys' forecasts are different. In the next section, we report the estimates of the inflation risk premium for the above inflation forecasts.

C Estimation of the Inflation Risk Premium (IRP)

In this section, we first test and reject the Fisher hypothesis that the inflation risk premium is zero. We then present the estimates of the inflation risk premium obtained using different measures of expected inflation.

To test the Fisher hypothesis, we run the following regression:

$$y_t(\tau) - y_t^*(\tau) - E_t \pi_{t+\tau}(\tau) = \alpha_0 + \alpha_1 [y_t(\tau) - y_t^*(\tau)] + u_{t+\tau}. \quad (24)$$

By definition, the left-hand side of this equation is $IRP_t(\tau)$. We can, therefore, examine to what extent the changes in the nominal-real yield spread $y_t(\tau) - y_t^*(\tau)$ are correlated with the inflation risk premium. If $IRP_t(\tau)$ were zero everywhere it would not be correlated with the yield spread. The empirical results (not reported) indicate that we strongly reject the Fisher hypothesis $IRP_t(\tau) = 0$. As such, the use of the Fisher equation with $IRP_t(\tau) = 0$ is challenged, and the inflation risk premium should be taken into account when computing real yields. The rejection of the Fisher hypothesis is widely documented in, e.g., Mundell (1963), Tobin (1965), Feldstein (1976), Fisher (1975), Fama and Gibbons (1992), and Fama (1990). Evans (1998) also documents this empirical regularity using U.K. data.

Next we proceed to estimate the inflation risk premium. First, we compute the IRP based on the average of the historical inflation. Second, we use VAR(1) model's inflation

forecasts. Third, we use inflation forecasts from the Survey of Professional Forecasters and the Blue Chip Economic Indicators to compute the IRP. The inflation risk premium estimates have to be treated cautiously with understanding that part of the $IRP_t(\tau)$ magnitude is due to liquidity problems.⁸ Table 6 reports the IRP estimates for our sample period and two subsamples.

The estimates of the inflation risk premium reveal the persistent pattern across different estimates associated with different inflation forecasts. Firstly, the IRP estimates are always increasing with the yields' maturity τ for all inflation forecasts. This result is robust for the whole sample and for the subsamples too.

Second, the CPI-based inflation risk premium is negative for $\tau = 57$ and 81 for all measures of expected inflation, for the whole sample, 2000-2007, and for the first subperiod, 2000-2003. Most of these estimates are statistically significant at the 1%. We cannot reject zero inflation risk premium in the case of one-year SPF inflation forecast for 2000-2007, and in the case of naive forecasts for 2000-2003. The situation is different when we consider the longer horizon, $\tau = 117$. Most of the forecasts result in the positive inflation risk premium when $\tau = 117$, and most of them are statistically significant either on 1% or 5% level. We cannot reject the zero inflation risk premium for the 10-year SPF forecast and the historical-based CPI forecast. In sum, the 10-year inflation risk premium based on CPI expected inflation is positive and is in the range of 10 to 22 basis points. In particular, VAR-based forecasts result in 11 basis points IRP, while Blue Chip Forecasts result in 13 basis points IRP. The inflation risk premium of the one-year inflation forecast by SPF is 22 basis points.

Third, all the IRP estimates but one are positive and most of them are statistically significant in the second half of the sample, 2004-2007. The CPI-based 10-year inflation risk premium varies between 29 basis points (Blue Chip Forecasts) and 48 basis points (Survey of Professional Forecasters). VAR-based expected inflation produces the IRP of 35 basis points, and historical-based forecasts produce the IRP of 36 basis points. The seven-year inflation premium is positive for all forecasts and statistically significant everywhere at the 1% level except for Blue Chips, where it is significant at the 5% level. The five-year inflation risk premium is positive and statistically significant only for SPF forecasts, equal to 24 basis points for the one-year inflation forecast horizon.

The inflation risk premium estimates based on Core inflation are also given in Table 6

⁸It remains to be determined how much of the difference between a breakeven rate and an expected inflation rate is due to inflation uncertainty and how big the liquidity component is.

for comparison. As a rule, they are higher than the estimates based on the CPI. This is natural since the Core inflation on average is lower than CPI inflation. In our sample, average Core inflation is 2.18%, while CPI inflation is 2.74% (see Table 1, Panel C).

