Empirical Evidence on Jumps and Large Fluctuations in Individual Stocks*

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Abstract

We make use of the extant testing methodology of Barndorff-Nielsen and Shephard (2006) and Aït-Sahalia and Jacod (2009a,b,c) to examine the importance of jumps, and in particular "large" and "small" jumps, using high frequency price returns on 25 stocks in the DOW 30 and S&P futures index. In particular, we examine jumps from both the perspective of their contribution to overall realized variation and their contribution to predictive regressions of realized volatility. We find evidence of jumps in around 22.8% of the days during the 1993-2000 period, and in 9.4% of the days during the 2001-2008 period, which implies more (jump induced) turbulence in financial markets in the previous decade than the current decade. Also, it appears that frequent "small" jumps of the 1990s have been replaced to some extent with relatively infrequent "large" jumps in recent years. Interestingly, this result holds for all of the stocks that we examine, supporting the notion that there is strong comovement across jump components for a wide variety of stocks, as discussed in Bollerslev, Law and Tauchen (2008). In our prediction experiments using the class of linear and nonlinear HAR-RV, HAR-RV-J and HAR-RV-CJ models proposed by Müller, Dacorogna, Davé, Olsen, Puctet, and Von Weizsäckeret (1997), Corsi (2004) and Andersen, Bollerslev and Diebold (2007), we find that the "linear" model performs well for only very few stocks, while there is significant improvement when instead using the "square root" model. Interestingly, the "log" model, which performs very well in their study of market indices, performs approximately equally as well as the square root model when our longer sample of market index data is used. Moreover, the log model, while yielding marked predictability improvements for individual stocks, can actually only be implemented for 7 of our 25 stocks, due to data singularity issues associated with the incidence of jumps at the level of individual stocks.

Keywords: Itô semi-martingale, realized volatility, jumps, quadratic volatility, multipower variation, tripower variation, truncated power variation, quarticity, infinite activity jumps.

JEL Classification: C58, C22, G17.

1 Introduction

In recent years, a sustained effort in the financial econometrics research community has been undertaken in order to further understand the underlying structure of asset returns. In one branch of this research, methods for testing whether log return processes have jumps has been formalized. A very few of the key contributions in this area include: Aït-Sahalia (2002), Carr et al., (2002), Carr and Wu (2003), Huang and Tauchen (2005), Barndorff-Nielsen and Shephard (BNS: 2006), Woerner (2006), Cont and Mancini (2007), Jiang and Oomen (2008), Lee and Mykland (2008), Tauchen and Todorov (2008), and Aït-Sahalia and Jacod (2009a,b,c).

In an important paper, Huang and Tauchen (2005) find evidence of jumps for S&P cash and future (log) returns from 1997 to 2002 in approximately 7% of the trading days. Their test requires the jump component to be a compound Poisson process. Several authors, including Cont and Mancini (2007), Tauchen and Todorov (2008) and Aït-Sahalia and Jacod (2009c) have taken the analysis of jumps one step further by developing tests to ascertain whether the process describing an asset contains "infinite activity jumps" - those jumps that are tiny and look similar to continuous movements, but whose contribution to the jump risk of the process is not negligible. Cont and Mancini (2007) implement their method of testing for the existence of infinite activity jumps using foreign exchange rate data, and find no evidence infinite activity jumps. Aït-Sahalia and Jacod (2009c) estimate that the degree of activity of jumps in Intel and Microsoft log returns is approximately 1.6, which implies evidence of infinite activity jumps for these, and possibly many other stocks. Anderson, Bollerslev and Diebold (ABD: 2007) find that separating out the volatility jump component results in improved out-of-sample volatility forecasting, and find that jumps are closely related to macroeconomic announcements. In summary, it is now generally accepted that many return processes contain jumps.

In this paper, we first add to the burgeoning literature on this topic by using a simple procedure for decomposing high frequency return jumps into "small" and "large" jumps, and by empirically examining the properties of these different types of jumps via examination of the degree of jump activity and the contribution of jump variation to total quadratic variation. The impetus for our research stems from the fact that while "small" jumps may or may not play an important role in financial decision making and asset allocation, "large" jumps almost certainly do play an important role, and hence directly testing for jumps and then characterizing their degree of activity and variation magnitude is of particular interest to applied practitioners. There are many precedents to our empirical analysis, although our paper is closest to Huang and Tauchen (2005) and ABD (2007) and Aït-Sahalia and Jacod (2009a), and indeed our empirical analysis is meant to build on the empirical findings of those papers. Thereafter, we revisit class of HAR-RV prediction models.proposed by Müller, Dacorogna, Davé, Olsen, Puctet, and Von Weizsäckeret (1997) and Corsi (2004) and the extension thereof to linear and nonlinear HAR-RV-J and HAR-RV-CJ models examined by ABD (2007). These models utilize realized measures of jump and continuous components of asset return in order to assess realized volatility predictability.

Our approach in this empirical paper is to implement recent theoretical advances in the areas of jump testing and the characterization of continuous time processes with jumps in order to isolate and examine jumps with magnitude larger than level γ (and smaller than γ), for some given constant value of γ. In particular, we first examine whether there are jumps in the process describing the dynamics of an asset return by using methodology in Huang and Tauchen (2005), BNS (2006), Jacod (2007), and Aït-Sahalia and Jacod (2009b). The idea underlying their methods is to track the distance between the variation of the continuous component and the overall quadratic variation of a given log return process. Of note is that BNS (2006) provides methodology appropriate for processes with finite activity jumps, although we also allow for infinite activity jumps in this paper, as we take advantage of the limit theory developed for this purpose in Jacod (2008) and Aït-Sahalia and Jacod (2009b). Once jumps are found, we truncate the process in order to isolate those jumps with size larger than γ, and construct a realized measure of the variational contribution of large and small jumps to total variation.

One potential use of our approach is in jump risk assessment and management. For example, financial managers may be interested in knowing not only the probability of jumps, but also the probability that jumps of certain pre-defined "large" magnitudes will occur. This is an important distinction, particularly given that, as shown by Aït-Sahalia and Jacod (2009a,c), infinite activity jumps are present in the dynamics of some asset returns. However, such jumps, when of small magnitude, may not only be difficult to distinguish (in practice) from the continuous component of the process, but may not be of as serious concern to financial planners as "large" jumps. In this sense, it may be of empirical interest not only to test for jumps in general, but also to check for jumps of varying magnitudes, and to characterize the contribution of such jumps to total variation. In particular, the partitioning of jumps into those that are "small" and "large" allows us to uncover empirical evidence concerning what type of jumps are contributing to overall jump variation. This is also potentially of interest in macroeconomics, for example, as it may turn out that larger but less frequent jumps characterize periods of economic recession, while smaller jumps characterize expansionary periods, say. More generally, jump frequency and magnitude (i.e. jump risk) may play an important role in dating business cycle turning points. Moreover, it is already known from ABD (2007) that many significant jumps are associated with specific macroeconomic news announcements, and our approach provides a simple framework from within which this finding can be further explored.

In our empirical analysis, we examine high frequency data for 25 stocks in the DOW 30, using 5 minute interval observations, and for the sample period from 1993 to 2008. Some of the stocks in our dataset, (e.g. Microsoft and Intel) have been found to be characterized by infinite activity jumps by Aït-Sahalia and Jacod (2009b,c), and therefore do not belong to the class of finite activity jump processes that BNS (2006) has often been applied to. This fact underscores the importance of the recent papers by Jacod (2008), Tauchen and Todorov (2008) and Aït-Sahalia and Jacod (2009a,b,c), where new limit theory applicable to infinite activity is implemented and developed.

In the first part of our empirical analysis, we find evidence of jumps in around 22.8% of the days in the 1993-2000 period, and 9.4% in the 2001-2008 period. This degree of jump activity implies more (jump induced) turbulence in financial markets in the previous decade than the current decade. However, and as expected, the prevalence of "large" jumps varies across these periods. (Note that we examine large jumps by picking 3 different fixed γ levels, corresponding to $50th, 75th$ and $90th$ percentiles of samples of the monthly maximum return increments, i.e. our monthly “abnormal event" samples.) In particular, large jump activity increases markedly during the 2001-2008 period, with respect to its contribution to the realized variation of jumps and with respect to the contribution of large jumps to the total variation of the (log) price process. This suggests that while the overall role of jumps is lessening, the role of large jumps has not decreased, and indeed, the relative role of large jumps, as a proportion of overall jumps has actually increased in the 2000s. Note that this result holds on avergae across all 25 stocks examined. In summary, it appears that frequent “small" jumps of the 1990s have been replaced with relatively infrequent "large" jumps in recent years. Interestingly, this result holds for all of the stocks that we examine, supporting the notion that their is strong comovement across jump components for a wide variety of stocks, as discussed in Bollerslev, Law and Tauchen (2008).

In the second part of our empirical analysis, we revisit the HAR-RV models discussed above. However, we examine predictive ability regressions for our 25 individual stocks rather than for stock market indices as ABD (2007) do. This allows us to assess whether their findings hold for individual stocks. We find that the "linear" model performs well for only very few stocks, while there is significant improvement when instead using the "square root" model. Interestingly, the "log" model, which performs very well in their study of market indices, performs approximately equally as well as the square root model when our longer sample of market index data is used. Moreover, the log model, while yielding marked predictability improvements for individual stocks, can actually only be implemented for 7 of our 30 stocks, due to data singularity issues that are likely associated with the incidence of jumps at the level of individual stocks.

The rest of the paper is organized as follows: Section 2 discusses the model and assumptions, and Section 3 summarizes results from the extant testing and prediction literatures that are used in the sequel. Section 4 contains the results of our empirical analysis of 25 of the DOW 30 stocks, and concluding remarks are contained Section 5.

2 Model and Assumptions

In this section, we follow the general set-up of Aït-Sahalia and Jacod (2009b). Consider the filtered probability space $(\Omega, F, (F_t)_{t\geq 0}, P)$, in which $(F_t)_{t\geq 0}$ is denoted as a filtration (i.e., a family of sub-sigma algebra F_t of F, being increasing $t : F_s \subset F_t$ if $s \leq t$). The log price process, $X_t = log(P_t)$, is assumed to be an Itô semimartingale process that can be written as:

$$X_t = X_0 + \int_0^t b_s ds + \int_0^t \sigma_s dB_s + \sum_{s\leq t} \Delta X_s, \tag{1}$$

where $X_0 + \int_0^t \mu_s ds + \int_0^t \sigma_s dB_s$ is the continuous semimartingale component of the process, which is the sum of a local martingale plus an adapted process with finite variation component. Additionally, ΔX_s is a jump at time s, defined as:

$$\Delta X_s = X_s - \lim_{\tau<s,\tau\to s} X_\tau.$$

Given this definition, the jump part of X_t in the time interval $[0, t]$ is defined to be $\sum_{s\leq t} \Delta X_s$. Note that when the jump is a Compound Poisson Process (CPP) - i.e. a finite activity jump process -

then it can be expressed as:

$$J_t = \sum_{s \le t} \Delta X_s = \sum_{i=1}^{N_t} Y_i,$$

where N_t is number of jumps in $[0, t]$, N_t follows a Poisson process, and the Y_i's are i.i.d. and are the sizes of the jumps. The CCP assumption has been widely used in the literature on modeling, forecasting, and testing for jumps. However, recent evidence suggests that processes may contain infinite activity jumps - i.e. infinite tiny jumps that look similar to continuous movements. In such cases, the CCP assumption is clearly violated, and hence we draw in such cases on the theory of Jacod (2008) and Aït-Sahalia and Jacod (2009b,c) when applying standard BNS (2006) type jump tests.