Figure 3 shows the plot of the term structure of the inflation risk premium based on the historical inflation with the estimation horizon $T = 5$ years. It graphically shows that the inflation risk premium is visibly negative and relatively volatile in the first half of the sample but becomes positive and less volatile in the second half of the sample. Volatilities of the inflation risk premium estimates (not reported) also confirm that the IRP is less volatile in 2004-2007 than in 2000-2003.

To summarize the discussion of the magnitudes of inflation premia, we find that the pattern of negative vs positive inflation risk premia in two subsamples is robust with respect to the choice of inflation proxy and its horizon and estimation period of the historical-based forecasts. Overall, the 10-year inflation risk premium estimates are positive for all inflation forecast series in our sample (and significant for most of them). Yet these estimates are considerably lower than the ones reported by Campbell and Shiller (1996), Buraschi and Jiltsov (2005), and Ang, Bekaert, and Wei (2008). In a related study, the long-run averages of inflation risk premium in D'Amico, Kim, and Wei (2006) can be positive or negative depending on the different series that they use to fit the three-factor term-structure model.⁹

Evidence on both averages and volatilities of the inflation risk premium over different horizons indicates that there seems to be a significant shift in the behavior of the premium around 2003-2004. Whether it is a structural break, and therefore, a sample is split between two regimes, remains an open question. Why there has been a shift in the inflation risk premia's magnitude, also remains an open question. We see two possibilities. One is the relative illiquidity of the TIPS market in the beginning of our sample period. It might have led the real yields to be relatively high during the first half of the sample, as we document in Table 3. Another possibility is the deflation scare period, over which deflation risks lead to relatively low nominal yields. Economic data seem to support this explanation. From August to December 2003 the target federal-funds rate was at 45-year low of 1%. At the same time, Core inflation decreased from 2% to about 1% between November 2002 and November 2003. Ip (2004), among others, in his article in the Wall Street Journal on September 17 provides evidence that addressing deflation scare was a first-rate priority of the Fed at that time: *"The Federal Reserve concluded that its new policy of talking more clearly about its interest-rate plans had a sizable impact on bond markets last year, helping*

⁹See Figures 4 through 6 in their paper.

to avert deflation when the Fed's main interest rate target approached zero." Both factors could have potentially contributed to the negative inflation risk premia we observe in the beginning of our sample.

Concerns of deflation in 2008 seem to drive the negative 5-year and nearly zero 10-year break-even rates that have been observed (as shown in Figure 4), especially given that the liquidity of the TIPS market should have improved over the past ten years. We expect the estimates of the inflation risk premium in 2008 to be negative as in 2000-2003.¹⁰

We have also looked how the TIPS market liquidity impacts the magnitude of the real yields. We find the strong negative relationship between the real yields and the TIPS trading volume indicating that real yields potentially contain the significant liquidity component. But since the market is relatively young, its liquidity proxies, such as average bid-ask spread and trading volume, are relatively noisy, so we do not report our results in the study.

5 Conclusion

This paper represents perhaps the first attempt to estimate the inflation risk premium directly using the prices of Treasury Inflation Protected Securities (TIPS). Using the market data on prices of TIPS over the period 2000-2007, we find that the 10-year average inflation risk premium ranges from 11 to 22 basis points. We also find that it is time-varying. More specifically, it is negative in 2000-2003 but positive in 2004-2007.

The negative inflation risk premium during 2000-2003 is due to either concerns of deflation or liquidity problems in the TIPS market. There seems to be more evidence that supports the former explanation. The estimated average 10-year inflation risk premium over the second half varies between 29 and 48 basis points, depending on the proxy used for the expected inflation. The estimates based on Blue Chips inflation forecast are the lowest (29 basis points), and the estimates based on one-year SPF are the highest (48 basis points). We also find that the inflation risk premium is considerably less volatile during 2004-2007, a finding consistent with the observations that inflation expectations became more stable during this period, investors became more familiar with the TIPS market, and the market liquidity has gradually improved.

Our empirical results on inflation risk premium estimated directly from TIPS should be valuable for practitioners, monetary authorities and policymakers alike because they help to assess the inflation expectations and the inflation risk premium of bond market investors.

¹⁰We will do the estimation once the data for 2008 become available.

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Table 1: Summary Statistics of Data

This table reports the summary statistics of the data. Panels A and B report the statistics on h -month Treasury Inflation Protected Security (TIPS) yields $y^{TIPS}(h)$ from Barclays and Federal Reserve data set. Panel C reports statistics on h -month nominal Treasury yields $y(h)$. Panel D reports the statistics of realized and expected inflation variables. Realized inflation variables are based on the CPI, seasonally-unadjusted Consumer Price Index for All Urban Consumers (CPI); and CPI Core, seasonally-adjusted Core CPI, which is the Consumer Price Index Less Food and Energy. We calculate the inflation measure at time t using $\log(P_t/P_{t-12})$, where P_t is the inflation index. Expected inflation variables are: (1) one-year ahead forecast based on CPI price deflator from SPF, (2) 10-year ahead forecast based on CPI price deflator from SPF, and (3) one-year ahead forecast from the Blue Chip Economic Indicators. Panel E reports real activity measures: the Index of Help Wanted Advertising in the Newspapers HELP, the growth rate of employment EMPLOY, the growth rate of industrial production index IP, and the unemployment rate UE. The growth rate variables are computed as $\log(I_t/I_{t-12})$, where I_t is the employment of the industrial production index. Source: Barclays Capital, Federal Reserve Board TIPS data set, Federal Reserve discount bonds data set, Federal Reserve Bank of St. Louis Economic Data Base, Federal Reserve Bank of Philadelphia, Conference Board and Aspen Publishers. The sample period for all variables is 2000:01-2007:12, monthly frequency, except SPF, whose forecasts are quarterly.

	Central Moments				Autocorrelations		
	Mean	Stdev	Skew	Kurt	Lag1	Lag2	Lag3
Panel A: TIPS Yields, Barclays Capital							
$y^{TIPS}(60)$	2.2086	0.9527	0.5267	2.3081	0.9771	0.9540	0.9341
$y^{TIPS}(84)$	2.4028	0.8538	0.6334	2.2862	0.9800	0.9599	0.9423
$y^{TIPS}(120)$	2.5745	0.7717	0.6527	2.1644	0.9821	0.9645	0.9487
Panel B: TIPS Yields, Federal Reserve							
$y^{TIPS}(60)$	2.1958	0.9575	0.4917	2.2946	0.9770	0.9536	0.9336
$y^{TIPS}(84)$	2.4035	0.8605	0.5995	2.2296	0.9801	0.9600	0.9424
$y^{TIPS}(120)$	2.5733	0.7596	0.6872	2.2073	0.9820	0.9645	0.9487
Panel C: Nominal Bond Yields							
$y(60)$	4.2356	0.9651	0.4629	2.9811	0.9822	0.9647	0.9477
$y(84)$	4.5374	0.7916	0.6787	3.2281	0.9835	0.9675	0.9525
$y(120)$	4.8903	0.6383	0.7920	3.1275	0.9850	0.9707	0.9574
Panel D: Inflation Variables							
CPI	2.7362	0.8218	-0.0745	2.2596	0.9738	0.9400	0.9160
Core CPI	2.1821	0.4274	-0.8045	3.0240	0.9880	0.9758	0.9630
PPI	2.8098	2.2224	-0.9951	3.4925	0.9446	0.8749	0.8322
SPF, CPI 1yr forecast	2.3375	0.2378	-0.8740	4.0971	0.9625	0.9286	0.8902
SPF, CPI 10yr forecast	2.4844	0.0410	-1.8655	5.8855	0.9695	0.9391	0.9087
Blue Chips, CPI 1yr forecast	2.4475	0.3989	-0.3741	2.1034	0.9873	0.9731	0.9589
Panel E: Real Activity Variables							
HELP	44.7396	17.4798	1.2913	3.6953	0.9795	0.9587	0.9382
EMPLOY	1.1230	0.9894	-0.4849	2.8177	0.9561	0.9217	0.8798
IP	1.4162	2.5913	-1.1828	4.0212	0.9545	0.9008	0.8332
UE	5.0427	0.6813	-0.0490	1.8173	0.9914	0.9833	0.9751

Table 2: VAR Correction to the real yields due to the 3-month indexation lag of TIPS

This table reports estimates of $\gamma_t(\tau)$ computed as

$$\gamma_t(\tau) = i_1' \left[\sum_{i=1}^{\tau} A^{\tau-i} \left(\sum_{j=1}^i A^{i-j} V(e_{t+j}|z_t) A^{i-j'} \right) \right] i_2,$$

where i_1 is the selection vector such that $\Delta p_t = i_1' z_t$ and $q_t(l) = i_2' z_t$. This equation shows how the covariance between $\Delta^\tau p_{t+\tau}$ and $q_{t+\tau}(l)$ conditioned on z_t relates to the dynamics of VAR through the coefficient matrix A , and the innovation variances, $V(e_{t+j}|z_t)$. $\Delta^\tau p_{t+\tau}$ is the log change in the price level from time t to time $t + \tau$. $q_{t+\tau}(l)$ is the log of prices at time $t + \tau$ for a nominal bond maturing in l months. Annualized negative $\gamma_t(\tau)$ represents the correction to the real yields.