The empirical evidence discussed in this paper involves examining the structure of the jump component of the log return process, X_t, using one historically observed price sample path $\{X_0, X_{\Delta_n}, X_{2\Delta_n} ... X_{n\Delta_n}\}$, where Δ_n is deterministic. The increment of the process at time $i\Delta_n$ is denoted by:

$$\Delta_i^n X = X_{i\Delta_n} - X_{(i-1)\Delta_n}.$$

For convenience, we consider the case $t = n\Delta_n$ in the sequel.[1] Moreover, note that for a given level of γ, $\gamma > 0$, equation (1) can be written as:

$$X_t = X_0 + \int_0^t \mu_s ds + \int_0^t \sigma_s dB_s + \sum_{s \le t} \Delta X_s I_{|\Delta X_s| \le \gamma} + \sum_{s \le t} \Delta X_s I_{|\Delta X_s| > \gamma}, \tag{2}$$

where $I_{|\Delta X_s \ge \gamma|}$ is an indicator which equals 1 for $|\Delta X_s| \ge \gamma$ and 0 otherwise. Thus, once the process is found to have jumps, the jumps process can be decomposed into 2 components. One contains jumps with size larger than γ (large jumps) and the other contains jumps with size smaller than γ (small jumps). In the next section, we summarize various important features of the extant literature on jump testing and the use of it in realized volatility forecasting.

3 Testing for Jumps and Decomposing Jumps

3.1 Testing for jumps

In this section, we review some theoretical results relating to testing for jumps, namely testing whether $J_t = \sum_{s \le t} \Delta X_s \ne 0$.

[1] See Jacod (2008) for further details.

In pioneering work, BNS (2006) proposes a robust and simple test for a class of Brownian Itô Semimartingales plus Compound Poisson jumps. In recent work, Aït-Sahalia and Jacod (2009b) among others develop a different test which applies to a large class of Itô-semimartingales, and allows the log price process to contain infinite activity jumps - small jumps with infinite concentrations around 0. In this paper, we follow the jump test methodology of Huang and Tauchen (2005) as well as Barndorff-Nielsen and Shephard (2006), which looks at the difference between the continuous component and total quadratic variation in order to test for jumps. However, we make use of the limit theorems developed and used by Jacod (2008) and Aït-Sahalia and Jacod (2009b) in order to implement the Barndorff -Nielsen and Shephard (2006) type test under the presence of both infinite activity and finite activity jumps (see Section 4 for further discussion).

A simplified version of the results of the above authors applied to (1) for the one-dimensional case is as follows. If the process X is continuous, let $f(x) = x^n$ (exponential growth), let ρ_{σ_s} be the law $N(0, \sigma_s^2)$, and let $\rho_{\sigma_s}(f)$ be the integral of f with respect to this law. Then:

$$\sqrt{\frac{1}{\Delta_n}}\left(\Delta_n \sum_{i=1}^{n} f(\frac{\Delta_i^n X}{\sqrt{\Delta_n}})^2 - \int_0^t \rho_{\sigma_s}(f)ds\right) \xrightarrow{L-S} \int_0^t \sqrt{\rho_{\sigma_s}(f^2) - \rho_{\sigma_s}^2(f)}dB_s \tag{3}$$

Here, $L - S$ denotes stable convergence in law, which also implies convergence in distribution. For $n = 2$, the above result is the same as BNS (2006). More generally:

$$\sqrt{\frac{1}{\Delta_n}}\left(\sum_{i=1}^{n}(\Delta_i^n X)^2 - \int_0^t \sigma_s^2 ds\right) \xrightarrow{D} N(0, \int_0^t \vartheta\sigma_s^4 ds) \tag{4}$$

or

$$\frac{\sqrt{\frac{1}{\Delta_n}}\left(\sum_{i=1}^{n}(\Delta_i^n X)^2 - \int_0^t \sigma_s^2 ds\right)}{\sqrt{\int_0^t \vartheta\sigma_s^4 ds}} \xrightarrow{D} N(0,1), \tag{5}$$

where ϑ is constant and where $\int_0^t \sigma_s^2 dt$ is known as the integrated volatility or the variation of the continuous component of the model and $\int_0^t \sigma_s^2 dt$ is integrated quarticity. From the above result, notice that if the process does not have jumps, then $\sum_{i=1}^{n}(\Delta_i^n X)^2$, which is an approximation of quadratic variation of the process, should be "close" to the integrated volatility. This is the key idea underlying the BNS (2006) jump test. A final crucial issue in this jump test is the estimation of $\int_0^t \sigma_s^2 dt$ and $\int_0^t \sigma_s^4 dt$ in the presence of both finite and infinite activity jumps. As remarked in BNS (2006), in order to ensure that tests have power under the alternative, intergrated volatility and integrated quarticity estimators should be consistent under the presence of jumps. The authors note that robustness to jumps is straightforward so long as there are a finite number of jumps, or in cases where the jump component model is a Lévy or non-Gaussian OU model (Barndorff-Nielsen, Shephard, and Winkel (2006)). Moreover, under infinite activity jumps, note that as pointed out in Jacod (2007), there are available limit results for volatility and quarticity estimators for the case of semimartingales with jumps.

Turning again to our discussion of volatility and quarticity, note that in a continuation of work initiated by Barndorff-Nielsen and Shephard (2004), Barndorff-Nielsen, Graverson, Jacod, Podolskij, and Shephard (2006) and Jacod (2007) develop general so-called multipower variation estimators of $\int_0^t \sigma_s^k ds$, in the case of continuous semimartingales and semimartingales with jumps, respectively, which are based on

$$V_{r_1, r_2 ..., r_j} = \sum_{i=2}^{n} |\Delta_i^n X|^{r_1} |\Delta_{i-1}^n X|^{r_2} .... |\Delta_{i-j}^n X|^{r_j}.$$

where $r_1, r_2, ..., r_j$ are positive, such that $\sum_1^j r_i = k$. For cases where $k = 2$ and $k = 4$, BNS (2006) use $V_{1,1}$ (bipower variation) and $V_{1,1,1,1}$. In our jump test implementation, we use $V_{\frac{2}{3},\frac{2}{3},\frac{2}{3}}$(tripower variation) and $V_{\frac{4}{5},\frac{4}{5},\frac{4}{5}}$. The reason we use tripower variation, $V_{\frac{2}{3},\frac{2}{3},\frac{2}{3}}$, instead of bipower variation, $V_{1,1}$, is that it is more robust to clustered jumps. Denote the estimators of $\int_0^t \sigma_s^2 ds$ and $\int_0^t \sigma_s^4 ds$ to be $\widehat{IV}$ and $\widehat{IQ}$, and note that:

$$\widehat{IV} = V_{\frac{2}{3},\frac{2}{3},\frac{2}{3}} \mu_{\frac{2}{3}}^{-3} \simeq \int_0^t \sigma_s^2 ds \tag{6}$$

and

$$\widehat{IQ} = \Delta_n^{-1} V_{\frac{4}{3},\frac{4}{3},\frac{4}{3}} \mu_{\frac{4}{5}}^{-5} \simeq \int_0^t \sigma_s^4 ds, \tag{7}$$

where $\mu_r = E(|Z|^r)$ and Z is a $N(0,1)$ random variable.

Regardless of the estimator that is used, the appropriate test hypotheses are:

H_0 : X_t is a continuous process

H_1 : the negation of H_0 (there are jumps)

If we use multi-power variation, under the null hypothesis the test statistic which directly follows from the CLT mentioned above is:

$$LS_{jump} = \frac{\sqrt{\frac{1}{\Delta_n}} \left(\sum_{i=1}^n (\Delta_i^n X)^2 - \widehat{IV} \right)}{\sqrt{\vartheta \widehat{IQ}}} \xrightarrow{D} N(0,1)$$

and the so-called jump ratio test statistic is:

$$RS_{jump} = \frac{\sqrt{\frac{1}{\Delta_n}}}{\sqrt{\vartheta \widehat{IQ}/(\widehat{IV})^2}} \left(1 - \frac{\widehat{IV}}{\sum_{i=1}^{n}(\Delta_i^n X)^2}\right) \xrightarrow{D} N(0,1).$$

Of note is that an adjusted jump ratio statistic has been shown by extensive Monte Carlo experimentation in Huang and Tauchen (2005), in the case of CCP jumps, to perform better than the two above statistics, being more robust to jump over-detection. This adjusted jump ratio statistic is:

$$AJ_{jump} = \frac{\sqrt{\frac{1}{\Delta_n}}}{\sqrt{\vartheta max(t^{-1}, \widehat{IQ}/(\widehat{IV})^2)}} \left(1 - \frac{\widehat{IV}}{\sum_{i=1}^{n}(\Delta_i^n X)^2}\right) \xrightarrow{D} N(0,1)$$

In general if we denote the daily test statistics to be $Z_{t,n}(\alpha)$, where n is the number of observations per day and α is the test significance level [2], then we reject the null hypothesis if $Z_{t,n}(\alpha)$ is in excess of the critical value Φ_α, leading to a conclusion that there are jumps. The converse holds if $Z_{t,n}(\alpha)$ is less than Φ_α. In our empirical application, $Z_{t,n}(\alpha)$ is the adjusted jump ratio statistic, and we calculate the percentage of days that have jumps, for the period from 1993 to 2008. We now turn to a discussion of large jumps and constructing measures of the daily variation due to continuous and jump components.

3.2 Large Jumps and Small Jumps

There is now clear evidence that jumps are prevalent in equity market. For example, Huang and Tauchen (2005) construct the above jump test statistics, and find that jumps contribute about 7% to the total variation of daily stock returns. Aït-Sahalia and Jacod (2009b) not only find jumps but given the existence of jumps, they look more deeply into the structure of the jumps, and for Intel and Microsoft returns they find evidence of the existence of infinite activity jumps.

An important focus in our paper is to the decomposition of jumps into "large" and "small" components so that we may assess their contributions to the overall variation of the price process. In particular, for some fixed level γ, define large and small jump components as follows, respectively:

$$LJ_t(\gamma) = \sum_{s \le t} \Delta X_s I_{|\Delta X_s| \ge \gamma}$$

[2] i.e., $\Delta_n = 1/n$

and

$$SJ_t(\gamma) = \sum_{s \le t} \Delta X_s I_{|\Delta X_s| < \gamma}.$$

The choice of γ may be data driven, but in this paper we are more concerned with scenarios where there is some prior knowledge concerning the magnitude of γ. For example, under various regulatory settings, capital reserving and allocation decisions may be based to a large extent on the probability of jumps or shocks occurring that are of a magnitude greater than some known value, γ. In such cases, planners may be interested not only in knowledge of jumps of magnitude greater than γ, but also in characterizing the nature of the variation associated with such large jumps. The procedure discussed in this section can readily be applied to uncover this sort of information.

3.3 Realized measures of daily jump variation

The partitioning of variation due to continuous and jump components can be done, for example, using truncation based estimators which have been developed by Mancini (2001,2004,2009) and Jacod (2008). One can also simply split quadratic variation into continuous and jump components by combining various measures of integrated volatility, such as bipower or tripower variation and realized volatility. Andersen, Bollerslev, and Diebold (2007) do this, and construct measures of the variation of the daily jump component as well as the continuous component. In this paper we use their method, but apply it to both small and large jumps. In particular, once jumps are detected, the following risk measures introduced by Andersen et al. (2007) are constructed:

$VJ_t = \textit{Variation of the jump component} = max\{0, RV_t - \widehat{IV}_t\} * I_{jump,t}$

$VC_t = \textit{Variation of continuous component} = RV_t - VJ_t$,

where $RV_t = \sum_{i=1}^{n} (\Delta_i^n X)^2$ is the daily realized volatility (i.e. a measure of the variation of the entire (log) stock return process), I_{jump} is an indicator taking the value 0 if there are no jumps and 1 otherwise, and n is the number of intra-daily observations. One can then calculate daily jump risk. Note that in these formulae, the variation of the continuous component has been adjusted (i.e. the variation of the continuous component equals realized volatility if there are no jumps and equals $\widehat{IV}_t$ if there are jumps). In addition, note that $\sum_{i=1}^{n} (\Delta_i^n X)^2 I_{|\Delta_i^n X| \ge \gamma}$ converges uniformly in probablity to $\sum_{s \le t} (\Delta X_s)^2 I_{|\Delta X_s| \ge \gamma}$, as n goes to infinity [3]. Thus, the contribution of the variation

[3] See Jacod (2008), Aït-Sahalia and Jacod (2009a) for further details.

of jumps with magnitude larger than γ and smaller than γ are denoted and calculated as follows:

Realized measure of large jump variation: $VLJ_{t,\gamma}$=min$\{VJ_t, (\sum_{i=1}^{n}(\Delta_i^n X)^2 I_{|\Delta_i^n X|\geq\gamma} * I_{jump,t})\}$,

Realized measure of small jump variation: $VSJ_{t,}\gamma = VJ_t - VLJ_{t,\gamma}$,

where I_{jump} is defined above and $I_{jump,\gamma}$ is an indicator taking the value 1 if there are large jumps and 0 otherwise. This condition simply implies that large jump risk is positive if the process has jumps and has jumps with magnitude greater than γ.

Now we can write the relative contribution of the variation of the different jump components to total variation in a variety of ways:

Relative contribution of continuous component $= \frac{VC_t}{RV_t}$

Relative contribution of jump component $= \frac{VJ_t}{RV_t}$

Relative contribution of large jump component $= \frac{VLJ_{t,\gamma}}{RV_t}$

Relative contribution of small jump component $= \frac{VLS_{t,\gamma}}{RV_t}$

Relative contribution of large jumps to jump variation $= \frac{VLJ_{t,\gamma}}{VJ_t}$

Relative contribution of small jumps to jump variation $= \frac{VLS_{t,\gamma}}{VJ_t}$

3.4 Linear and Nonlinear HAR-RV, HAR-RV-J AND HAR-RV-CJ Models

The realized measures summarized in previous section have been utilized to forecast future realized volatility by several authors. In a key paper in this forecasting literature, ABD (2007) develop Linear, Square Root and Log HAR-RV, HAR-RV-J and HAR-RV-CJ classes of models (see above for further details). The HAR-RV formulation is based on an extension of the so-called Heterogeneous ARCH, or HARCH, class of models analyzed by Müller et al. (1997), in which the conditional variance of the discretely sampled returns is parameterized as a linear function of the lagged squared returns over the identical return horizon together with the squared returns over shorter return horizons. The authors find that there is an improvement by incorporating jumps in these models, and their class of log HAR-RV, HAR-RV-J and HAR-RV-CJ models performs the best. We revisit this class of models but focus on the predictive performance of the models applied to individual stock returns, as opposed to market indices. The models are specified as follows:

First, define the multi-period normalized realized variation for jump and continuous components as the sum of the corresponding one-period measures. Namely:

$RV_{t,t+h} = h^{-1}[RV_{t+1} + RV_{t+2} + ... + RV_{t+h}]$

$VC_{t,t+h} = h^{-1}[VC_{t+1} + VC_{t+2} + ... + VC_{t+h}]$

$$VJ_{t,t+h} = h^{-1}[VJ_{t+1} + VJ_{t+2} + ... + VJ_{t+h}],$$

where $h = 1, 2, ...$ Note that in the case where $h = 1$, $RV_{t,t+1} = RV_{t+1}$. Also, $h = 5$ and $h = 22$ refer to measures of weekly and monthly volatilities, respectively. The class of linear models includes:

HAR-RV Model (Model Type 1)

$$RV_{t,t+h} = \beta_0 + \beta_d RV_t + \beta_w RV_{t,t-5} + \beta_m RV_{t,t-22} + \epsilon_{t+h}$$

HAR-RV-J Model (Model Type 2)

$$RV_{t,t+h} = \beta_0 + \beta_d RV_t + \beta_w RV_{t,t-5} + \beta_m RV_{t,t-22} + \beta_j VJ_t + \epsilon_{t+h}$$

HAR-RV-CJ Model (Model Type 3)

$$RV_{t,t+h} = \beta_0 + \beta_{cd} VC_t + \beta_{cw} VC_{t,t-5} + \beta_{cm} VC_{t,t-22} + \beta_{jd} VJ_t + \beta_{jw} VJ_{t,t-5} + \beta_{jm} VJ_{t,t-22} + \epsilon_{t+h}$$

The class of square root models includes:

Square Root HAR-RV Model (Model Type 1)

$$(RV_{t,t+h})^{1/2} = \beta_0 + \beta_d (RV_t)^{1/2} + \beta_w (RV_{t,t-5})^{1/2} + \beta_m (RV_{t,t-22})^{1/2} + \epsilon_{t+h}$$

HAR-RV-J Model (Model Type 2)

$$(RV_{t,t+h})^{1/2} = \beta_0 + \beta_d (RV_t)^{1/2} + \beta_w (RV_{t,t-5})^{1/2} + \beta_m (RV_{t,t-22})^{1/2} + \beta_j (VJ_t)^{1/2} + \epsilon_{t+h}$$

HAR-RV-CJ Model (Model Type 3)

$$(RV_{t,t+h})^{1/2} = \beta_0 + \beta_{cd} (VC_t)^{1/2} + \beta_{cw} (VC_{t,t-5})^{1/2} + \beta_{cm} (VC_{t,t-22})^{1/2} + \beta_{jd} (VJ_t)^{1/2} + \\ + \beta_{jw} (VJ_{t,t-5})^{1/2} + \beta_{jm} (VJ_{t,t-22})^{1/2} + \epsilon_{t+h}$$

Finally, the class of log linear models includes:

HAR-RV Model (Model Type 1)

$$\log(RV_{t,t+h}) = \beta_0 + \beta_d \log(RV_t) + \beta_w \log(RV_{t,t-5}) + \beta_m \log(RV_{t,t-22}) + \epsilon_{t+h}$$

HAR-RV-J Model (Model Type 2)

$$\log(RV_{t,t+h}) = \beta_0 + \beta_{cd} \log(RV_t) + \beta_{cw} \log(RV_{t,t-5}) + \beta_{cm} \log(RV_{t,t-22}) + \beta_j \log(VJ_t) + \epsilon_{t+h}$$

HAR-RV-CJ Model (Model Type 3)

$$\log(RV_{t,t+h}) = \beta_0 + \beta_{cd} \log(VC_t) + \beta_{cw} \log(VC_{t,t-5}) + \beta_{cm} \log(VC_{t,t-22}) + \beta_{jd} \log(VJ_t) + \\ + \beta_{jw} \log(VJ_{t,t-5}) + \beta_{jm} \log(VJ_{t,t-22}) + \epsilon_{t+h}$$

We now turn to the results of our empirical investigation using the above methodology.

4 Empirical Findings

4.1 Data description

We use a large tick by tick dataset of 25 DOW 30 stocks available for the period 1993-2008. The data source is the TAQ database. We use only 25 stocks because we purge our dataset of those stocks that not frequently traded or are not available across the entire sample period. For the market index, we follow several other papers and look at S&P futures. We also follow the common practice in the literature of eliminating from the sample those days with infrequent trades (less than 60 transactions at our 5 minute frequency).

One problem in data handling involves determining the method to filter out an evenly-spaced sample. In the literature, two methods are often applied - *previous tick* filtering and *interpolation* (Dacorogna, Gencay, Müller, Olsen, and Pictet (2001)). As shown in Hansen and Lund (2006), in applications using quadratic variation, the *interpolation* method should not be used, as it leads to realized volatility with value 0 (see *Lemma 3* in their paper). Therefore, we use the *previous tick* method (i.e. choosing the last price observed during any interval). We restrict our dataset to regular time (i.e. 9:30am to 4:00pm) and ignore ad hoc transactions outside of this time interval. To reduce microstructure effects, the suggested sampling frequency in the literature is from 5 minutes to 30 minutes[4]. As mentioned above, we choose the 5 minute frequency, yielding a maximum of 78 observations per day.

4.2 Jump and Large Jump Results

We implement our analysis in two stages. In the first stage we test for jumps and in second stage we examine large jump properties, in cases where evidence of jumps is found. The list of the companies for which we examine asset returns is given in Table 1, along with a summary of our jump test findings. The rest of the tables and figures summarize the results of our empirical investigation. Before discussing our findings, however, we briefly provide some details about the calculations that we have carried out.

All daily statistics are calculated using the formulae in Sections 3 with:

[4] See Aït-Sahalia, Y., Mykland, P. A., and Zhang, L. (2005)

$$\Delta_n = \frac{1}{n} = \frac{1}{\text{\# of 5 minute transactions / day}}$$

Therefore, $\Delta_n = 1/78$ for most of the stocks in the sample, except during various shortened and otherwise nonstandard days, and except for some infrequently traded stocks. This also implies the choice of time to be the interval $[0, 1]$, where the time from $[0, 1]$ represents the standardizing time with beginning (9 am) set to 0 and end (4.30 pm) set to 1. In our calculations of estimates of integrated volatility and integrated quarticity, we use multipower variation, as given in (6) and (7). Recall also that $\Delta_i^n X = X_{i\Delta_n} - X_{(i-1)\Delta_n}$ is simply the incremental return of $X_{i\Delta_n}$. For any trading day, X_0 and X_1 correspond to the first and the last observations of the day. Denote T as the number of days in the sample. We construct the time series $\{Z_{t,n}(\alpha)\}_{t=0}^{T}$ and $\left\{\frac{VC_t}{RV_t}, \frac{VJ_t}{RV_t}, \frac{VLJ_{t,\gamma}}{RV_t}, \frac{VSJ_{t,\gamma}}{RV_t}\right\}_{t=0}^{T}$. The number of days and proportion of days identified as containing jumps can easily be calculated as:

Number of days identified as jumps $= \sum_{i=0}^{T} I(Z_{i,n}(\alpha) > \Phi_\alpha)$.

Proportion of days identified as jumps $= \frac{\sum_{i=0}^{T} I(Z_{t,n}(\alpha) > \Phi_\alpha)}{T}$.

In addition, we construct the following monthly time series

Proportion of days identified as jumps in a month$= \frac{\sum_{i=m}^{m+h} I(Z_{t,n}(\alpha) > \Phi_\alpha)}{h}$

Monthly average relative contribution of jump component $= \frac{\sum_{i=m}^{m+h} \frac{VJi}{RVi}}{h}$

Monthly average relative contribution of large jump component truncated at level $\gamma = \frac{\sum_{i=m}^{m+h} \frac{VLJ_{i,\gamma}}{RV_t}}{h}$,

where m is the starting date and h is the number of days in each month. On average, there are 22 business days per month. Note that there are 12 statistics each year for each time series.

Here, $I(\cdot)$ denotes the indicator function. The average relative contribution of continuous, jump, and large jump components to the variation of the process is reported using the mean of the sample (i.e. we report the means of $\frac{VC_t}{RV_t}, \frac{VJ_t}{RV_t}, \frac{VLJ_{t,\gamma}}{RV_t}$, and$\frac{VSJ_{t,\gamma}}{RV_t}$).

In addition to reporting findings based on examination of the entire sample period, we also split the sample into two periods. The first period is from 1993 to 2000 and the second period is from 2001 to 2008. The reason for doing this is that we would like to see whether jump activity changes over time. Moreover, these subsamples correspond roughly to break dates for financial data found in Cai and Swanson (2010).

In the sequel, we provide figures for representative individual stocks in our sample (i.e. Walmart, IBM, Bank of America and Citigroup). These stocks are chosen on the basis of their market systematic risk beta. Namely, Walmart has low beta of around 0.3, IBM has a beta close to 0.7, and Bank of America and Citigroup are more risky stocks with betas of around 2.6 and 2.8.

Turning now to our results, a first sense of the prevalence of jumps can be formulated by inspecting Panels A,B, C and D of Figure 1, where statistics higher than 3.9 (i.e. the 0.001 significance level critical value) are presented for the entire sample from 1993 to 2008. It is obvious that jumps are prevalent. Additionally, it should be noted that there is a marked difference in jump frequency between 1993-2000 and 2001-2008, where the first period is much more densely populated with jumps than the latter period. The highest statistic values are around 11, for Walmart in 1997, 11 for IBM in 1994, 10 for Bank of America in 1996 and 7 for Citigroup from 1996 to 1998. Post 2000, the highest statistics are consistently located in 2002 and 2006-2008. Moreover, a simple visual check of the statistic magnitudes in this figure suggests that jumps are more prevalent in the earlier sample period, with respect to both frequency and significance level (more will be said on this later).