Maturity τ (month)	Correction to the real yields	
	due to the 3-month indexation lag of TIPS	
	$\gamma(\tau)$	annualized $-\gamma(\tau)$,
12	-0.0001	0.0112
24	-0.0005	0.0274
36	-0.0014	0.0482
57	-0.0043	0.0897
60	-0.0048	0.0959
81	-0.0093	0.1379
84	-0.0101	0.1438
117	-0.0198	0.2026
120	-0.0207	0.2075

Table 3: TIPS and Real Yields Statistics

This table reports statistics for TIPS yields, $y_t^+(h)$, and implied real yields with h months to maturity. $y^{real}(h)$ is the real yield on an h -month zero-coupon bond, estimated using the following formula:

$$y^*(\tau) = \frac{h}{\tau} y_t^+(h) - \frac{l}{\tau} f_t(\tau, l) + \frac{1}{\tau} \gamma_t(\tau).$$

Sample period: 2000:01 to 2007:12, monthly frequency. Statistics is reported for the whole sample and for two subsamples. various subsamples. The estimates are based on the seasonally-unadjusted CPI, used to estimate the index-linked premium $\gamma_t(\tau)$ reported in Table 2.

Sample period	Horizon τ (month)	TIPS Yields				Implied Real Yields			
		Mean	Stdev	Skew	Kurt	Mean	Stdev	Skew	Kurt
2000-2007	57	2.196	0.957	0.492	2.295	1.956	0.981	0.471	2.292
	81	2.403	0.861	0.600	2.230	2.150	0.877	0.592	2.238
	117	2.573	0.760	0.687	2.207	2.285	0.771	0.694	2.226
2000-2003	57	2.678	1.031	-0.166	1.751	2.440	1.059	-0.157	1.742
	81	2.922	0.879	-0.228	1.747	2.670	0.903	-0.219	1.736
	117	3.086	0.735	-0.223	1.762	2.799	0.754	-0.213	1.751
2004-2007	57	1.713	0.562	-0.192	1.744	1.473	0.590	-0.211	1.747
	81	1.885	0.419	-0.099	1.722	1.631	0.436	-0.116	1.726
	117	2.061	0.299	0.021	1.767	1.772	0.308	0.020	1.762

Table 4: Expected Inflation based on Historical Average

This table reports the estimates of the expected inflation based on historical average. $E_t \pi_{t+\tau}(\tau)$ is the expected rate of inflation computed as T -year historical average of $\pi_{t+\tau}(\tau)$, τ -period realized inflation, computed as

$$E_t \pi_{t+\tau}(\tau) = \frac{1}{T} \sum_{k=1}^T \frac{1}{\tau} (p_{t-k} - p_{t-k-\tau}).$$

Panel A reports expected inflation based on CPI, the Consumer Price Index for All Urban Consumers, seasonally-unadjusted as a measure of realized inflation. Panel B reports expected inflation based on historical average of Core CPI, the Consumer Price Index Less Food and Energy, seasonally adjusted. Volatilities are reported in parenthesis below the estimates. Estimation period is 1980:01 to 2007:12, monthly frequency.