Regarding our choice of the large jumps, an important step is to choose truncation levels, γ. If we choose arbitrarily large truncation levels, then clearly we will not find evidence of large jumps. Also one may easily proceed by just picking the truncation level based on the percentiles of the entire historical sample of the 5 minute log return. However, results could then turn out to be difficult to interpret, as in one case the usual choice of $90th$ or $75th$ percentiles leads to virtually no large jumps while the choice $25th$ or $10th$ percentiles leads to a very large number of large jumps. In addition, "large" jumps are often thought of as abnormal events that arise at a frequency of one in several months or even years. Therefore, a reasonable way to proceed is to pick the truncation level on the basis of the sample of the monthly maximum increments - monthly based abnormal events. Specifically, we set three levels $\gamma = 1, 2, 3$ to be the $50th, 75th$, and $90th$ percentiles of the entire sample from 1993 to 2008. Panels A,B,C, and D of Figure 2 depict the monthly largest absolute increments and the jump truncation levels used in our calculations of the variation of large and small jump components. Again, it is quite obvious that the monthly maximum increments are dominant in the previous decade. The larger monthly increments in current decades are mostly located in 2006-2008 and 2002-2004. As a result, the fixed truncation levels which are chosen across the entire sample result in more "hits" in previous decade than in the current one. The truncation level of Citigroup is the largest of the four stocks depicted (for example at $\gamma = 3$ the level is approximately 0.04 for Citigroup and 0.025 for IBM).

Notice that the graphs in Figures 3A and 3B depict magnitudes of the variation of continuous, jump, and truncated jump components of returns for our 4 sample stocks. Namely, the plots are of daily realized volatility, and realized variance of continuous, jump and large jump components at different truncation levels. As might be expected, inspection of the graphs suggests a close linkage between the greater number of jumps in the first decade of the sample and the and large jump risk over the same period. For example, in the case of IBM, the variation of the jump components is clearly dominant in the earlier decade. The highest daily jump risk occurs in late 1998, and is above 0.018. Indeed, at jump truncation level 3, we only see large jump risk for the years 1994, 1996, 1998, 2000 and 2008. Combined with the results of Figure 1, this again strongly suggests that there was much more turbulence in the earlier decade.

Turning now to our tabulated results, first recall that Table 1 reports the proportion of days identified as having jumps, at 6 different significant levels, $\alpha = \{0.1, 0.05, 0.01, 0.005, 0.001, 0.0001\}$. Again, there is clear evidence of jumps in both periods. However, the jump frequency in the 1993-2000 sample is significantly higher than that in the 2001-2008 sample, across all stocks and test significance levels. For example, at the $\alpha = 0.005$ and 0.001 levels, the average daily jump frequencies are 46.9% and 22.8% during the 1993-2000 period, as compared with 16.8% and 9.4% during the 2001-2008 period, respectively. When considering individual stocks, the story is much the same. As illustrated in Figures 1, and tabulated in Table 1, the proportion of "jump-days" for IBM and for the Bank of America are 5.9% and 8.8% during the 2000s, which is much smaller than the value of 19.2% and 21.3% for the two stocks during the 1990s, based on tests implemented using a significance level of $\alpha = 0.001$.

Of course, when calculating jump frequencies, we ignore the magnitudes of the jumps. Table 3 addresses this issue by summarizing another measure of jumps - namely the average percentage contribution of jumps to daily realized variance. Details of the measures reported are given above and in Section 3. In support of our earlier findings, it turns out that jumps account for about 15.6% and 8.1% of total variation at significance levels $\alpha = 0.005$ and 0.001, respectively, when considering the entire sample period from 1993-2008. Moreover, analogous statistics for the period 1993-2000 are 25.1% and 12.7%, while those for the 2001-2008 period are 7% and 5%. The statistics for IBM and Bank of America are 25.3% and 10.7% for the period 1993-2000 and 3.5% and 2.3% for the period 2001-2008 while those for the entire samples are 7.9% and 6.6%. This result is consistent with our earlier findings through figure analysis.

In summary, without examining the impact of large jumps, we already have evidence that: (i) There is clear evidence that jumps characterize the structure of the returns of all of the stocks that we examine. (ii) The 1990s are characterized by the occurrence of more jumps than the 2000s. (iii) The contribution of jumps to daily realized variance is substantively higher during the 1990s than the 2000s. (iv) Our results are consistent across all stocks, suggesting the importance of jump risk comovement during turbulence periods.

In our empirical analysis of large jumps, we carry out the same steps as those employed above when examining overall jump activity. Results are reported in Tables 4A-C are for truncation levels $\gamma = 1, 2, 3$ at 6 different significant levels, $\alpha = \{0.1, 0.05, 0.01, 0.005, 0.001, 0.0001\}$. As mentioned earlier, Figures 1 and 3 contain plots of jump test statistics and realized variation not only for overall jump activity, as discussed above, but also for large jumps. Examination of these tables suggest a number of conclusions.

Across the entire sample, there is evidence of large jumps at all levels by measure of variation. Table 4A reports the proportion of days identified as having large jumps for truncation level $\gamma = 1$. It can be seen that the proportion of variation due to large jumps at truncation level $\gamma = 1$ accounts for about 0.9% and 0.6% of total variation (regardless of stock), at significance levels $\alpha = 0.005$ and 0.001, respectively. Values at significance level 0.001 for the periods 1993-2000 and 2001-2008 are around 0.8% and 0.4%, respectively. For $\gamma = 2$, values are 0.4% and 0.3% at significant levels $\alpha = 0.005$ and 0.001, respectively, when considering the entire sample. Values at significance level 0.001 for the periods 1993-2000 and 2001-2008 are around 0.4% and 0.2% for period 1993-2000 and 2001-2008, respectively. A similar result obtains for $\gamma = 3$, suggesting that large jump variability is around twice as big (as a proportion of total variability) for the latter sub-sample, regardless of truncation level. As previously, these results are surprisingly stable across stocks. Although not included here, our analysis of the market index data discussed above yielded a similar result. Further examination of the statistics in the Tables 4A-C also yields another interesting finding. In particular, though proportions of jumps and large jumps at truncation level $\gamma = 1, 2, 3$ are all larger in the previous decade, the difference is smaller and increasingly narrows as higher truncation levels are considered, when examining large jumps. This result, which is true for many of our stocks, suggests an increased role of large jumps in explaining daily realized variance during the latter sub-sample. To illustrate this point, which is apparent upon inspection of average statistics constructed for all 25 stocks, we investigate the case of of ExxonMobil, where we look at all statistics at significance level $\alpha = 0.001$. The proportion of variation of jumps to total variation is 17% for the period 1993-2000 (as shown in Table 3), almost 3 times as much as the corresponding value of 6.2% in 2001-2008. However for large jumps at truncation level $\gamma = 1$, the analogous value is 0.6% for 1993-2008, which is just 1.5 times as much as the 0.4% value during 2001-2008. Similarly at truncation level $\gamma = 2$, the value is 0.4% for 1993-2008 and 0.2% for 2001-2008. Interestingly, at truncation level $\gamma = 3$, the proportion of variation of jumps is 0 for period 1993-2000 while it is 0.1% for period 2001-2008. Therefore, with respect to large jump we find that: (i) Large jumps incidence and magnitudes are consistent with our earlier finding that the 1990s are much more turbulent than the 2000s. (ii) However, for higher truncation levels, the contribution of jump risk during the two periods becomes much closer, and indeed the contribution during the latter period can actually become marginally greater. This suggests that while the overall role of jumps is lessening, the role of large jumps has not decreased, and indeed, the relative role of large jumps, as a proportion of overall jumps has actually increased in the 2000s.

4.3 Realized Jump Measures and Realized Volatility Prediction

Equations for all of the prediction models for which results are discussed in this section were presented in Section 3. Note that the HAR-RV model does not include a jump variable, while HAR-RV-J incorporates jump variation into the HAR-RV model. The HAR-RV-CJ model goes one step further, and separates continuous and jump variation components. The empirical analyses of exchange rates, equity index returns, and bond yields in ABD (2007) suggests that the volatility jump component is both highly important and distinctly less persistent than the continuous component, and that separating the "rough" jump movements from the smooth continuous movements results in significant in-sample volatility forecast improvements (i.e. the linear and nonlinear HAR-RV-CJ models perform better than the other two classes of models). These models, which are both simple and convenient, have been widely referred to in RV forecasting literature, and we revisit them in the context of our individual stocks over our long sample period from 1993 to 2008. We also provide a brief discussion on the performance of the models for S&P futures. Note that within the scope of our paper, the in-sample predictive performance of a model is measured by its R^2, which is similar to approach taken in ABD (2007).

Turning to our regression results, Table 5A report adjusted R^2 values for the linear HAR -RV, HAR-RV-J and HAR-RV-CJ models when used to forecast Realized Volatility of log stock returns at daily $h = 1$, weekly $h = 5$ and monthly $h = 22$ horizons. It is clear that except for JPM, Proctor and Gamble, Verizon and Exxon Mobile (values in bold in the table), R^2 values are very low, at all forecasting horizons and for all HAR realized models. For example, in the case of Intel, the value for $h = 1$ is 0.0716 for the RV model, while it is 0.0766 for the RV-J model and 0.0833 for the RV-CJ. At the weekly horizon the corresponding values are all even smaller (0.0612, 0.0612, and 0.0674), and again at the monthly horizon (values are 0.0430,0.0438 and 0.0470). It is quite obvious from these low values that the predictive performance is poor regardless of forecasting horizon, although there is some improvement when switching from the RV model to the RV-CJ model. The same result holds for all stocks. Moreover, though the improvement is small in magnitude, it turns out to be quite a significant percentage. In the case of Intel, for example, the percentage improvement is 16% when $h = 1$.

Table 5B reports analogous results for the nonlinear square root RV class of models. As might be expected given the results of ABD (2007), there is substantive improvement in the predictive performance of this class of non-linear models for all stocks in our sample. In contrast to the linear model, 20 stocks have R^2 values greater than 0.2. Stocks with values larger than 0.3 are highlighted. Again focussing on Intel, for $h = 1$ the R^2 value is 0.4028 for the RV model, 0.4069 for the RV-J model and 0.4212 for the RV-CJ model. For $h = 5$, values are 0.3338, 0.3344, and 0.3452. Finally for $h = 22$, values are 0.2576,0.2578 and 0.2670, suggesting that the RV plus jump model is preferable to the one that separates the continuous and jump components. This finding is similar to that of ABD (2007) for market indices. Overall, the huge improvement in the predictive performance points to strong non-linearity in the dependence of future RV on the past RV, VC and VJ variables, across different horizons. In addition, and similar to the more poorly performing linear models, predictive performance is better at shorter horizons.

The last class of models (see Table 5C) that we investigate includes our nonlinear log models, which are found by ABD (2007) to yield the best predictive performance. Note that the focus of our paper is on individual stocks in the DOW 30, which is quite different from examining the S&P futures index. Surprisingly, the HAR-RV-CJ just only works for 7 stocks in our sample, otherwise yielding data matrix singularities (i.e. due to too many zero values) leading to a failure of least squares. (Note that the market index is constructed as the weighted average of individual stocks returns, and therefore contains much more jump activity than single stock.) The stocks for which the model works, and for which results are reported include Citigroup, Dupont, Home Depot, Intel, Microsoft, Verizon and Exxon Mobil. Interestingly, for these models, there are significant overall improvements in regression fit, with R^2 values increasing to the 0.65 - 0.75 range for the log RV model when $h = 1$,for example. To illustrate, for the the case of Intel, values for the log RV-CJ model are 0.6202 for $h = 1$, 0.5329 for $h = 5$ and 0.4438 for $h = 22$. In order to contrast S&P futures results with the above findings, please refer to Panels D of Table 5. The performance of linear HAR-RV, HAR-RV-J and HAR-RV-CJ models are much better than in the case of individual stocks. Indeed R^2 value of approximately 0.38 for $h = 1$, 0.37 for $h = 5$ and 0.33 for $h = 22$ obtain. As before, nonlinear models are better than linear models, regardless of forecast horizon. However, the gain is not as pronounced as found when examining individual stocks. Again as before, the models perform best at lesser forecast horizons. Finally, and most surprisingly, note that the predictive performance of the log model is close to that of the square root model. Indeed, the square root model actually yields slightly higher R^2 values, regardless of forecast horizon. For example, for $h = 1$ the linear RV-CJ model value is 0.3800, while it is 0.4741 and 0.4592 for the square root and log models, respectively. These findings are somewhat different from ABD (2007) where the log model is always the best. This difference may be accounted for by the fact that our dataset includes recent data spanning the period 2002 to 2008.

In summary, we find that: (i) The class of linear models performs poorly for individual stocks, and there is much improvement when using more complicated models. (ii) Forecasts are better at shorter horizon for all models, regardless of stock and forecasting horizon (iii) There is huge improvement when moving from the linear model to the square root model and from the square root model to the log model (when the log model is "well-defined"), in the case of individual stocks. This improvement is much higher than that obtaining when examining market level data.

5 Concluding Remarks

In this paper, we review the recent literature on assessing the variational contribution of large jumps - those jumps that are far in the tails of a return distribution, and then undertake an extensive empirical investigation of 25 stocks in DOW 30 (as well as the S&P futures market index).

Our investigation provides new and clear evidence of jumps in individual log price processes. Moreover, there are clearly comovements during turbulent times, for all stocks. More noticably, jump incidence is noticably greater during the 1990s than during the 2000s, although the incidence of "very large" jumps is similar across both decades, and the relative importance of large jumps has increased. In a series of predictive experiments, we find similar results as those in ABD (2007). Namely, non-linear square root and log models yield substantial improvements in fit relative to their linear counterparts, when predicting realized volatility. Furthermore, including jump components in the regressions further improves predictive performance. However, the log model, though superior to the other two classes of models considered, is ill-defined for three quarters of our stocks. Moreover, when we re-consider a market index similar to that examined in ABD (2007), we find that the square root model is actually marginally superior to the log model, likely due to the fact that we use an extended sample of data that spans the 1990s and 2000s.

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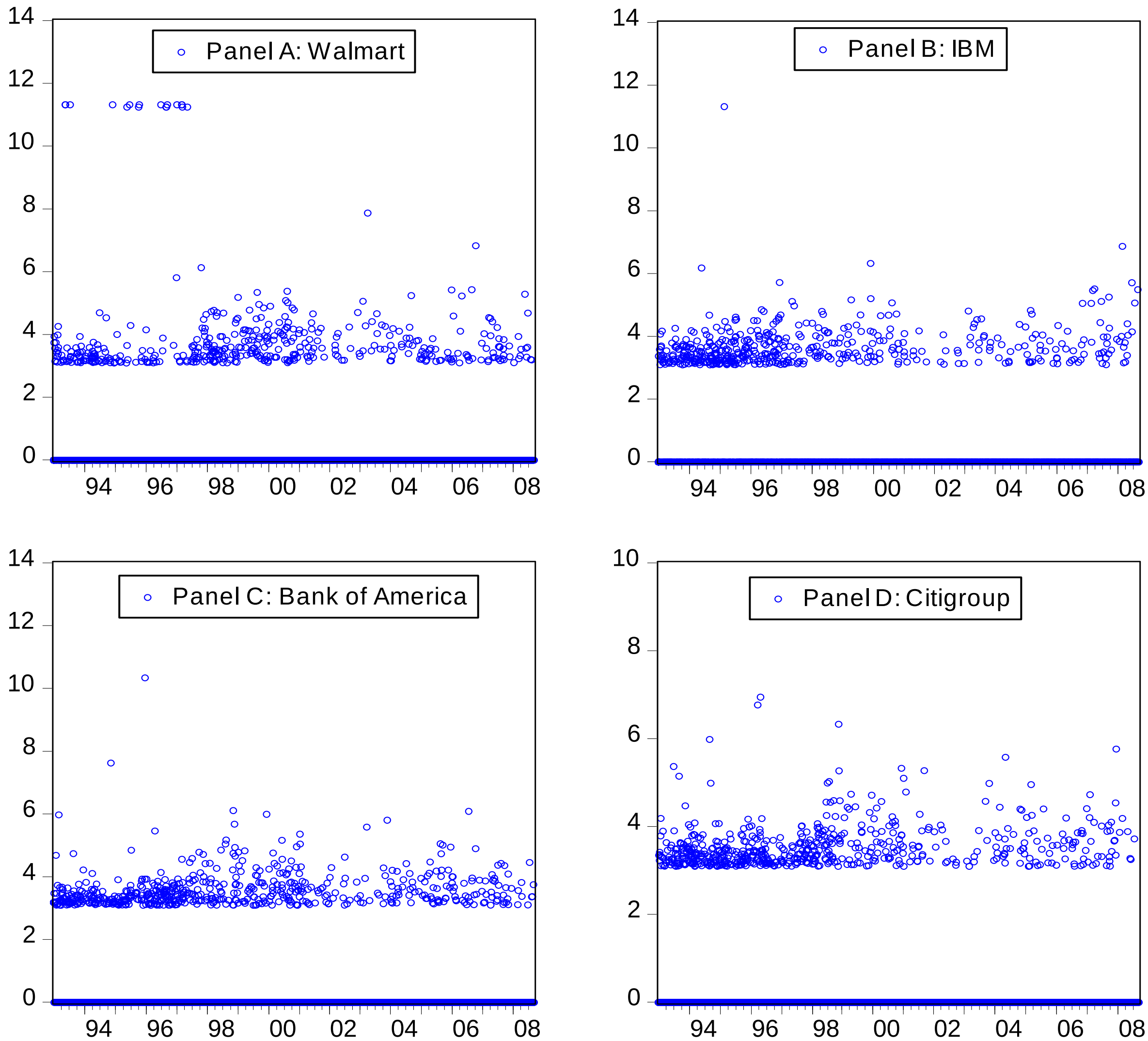
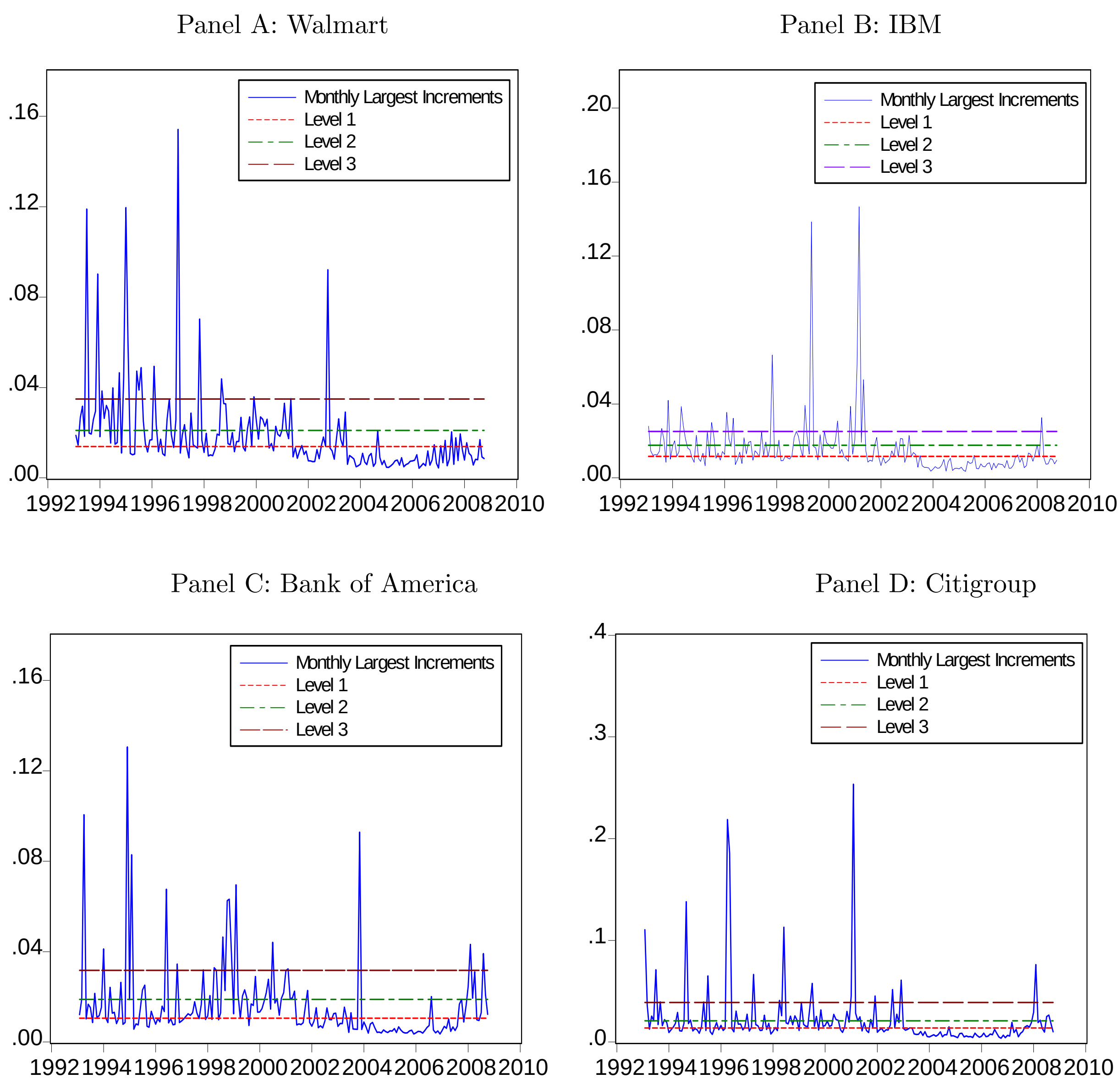
Figure 1: Jump Test Statistics of Days Identified as Having Jumps of (Log) Stock Prices: Sample Period 1993-2008 *



* Panel A, B, C, D depict daily test statistics of days identified as having jumps for Walmart, IBM, Bank of America, Citigroup (Log) Stock Price using 0.001 significant level. Specifically, all statistics in the figure are larger than 3.09. See section 4 for further details.