Horizon τ (month)	Estimation period T (year)				
	1	3	5	7	10
Panel A: Based on CPI					
57	2.470 (0.132)	2.436 (0.069)	2.454 (0.068)	2.526 (0.146)	2.743 (0.289)
81	2.480 (0.147)	2.465 (0.088)	2.525 (0.146)	2.647 (0.244)	2.871 (0.301)
117	2.539 (0.132)	2.617 (0.216)	2.738 (0.288)	2.868 (0.301)	3.067 (0.282)
Panel B: Based on Core CPI					
57	2.188 (0.148)	2.260 (0.163)	2.360 (0.204)	2.501 (0.293)	2.789 (0.406)
81	2.282 (0.164)	2.363 (0.210)	2.500 (0.291)	2.675 (0.371)	2.969 (0.417)
117	2.467 (0.289)	2.613 (0.361)	2.786 (0.405)	2.967 (0.418)	3.242 (0.410)

Table 5: Forecasting Errors of Expected Inflation

This table reports the forecasting errors in percentages for the following forecasts of inflation: (1) one-year historical-based forecasts of CPI and Core inflation, (2) VAR-based forecasts of CPI and Core inflation, and (3) forecasts by the Survey of Professional Forecasters (SPF) and Blue Chip Economic Indicators (BCF) of seasonally-unadjusted CPI. One-year historical-based forecasts are computed using the formula (19) and VAR-based forecasts are computed using the formula (21). Seasonally-unadjusted CPI and Core inflation are benchmarks. If \hat{I}_t is the benchmark inflation and I_t is the forecast series in question, then the root-mean-squared-error RMSE is given by:

$$RMSE_{I,T} = \sqrt{\frac{1}{T} \sum_{t=1}^T (I_t - \hat{I}_t)^2}$$

where T is the sample size. We report quarterly estimates in the case of SPF and monthly estimates in all other cases. Sample period: 2000:01 to 2007:12.

Expected Inflation	Horizon τ (month)	Benchmark used	
		CPI	Core
Historical-Based	57	0.1416	0.0543
	81	0.1185	0.0523
	117	0.0834	0.0793
VAR-based	57	0.2073	0.1683
	81	0.1868	0.2186
	117	0.1242	0.3647
Surveys:			
SPF	1 yr	0.4215	
SPF	10 yrs	0.2746	
BCF	1 yr	0.2887	

Table 6: Inflation Risk Premium

This table reports average estimates for the inflation risk premium in percentages based on the following forecasts of inflation: (1) one-year historical-based forecasts of CPI and Core inflation, (2) VAR-based forecasts of CPI and Core inflation, and (3) forecasts by the Survey of Professional Forecasters (SPF) and Blue Chip Economic Indicators (BCF) of seasonally-unadjusted CPI. Sample period: 2000:01 to 2007:12. We report quarterly estimates in the case of SPF and monthly estimates in all other cases. Statistics is reported for various subsamples. Estimates at the 1% level of statistical significance are reported in bold, while estimates at the 5% level of statistical significance are marked with **.

Sample period	Horizon τ (month)	Historical-based		VAR-based		Surveys-based (CPI)		
		CPI	Core	CPI	Core	SPF, 1yr	SPF, 10yr	BCF
2000-2007	57	-0.246	0.035	-0.382	0.175	-0.142	-0.289	-0.224
	81	-0.128	0.070	-0.219	0.273	-0.015	-0.162	-0.096**
	117	0.039	0.110**	0.105	0.458	0.219	0.072	0.130
2000-2003	57	-0.529	-0.473	-0.740	-0.219	-0.528	-0.691	-0.506
	81	-0.392	-0.383	-0.508	-0.044	-0.336	-0.499	-0.312
	117	-0.284	-0.380	-0.138	0.211	-0.041	-0.204	-0.029
2004-2007	57	0.037	0.543	-0.016	0.577	0.244	0.113	0.058
	81	0.135	0.522	0.076	0.597	0.307	0.175	0.121**
	117	0.361	0.601	0.354	0.711	0.479	0.347	0.289