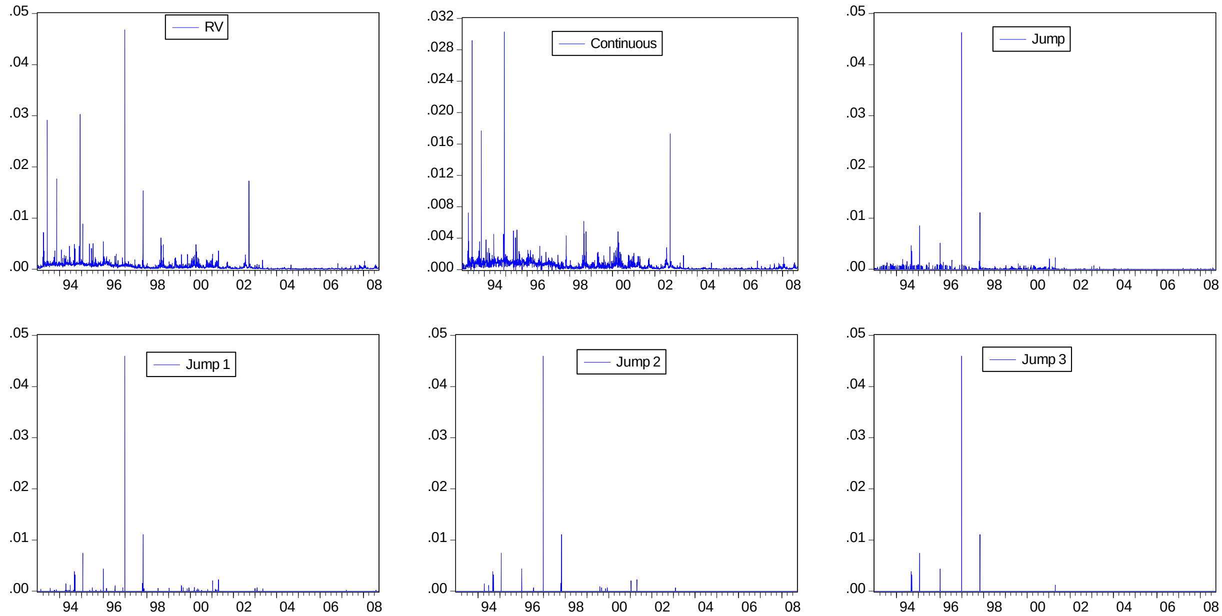
Figure 2: Monthly Largest Increments and Truncation Levels $\gamma = 1, 2, 3$: Sample Period 1993-2008 *



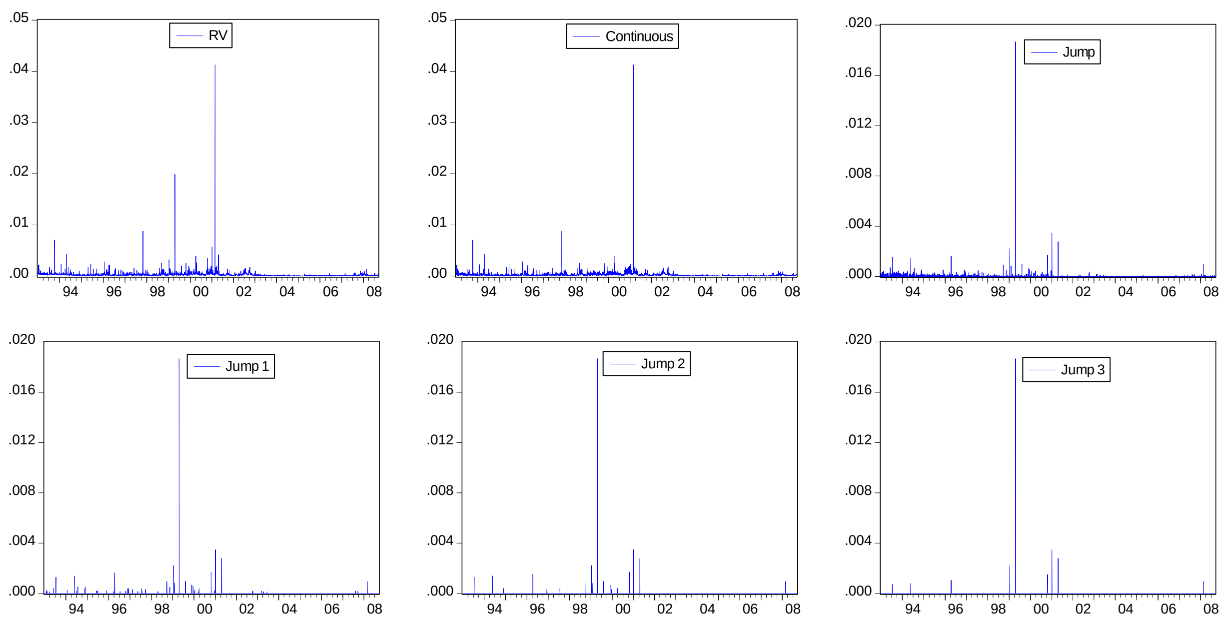
* Panel A,B,C,D depict the monthly largest absolute increments and the jump truncation levels used as thresholds in our calculations of the variations of large and small jump components, where level γ=1 corresponds to the median of monthly maximum increments , level γ=2 corresponds to 75th percentile of monthly maximum increments, and level γ=3 corresponds to 90th percentile monthly maximum increments of(log) stock prices of Walmart, IBM, Bank of America and Citigroup for the sample period is from 1993 to 2008.

Figure 3A: Daily Realized Volatility (RV) and Realized Variation of Continuous, Jump and Truncated Jump Components (Log) Stock Prices for Truncation Levels $\gamma = 1, 2, 3$ *

Panel A: Walmart



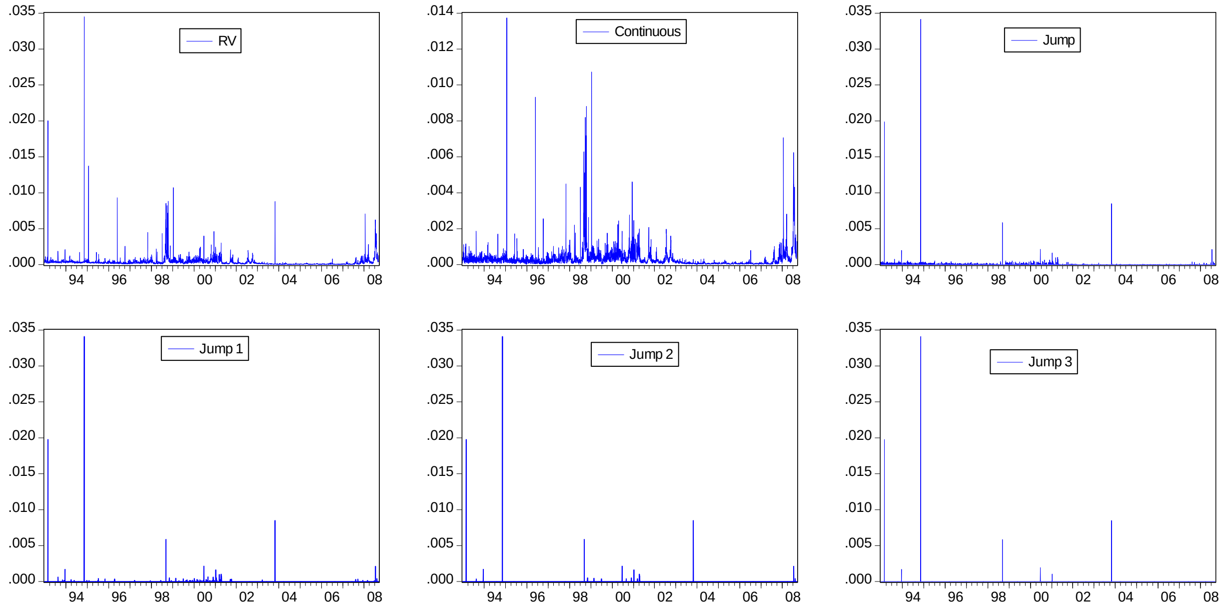
Panel B: IBM



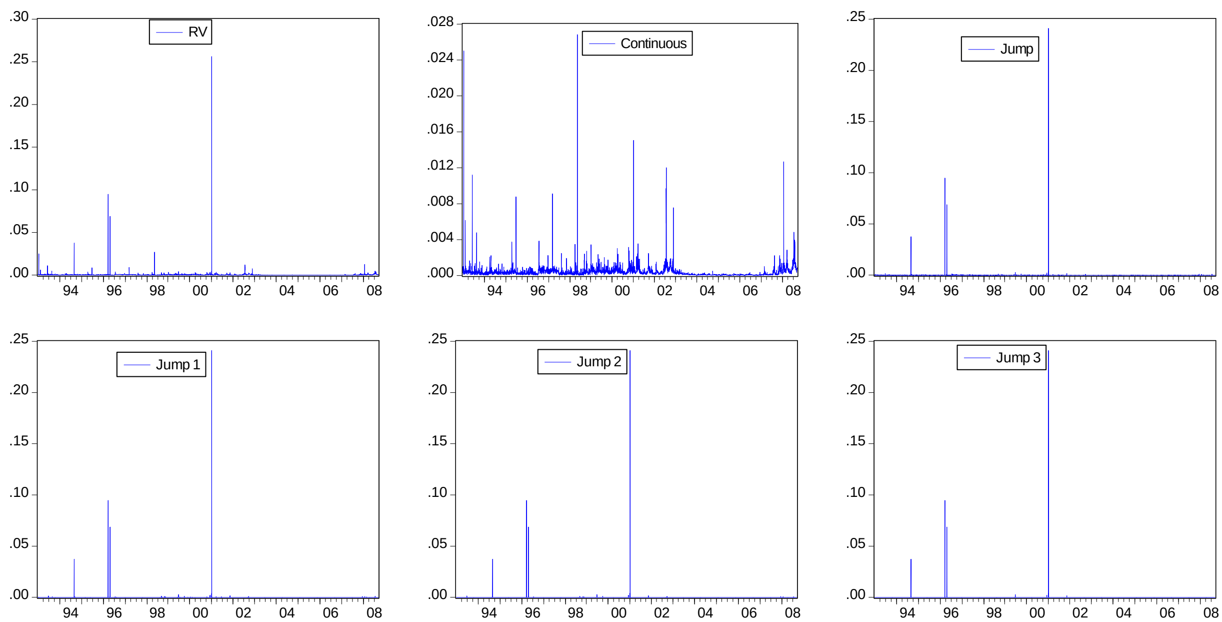
* See Figure 2 for details about the jump truncation levels. The above panels plot daily realized volatility, realized measures of the variation of continuous, jump and large jump components at truncation levels $\gamma = 1, 2, 3$, which are shortly referred to as jump 1, jump 2 and jump 3 for the period 1993-2008. The realized measures of variations are calculated as discussed in Section 3 and 4.

Figure 3B: Daily Realized Volatility (RV) and Realized Variation of Continuous, Jump and Truncated Jump Components of (Log) Stock Prices for Truncation Levels $\gamma = 1, 2, 3$ *

Panel C: Bank of America



Panel D: Citigroup



* See notes in Figure 3A.

Table 1: Percentage of Days Identified as Having Jumps Using Daily Statistics *

Stock Name	*Panel A: Sample Period 1993-2000 ($T \simeq 2000$)*						*Panel B: Sample Period 2001-2008 ($T \simeq 1900$)*					
Significant Level α	0.1	0.05	0.01	0.005	0.001	0.0001	0.1	0.05	0.01	0.005	0.001	0.0001
Alcoa	88	80.9	62.1	52	26.3	8.7	49.2	39.3	21.4	16.5	9.7	4.3
American Express	82.9	76.2	56.4	46.6	19.3	4.6	47.4	36.8	20.2	14.7	8.0	4.0
Bank of America	81.8	75.6	56.7	46.4	21.3	5.0	45.1	34.1	20.1	15.7	8.8	3.1
Citigroup	86.3	80.5	63.3	51.9	23.3	4.9	43.6	32.9	17.9	14.6	7.1	2.6
Caterpillar	87.2	81.5	61.8	51	25.9	7.3	46	35.3	19.9	16.3	9.5	4.3
Dupont	83.8	76.5	57.2	48.3	24.2	5.6	49.5	38.8	21.8	17.1	9.5	3.9
Walt Disney	89.3	83.9	65.9	56.0	27.3	5.3	55.6	43.9	23.9	17.6	10.1	3.9
General Electric	79.6	73.5	54.5	45.5	22.3	4.5	49.2	39.3	21.8	16.2	9.4	3.9
GM	88.1	83.1	65.4	54	25.4	6.2	51.8	40.4	22.8	17.8	10.5	4.7
Home Depot	87.7	81	62.1	51.4	24.6	5.1	49.5	38.5	22.1	16.8	10	4.3
IBM	73.8	65	47.3	39.6	19.2	5.9	39.9	30.1	15.1	11.7	5.9	2.8
Intel	69.2	58.9	39.5	33.0	18.0	6.3	51.7	41.4	23.6	18.7	11.3	4.7
Johnson & Johnson	86.7	81.2	62.8	52.5	25.2	5.7	47.5	37.7	22.1	18.0	10.9	4.6
JPM	79.5	73.2	55.7	47.6	21.4	5.0	47.9	35.9	20.8	16.1	9.0	3.3
Coca Cola	86.4	80.8	63.3	54.2	23.9	4.8	52.5	41.9	23.3	18.5	10.2	4.6
McDonald's	90.5	85	66.1	55.9	25.8	4.9	51.3	40.8	24.6	19.8	11.5	4.8
3M	85.7	78.8	59.2	49.9	25.6	6.9	43.1	33.1	18.8	14.2	7.9	3.6
Microsoft	68.5	58.7	38.6	30.5	16.4	7.0	56.3	44.8	25.7	21.5	11.1	4.4
Pfizer	82.6	75.4	56.6	49.1	26.3	6.5	50	40	23.5	17.7	9.4	4.1
Procter & Gamble	80.1	72.4	55.6	46.4	25.5	6.4	46.9	35.6	18.5	14.4	7.2	2.8
AT & T	89.3	83.3	65.8	54.7	23.1	4.4	58.8	48.4	29	22.8	13.8	6.1
United Tech.Corp.	84.2	77.1	54.3	43.9	22.8	8.2	46.3	36.3	20.5	16.0	9.1	3.6
Verizon	81.5	67.7	46	39.5	24.2	8.1	51.4	40.9	24.5	19.4	11.2	5.0
Walmart	86.7	81.5	59.8	46.9	15.5	5.1	44.7	34.3	18.7	14.0	7.4	2.6
ExxonMobil	61.3	49.8	32.8	26.2	17	5.2	44.2	33.6	17.5	12.9	6.2	2.9
Average	82.4	75.3	56.4	46.9	22.8	5.9	48.8	38.2	21.5	16.8	9.4	4.0

* See notes to Figure 1. Entries denote the percentage of days identified as having jumps based on the calculation of daily statistics. Statistics are the adjusted ratio jump statistics of Barndorff-Nielsen and Shephard (2006) and Huang and Tauchen (2005), as discussed in Section 3. Test results are summarized in Panel A for the sample period from 1993-2000 and for the sample period 2001-2008 in Panel B. These sample periods have approximately 2000 and 1900 daily statistics, respectively. Statistics are reported for six different significance levels, $\alpha = 0.1, 0.05, 0.01, 0.005, 0.001, 0.0001$.

Table 2: Daily Realized Variation: Ratio of Continuous to Total Variation *

Stock Name	*Panel A: Sample Period 1993-2000 ($T \simeq 2000$)*						*Panel B: Sample Period 2001-2008 ($T \simeq 1900$)*					
Significant Level α	0.1	0.05	0.01	0.005	0.001	0.0001	0.1	0.05	0.01	0.005	0.001	0.0001
Alcoa	58.8	60.5	67.4	71.6	85.2	94.8	86.4	88.2	92.3	93.6	95.9	97.9
American Express	60.5	61.9	69.5	73.8	89.2	97.4	87.3	89.1	92.9	94.3	96.5	98.1
Bank of America	62.8	64.0	70.3	74.7	87.9	97.1	88.4	90.2	93.2	94.4	96.5	98.5
Citigroup	58.5	59.7	65.9	70.8	86.3	97.0	89.0	90.7	94.0	94.9	97.2	98.8
Caterpillar	60.0	61.4	68.0	72.5	85.3	95.5	87.7	89.5	92.9	93.8	96.0	98.0
Dupont	63.2	64.6	70.9	74.2	86.7	96.8	86.7	88.5	92.4	93.6	96.0	98.2
Walt Disney	58.0	59.2	65.1	69.3	84.2	96.6	84.7	86.8	91.4	93.1	95.8	98.2
General Electric	66.0	67.2	72.9	76.2	87.7	97.5	86.8	88.4	92.3	93.9	96.2	98.2
GM	57.5	58.6	64.8	69.8	85.3	96.1	85.4	87.5	91.7	93.1	95.6	97.8
Home Depot	60.0	61.4	67.7	72.1	85.9	96.9	86.7	88.5	92.2	93.7	95.9	98.0
IBM	69.9	71.7	76.9	79.6	89.3	96.5	90.2	91.8	94.9	95.8	97.7	98.7
Intel	76.0	78.0	83.2	85.3	91.1	96.5	86.1	87.8	91.7	93.1	95.4	97.8
Johnson &Johnson	60.8	61.9	67.7	71.7	85.8	96.6	87.0	88.7	92.2	93.3	95.6	97.9
JPM	64.0	65.3	70.8	74.4	88.0	97.2	87.4	89.4	92.8	94.1	96.3	98.4
Coca Cola	60.1	61.2	66.9	70.6	86.3	97.2	86.0	87.7	91.8	93.2	95.8	97.9
McDonald's	56.1	57.2	64.0	68.5	84.9	96.9	85.1	86.9	90.8	92.2	95.0	97.7
3M	61.0	62.5	69.1	72.8	85.6	95.9	89.0	90.6	93.8	94.9	96.8	98.4
Microsoft	76.8	78.6	83.9	86.5	92.0	96.3	84.9	86.8	91.0	92.2	95.5	98.0
Pfizer	64.5	65.9	71.8	74.6	85.3	96.1	86.2	87.9	91.7	93.3	96.1	98.1
Procter &Gamble	66.1	67.6	72.2	75.5	85.8	96.3	88.1	89.9	93.7	94.8	97.0	98.6
AT &T	56.2	57.4	64.0	68.9	86.5	97.2	82.3	84.1	88.9	90.7	93.9	97.0
United Tech.Corp.	62.9	64.5	72.4	77.0	87.6	95.2	87.8	89.5	93.0	94.2	96.4	98.4
Verizon	70.6	73.2	79.0	81.2	88.0	95.6	85.6	87.4	91.3	92.6	95.3	97.6
Walmart	55.6	56.8	66.2	72.6	90.9	96.6	88.5	90.2	93.7	94.9	97.0	98.8
ExxonMobil	80.7	82.9	87.3	89.1	92.4	97.4	89.2	90.9	94.3	95.4	97.5	98.7

	Panel C: Sample Period 1993-2008 ($T \simeq 3900$)					
Significant Level α	0.1	0.05	0.01	0.005	0.001	0.0001
Alcoa	73.3	75.0	80.5	83.1	90.8	96.4
American Express	73.6	75.2	80.9	83.8	92.8	97.7
Bank of America	75.3	76.8	81.5	84.3	92.1	97.8
Citigroup	73.4	74.9	79.7	82.6	91.7	97.9
Caterpillar	73.8	75.3	80.4	83.1	90.6	96.7
Dupont	74.6	76.3	81.4	83.7	91.2	97.5
Walt Disney	71.0	72.7	78.0	80.9	89.8	97.4
General Electric	76.2	77.5	82.4	84.9	91.8	97.8
GM	71.1	72.7	77.9	81.2	90.3	96.9
Home Depot	73.0	74.7	79.7	82.6	90.8	97.5
IBM	79.8	81.5	85.7	87.5	93.4	97.6
Intel	81.0	82.8	87.4	89.1	93.2	97.1
Johnson &Johnson	73.6	75.0	79.7	82.3	90.6	97.2
JPM	75.4	77.0	81.6	84.0	92.1	97.8
Coca Cola	72.7	74.1	79.1	81.6	91.0	97.5
McDonald's	69.6	71.0	76.5	79.5	89.6	97.3
3M	74.0	75.6	80.6	83.1	90.8	97.1
Microsoft	80.6	82.4	87.2	89.2	93.7	97.1
Pfizer	75.1	76.7	81.5	83.7	90.6	97.1
Procter &Gamble	76.8	78.5	82.7	84.9	91.3	97.4
AT &T	68.8	70.4	76.1	79.5	90.1	97.1
United Tech.Corp.	76.3	77.9	83.5	86.2	92.3	96.9
Verizon	84.7	86.5	90.5	91.9	94.9	97.5
Walmart	71.7	73.1	79.6	83.5	93.9	97.7
ExxonMobil	88.1	89.9	93.4	94.6	96.9	98.5

* The entries in the table denote the average percentage of daily variation of the continuous component relative to daily realized variance for the sample periods 1993-2000, 2001-2008 and 1993-2008. The realized measure of variation of the continuous component is calculated as discussed in Section 3. Entries are caculcated accross 6 different significant levels, $\alpha = 0.1, 0.05, 0.01, 0.005, 0.001, 0.0001$.

Table 3: Daily Realized Variation: Ratio of Jump to Total Variation *

Stock Name	*Panel A: Sample Period 1993-2000 ($T \simeq 2000$)*						*Panel B: Sample Period 2001-2008 ($T \simeq 1900$)*					
Significant Level α	0.1	0.05	0.01	0.005	0.001	0.0001	0.1	0.05	0.01	0.005	0.001	0.0001
Alcoa	41.2	39.5	32.6	28.4	14.8	5.2	13.6	11.8	7.7	6.4	4.1	2.1
American Express	39.5	38.1	30.5	26.2	10.8	2.6	12.7	10.9	7.1	5.7	3.5	1.9
Bank of America	37.2	36.0	29.7	25.3	12.1	2.9	11.6	9.8	6.8	5.6	3.5	1.5
Citigroup	41.5	40.3	34.1	29.2	13.7	3.0	11.0	9.3	6.0	5.1	2.8	1.2
Caterpillar	40.0	38.6	32.0	27.5	14.7	4.5	12.3	10.5	7.1	6.2	4.0	2.0
Dupont	36.8	35.4	29.1	25.8	13.3	3.2	13.3	11.5	7.6	6.4	4.0	1.8
Walt Disney	42.0	40.8	34.9	30.7	15.8	3.4	15.3	13.2	8.6	6.9	4.2	1.8
General Electric	34.0	32.8	27.1	23.8	12.3	2.5	13.2	11.6	7.7	6.1	3.8	1.8
GM	42.5	41.4	35.2	30.2	14.7	3.9	14.6	12.5	8.3	6.9	4.4	2.2
Home Depot	40.0	38.6	32.3	27.9	14.1	3.1	13.3	11.5	7.8	6.3	4.1	2.0
IBM	30.1	28.3	23.1	20.4	10.7	3.5	9.8	8.2	5.1	4.2	2.3	1.3
Intel	24.0	22.0	16.8	14.7	8.9	3.5	13.9	12.2	8.3	6.9	4.6	2.2
Johnson &Johnson	39.2	38.1	32.3	28.3	14.2	3.4	13.0	11.3	7.8	6.7	4.4	2.1
JPM	36.0	34.7	29.2	25.6	12.0	2.8	12.6	10.6	7.2	5.9	3.7	1.6
Coca Cola	39.9	38.8	33.1	29.4	13.7	2.8	14.0	12.3	8.2	6.8	4.2	2.1
McDonald's	43.9	42.8	36.0	31.5	15.1	3.1	14.9	13.1	9.2	7.8	5.0	2.3
3M	39.0	37.5	30.9	27.2	14.4	4.1	11.0	9.4	6.2	5.1	3.2	1.6
Microsoft	23.2	21.4	16.1	13.5	8.0	3.7	15.1	13.2	9.0	7.8	4.5	2.0
Pfizer	35.5	34.1	28.2	25.4	14.7	3.9	13.8	12.1	8.3	6.7	3.9	1.9
Procter &Gamble	33.9	32.4	27.8	24.5	14.2	3.7	11.9	10.1	6.3	5.2	3.0	1.4
AT &T	43.8	42.6	36.0	31.1	13.5	2.8	17.7	15.9	11.1	9.3	6.1	3.0
United Tech.Corp.	37.1	35.5	27.6	23.0	12.4	4.8	12.2	10.5	7.0	5.8	3.6	1.6
Verizon	29.4	26.8	21.0	18.8	12.0	4.4	14.4	12.6	8.7	7.4	4.7	2.4
Walmart	44.4	43.2	33.8	27.4	9.1	3.4	11.5	9.8	6.3	5.1	3.0	1.2
ExxonMobil	19.3	17.1	12.7	10.9	7.6	2.6	10.8	9.1	5.7	4.6	2.5	1.3
Average	36.5	35.1	28.9	25.1	12.7	3.5	13.1	11.3	7.6	6.3	3.9	1.9

Panel C: Sample Period 1993-2008 ($T \simeq 3900$)