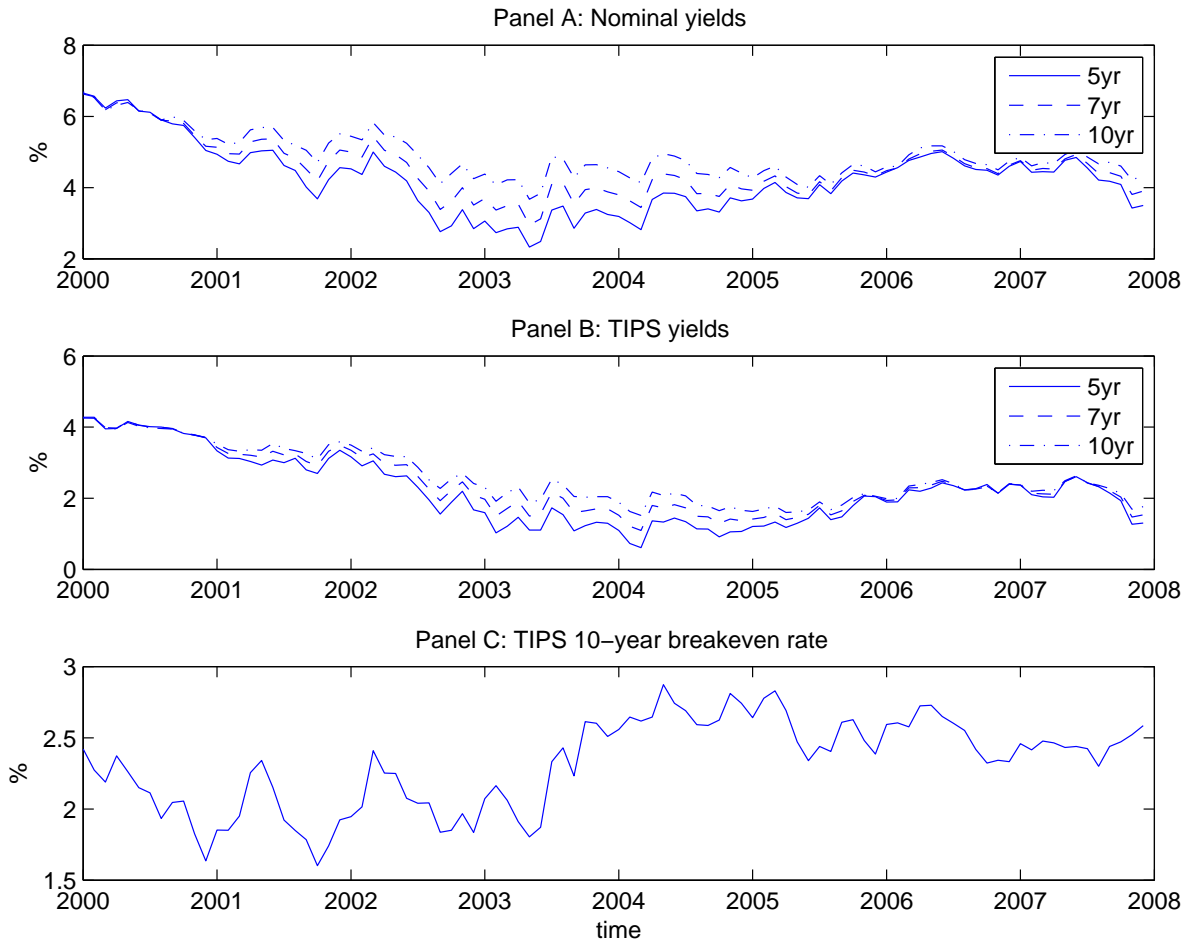


Figure 1: **Zero-coupon nominal and TIPS yields, and 10-year TIPS breakeven rate.**

This figure plots zero-coupon nominal and TIPS yields of 5, 7, and 10 year maturities and 10-year TIPS breakeven rate based on zero-coupon yield curves over the period January 2000 to December 2007.

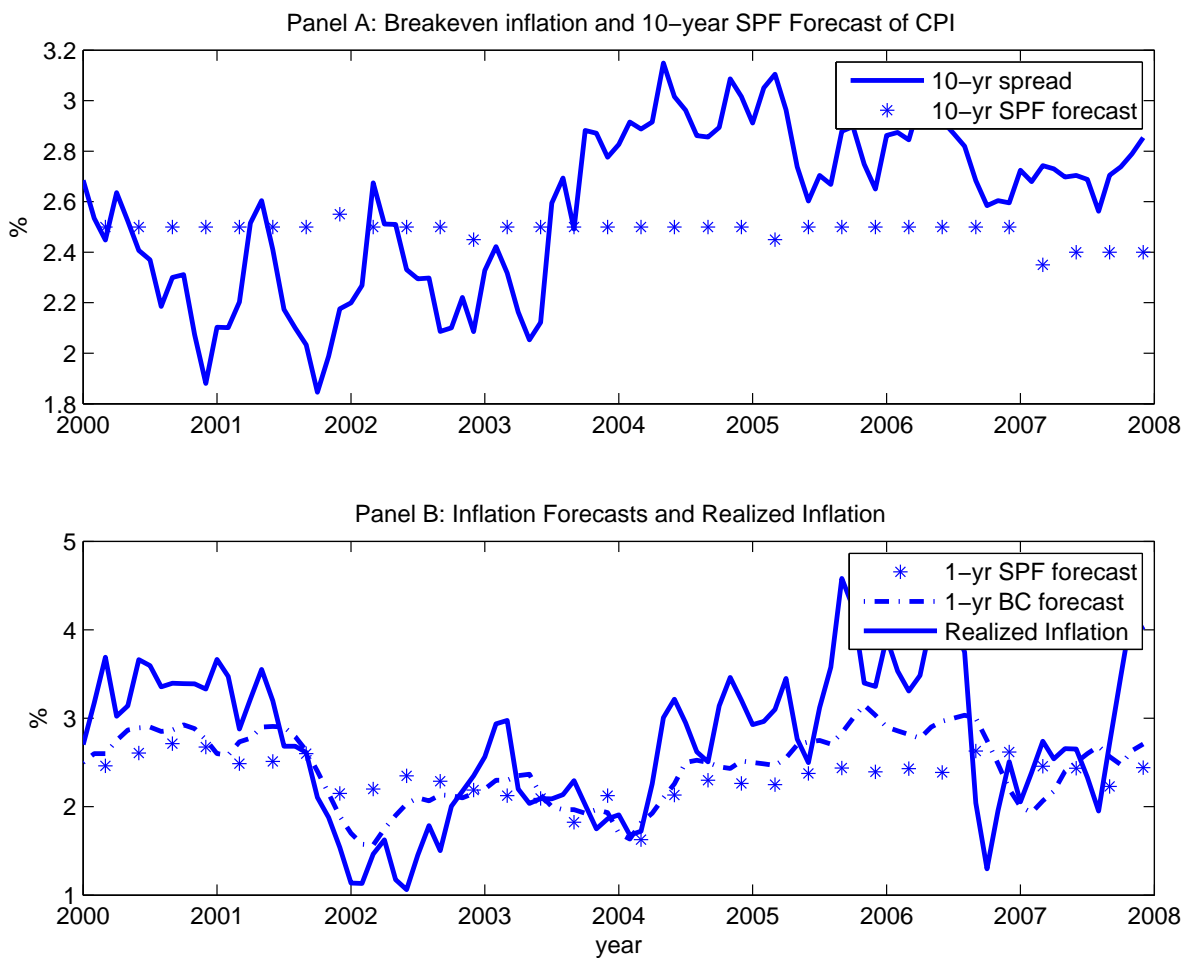


Figure 2: **The 10-year breakeven inflation rate, inflation forecasts based on surveys, and realized one-year inflation.**

Panel A shows the 10-year breakeven inflation rate and the Survey of Professional Forecasters 10-year ahead inflation forecast. Panel B plots one-year forecasts by the Survey of Professional Forecasters and Blue Chip Forecasters, and realized one-year inflation.

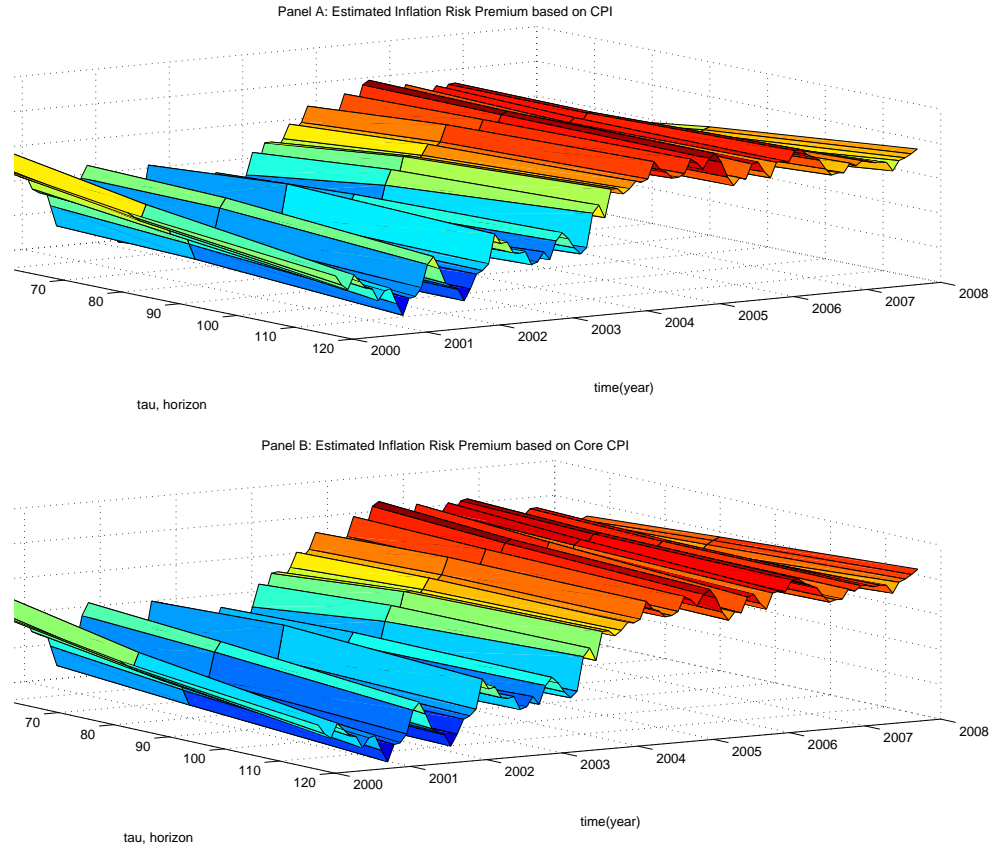


Figure 3: **The term structure of the inflation risk premium**

This figure displays the term structure of the inflation risk premium estimated using the historical-based inflation forecast reported in Table 4. Estimation period of inflation forecast is $T = 5$ years. Panel A shows the inflation risk premium based on the CPI, seasonally un-adjusted Consumer Price Index for All Urban Consumers. Panel B plots the inflation risk premium based on the seasonally adjusted Core CPI, Consumer Price Index Less Food and Energy. Sample period is 2000:01 to 2007:12, monthly frequency.

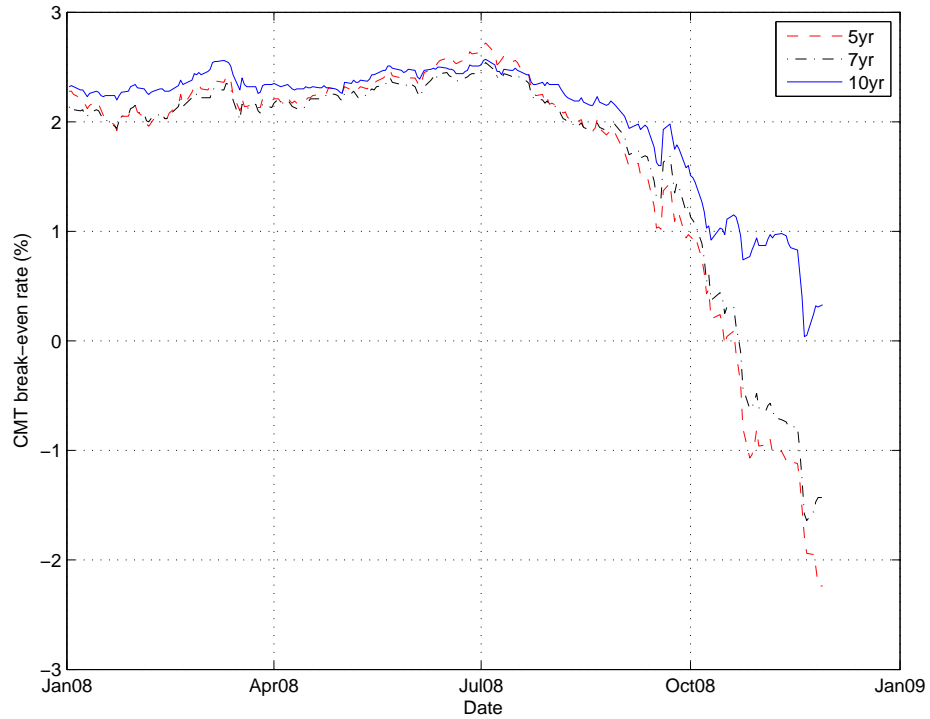


Figure 4: **The CMT break-even rate.**

This figure displays the breakeven rate computed as a difference between nominal and real Constant Maturity Treasury (CMT) rates of 5-, 7-, and 10-year maturities. Source: The U.S. Department of the Treasury. Sample period is 2008:01:02 to 2008:11:28, daily frequency.