Significant Level α	0.1	0.05	0.01	0.005	0.001	0.0001
Alcoa	26.7	25.0	19.5	16.9	9.2	3.6
American Express	26.4	24.8	19.1	16.2	7.2	2.3
Bank of America	24.7	23.2	18.5	15.7	7.9	2.2
Citigroup	26.6	25.1	20.3	17.4	8.3	2.1
Caterpillar	26.2	24.7	19.6	16.9	9.4	3.3
Dupont	25.4	23.7	18.6	16.3	8.8	2.5
Walt Disney	29.0	27.3	22.0	19.1	10.2	2.6
General Electric	23.8	22.5	17.6	15.1	8.2	2.2
GM	28.9	27.3	22.1	18.8	9.7	3.1
Home Depot	27.0	25.3	20.3	17.4	9.2	2.5
IBM	20.2	18.5	14.3	12.5	6.6	2.4
Intel	19.0	17.2	12.6	10.9	6.8	2.9
Johnson &Johnson	26.4	25.0	20.3	17.7	9.4	2.8
JPM	24.6	23.0	18.4	16.0	7.9	2.2
Coca Cola	27.3	25.9	20.9	18.4	9.0	2.5
McDonald's	30.4	29.0	23.5	20.5	10.4	2.7
3M	26.0	24.4	19.4	16.9	9.2	2.9
Microsoft	19.4	17.6	12.8	10.8	6.3	2.9
Pfizer	24.9	23.3	18.5	16.3	9.4	2.9
Procter &Gamble	23.2	21.5	17.3	15.1	8.7	2.6
AT &T	31.2	29.6	23.9	20.5	9.9	2.9
United Tech.Corp.	23.7	22.1	16.5	13.8	7.7	3.1
Verizon	15.3	13.5	9.5	8.1	5.1	2.5
Walmart	28.3	26.9	20.4	16.5	6.1	2.3
ExxonMobil	11.9	10.1	6.6	5.4	3.1	1.5
Average	24.7	23.1	18.1	15.6	8.1	2.6

* See notes to Figure 2. The entries in the table denote the average percentage of daily variation of the jump component relative to daily realized variance for the sample periods 1993-2000, 2001-2008 and 1993-2008. The realized measure of variation of the jump component is calculated as discussed in Section 3. In addition to frequency of jumps, realized measures of variations also take the magintude of jumps into account. Entries are caculcated accross 6 different significant levels, $\alpha = 0.1, 0.05, 0.01, 0.005, 0.001, 0.0001$.

Table 4A: Daily Realized Variation: Ratio of Truncation Jump to Total Variation, Jump Truncation Level $\gamma = 1$ *

Stock Name	*Panel A: Sample Period 1993-2000 ($T \simeq 2000$)*						*Panel B: Sample Period 2001-2008 ($T \simeq 1900$)*					
Significant Level α	0.1	0.05	0.01	0.005	0.001	0.0001	0.1	0.05	0.01	0.005	0.001	0.0001
Alcoa	1.8	1.7	1.2	1.1	0.9	0.6	1.3	1.2	0.9	0.8	0.5	0.4
American Express	1.5	1.3	1.0	0.8	0.6	0.4	1.2	1.1	0.8	0.6	0.5	0.3
Bank of America	3.0	2.7	1.8	1.5	1.0	0.7	1.0	0.9	0.8	0.6	0.4	0.2
Citigroup	2.1	1.9	1.2	1.0	0.6	0.4	0.8	0.7	0.5	0.4	0.3	0.1
Caterpillar	2.3	2.2	1.6	1.5	1.0	0.6	0.7	0.6	0.5	0.5	0.3	0.2
Dupont	2.1	1.9	1.2	1.0	0.7	0.3	0.9	0.9	0.6	0.5	0.4	0.2
Walt Disney	2.1	1.8	1.1	0.9	0.6	0.4	1.6	1.4	1.0	0.9	0.6	0.3
General Electric	1.5	1.4	0.8	0.7	0.4	0.2	1.3	1.2	0.9	0.6	0.3	0.2
GM	1.5	1.4	1.0	0.8	0.6	0.4	1.3	1.2	0.8	0.7	0.5	0.2
Home Depot	1.9	1.7	1.3	1.1	0.6	0.3	0.7	0.6	0.5	0.3	0.2	0.1
IBM	2.3	2.1	1.7	1.6	1.0	0.7	0.5	0.5	0.4	0.4	0.2	0.1
Intel	2.3	2.1	1.6	1.2	0.8	0.5	0.7	0.7	0.4	0.4	0.3	0.2
Johnson &Johnson	2.2	2.0	1.5	1.3	0.9	0.6	0.6	0.6	0.4	0.3	0.2	0.1
JPM	1.3	1.1	0.7	0.6	0.3	0.2	1.7	1.5	1.0	0.8	0.6	0.3
Coca Cola	2.3	2.1	1.3	1.2	0.8	0.5	0.8	0.8	0.6	0.5	0.4	0.3
McDonald's	1.8	1.6	1.2	0.9	0.7	0.4	1.0	1.0	0.7	0.6	0.4	0.2
3M	2.1	2.0	1.3	1.1	0.7	0.5	0.6	0.6	0.4	0.4	0.4	0.3
Microsoft	2.9	2.7	2.0	1.7	1.0	0.5	0.6	0.6	0.5	0.4	0.2	0.1
Pfizer	2.0	1.9	1.3	1.1	0.8	0.5	0.7	0.6	0.5	0.4	0.3	0.2
Procter &Gamble	2.4	2.2	1.7	1.4	0.9	0.5	0.8	0.7	0.5	0.4	0.3	0.2
AT &T	2.3	2.2	1.6	1.3	0.9	0.7	1.7	1.5	1.2	1.1	0.7	0.4
United Tech.Corp.	3.2	2.9	2.1	1.9	1.2	0.6	1.3	1.1	0.8	0.7	0.5	0.2
Verizon	6.9	6.3	5.0	4.4	2.7	1.1	1.5	1.3	1.0	0.9	0.6	0.4
Walmart	2.7	2.4	1.4	1.2	0.9	0.6	0.6	0.6	0.4	0.4	0.3	0.1
ExxonMobil	2.1	1.8	1.3	1.0	0.6	0.4	1.1	0.9	0.7	0.6	0.4	0.2
Average	2.3	2.1	1.5	1.3	0.8	0.5	1.0	0.9	0.7	0.6	0.4	0.2

Panel C: Sample Period 1993-2008 ($T \simeq 3900$)

Significant Level α	0.1	0.05	0.01	0.005	0.001	0.0001
Alcoa	1.6	1.4	1.0	0.9	0.7	0.5
American Express	1.3	1.2	0.9	0.7	0.5	0.4
Bank of America	2.0	1.9	1.3	1.1	0.7	0.5
Citigroup	1.4	1.3	0.8	0.7	0.5	0.3
Caterpillar	1.5	1.4	1.1	1.0	0.7	0.4
Dupont	1.5	1.4	0.9	0.8	0.5	0.2
Walt Disney	1.8	1.6	1.1	0.9	0.6	0.4
General Electric	1.4	1.3	0.8	0.7	0.4	0.2
GM	1.4	1.3	0.9	0.8	0.5	0.3
Home Depot	1.3	1.2	0.9	0.7	0.4	0.2
IBM	1.4	1.3	1.0	1.0	0.6	0.4
Intel	1.5	1.4	1.0	0.8	0.6	0.3
Johnson &Johnson	1.4	1.3	1.0	0.8	0.6	0.4
JPM	1.5	1.3	0.9	0.7	0.5	0.3
Coca Cola	1.6	1.5	1.0	0.8	0.6	0.4
McDonald's	1.4	1.3	1.0	0.8	0.5	0.3
3M	1.4	1.3	0.9	0.8	0.6	0.4
Microsoft	1.8	1.7	1.3	1.1	0.6	0.3
Pfizer	1.3	1.2	0.9	0.8	0.6	0.4
Procter &Gamble	1.6	1.5	1.1	0.9	0.6	0.3
AT &T	2.0	1.9	1.4	1.2	0.8	0.5
United Tech.Corp.	2.1	2.0	1.4	1.3	0.8	0.4
Verizon	1.8	1.6	1.2	1.1	0.7	0.4
Walmart	1.7	1.5	0.9	0.8	0.6	0.3
ExxonMobil	1.2	1.1	0.7	0.6	0.4	0.2
Average	1.6	1.4	1.0	0.9	0.6	0.3

* See notes to Figure 2. Entries in the table denote the average percentage of daily variation due to jumps constructed using truncation level $\gamma = 1$, relative to the daily realized variance, for the sample periods 1993-2000, 2001-2008 and 1993-2008. The realized measure of variation of the jump component is calculated as discussed in Section 3. Entries are caculcated accross 6 different significance levels (α =0.1, 0.05, 0.01, 0.005, 0.001, 0.0001).

Table 4B: Daily Realized Variation: Ratio of Truncation Jump to Total Variation, Jump Truncation Level $\gamma = 2$ *

Stock Name	*Panel A: Sample Period 1993-2000 ($T \simeq 2000$)*						*Panel B: Sample Period 2001-2008 ($T \simeq 1900$)*					
Significant Level α	0.1	0.05	0.01	0.005	0.001	0.0001	0.1	0.05	0.01	0.005	0.001	0.0001
Alcoa	0.8	0.8	0.6	0.5	0.5	0.3	0.5	0.5	0.4	0.4	0.2	0.1
American Express	0.8	0.7	0.6	0.5	0.3	0.3	0.5	0.4	0.3	0.2	0.2	0.1
Bank of America	0.9	0.9	0.5	0.4	0.3	0.2	0.4	0.4	0.3	0.3	0.2	0.1
Citigroup	1.0	1.0	0.7	0.5	0.4	0.3	0.3	0.3	0.2	0.2	0.2	0.1
Caterpillar	1.0	1.0	0.8	0.8	0.5	0.3	0.3	0.3	0.3	0.3	0.2	0.2
Dupont	0.9	0.8	0.4	0.3	0.2	0.1	0.4	0.4	0.2	0.2	0.1	0.0
Walt Disney	1.0	0.9	0.4	0.3	0.3	0.2	0.5	0.5	0.3	0.3	0.2	0.1
General Electric	0.7	0.7	0.4	0.4	0.2	0.1	0.6	0.6	0.5	0.3	0.1	0.1
GM	0.7	0.7	0.4	0.3	0.2	0.2	0.7	0.6	0.4	0.4	0.3	0.1
Home Depot	1.0	0.9	0.7	0.6	0.3	0.2	0.2	0.2	0.2	0.1	0.0	0.0
IBM	1.0	0.9	0.8	0.7	0.5	0.3	0.2	0.2	0.2	0.2	0.1	0.1
Intel	0.9	0.9	0.7	0.6	0.4	0.2	0.2	0.2	0.1	0.1	0.1	0.0
Johnson &Johnson	0.9	0.8	0.6	0.5	0.4	0.3	0.3	0.3	0.2	0.2	0.2	0.1
JPM	0.4	0.4	0.2	0.2	0.1	0.0	0.7	0.7	0.5	0.4	0.3	0.2
Coca Cola	0.9	0.9	0.4	0.4	0.2	0.2	0.4	0.4	0.2	0.2	0.2	0.2
McDonald's	0.8	0.7	0.5	0.4	0.3	0.3	0.5	0.5	0.4	0.2	0.2	0.0
3M	1.0	0.9	0.5	0.4	0.3	0.2	0.2	0.1	0.1	0.1	0.1	0.1
Microsoft	1.1	1.0	0.8	0.6	0.3	0.1	0.2	0.2	0.2	0.2	0.1	0.0
Pfizer	0.8	0.8	0.6	0.6	0.4	0.3	0.3	0.3	0.2	0.2	0.1	0.1
Procter &Gamble	1.1	1.0	0.8	0.6	0.4	0.3	0.3	0.2	0.2	0.2	0.1	0.1
AT &T	1.1	1.0	0.7	0.6	0.4	0.4	0.7	0.6	0.6	0.5	0.3	0.1
United Tech.Corp.	1.1	1.0	0.7	0.6	0.5	0.3	0.4	0.4	0.3	0.3	0.2	0.1
Verizon	2.8	2.6	2.2	1.9	0.7	0.2	0.5	0.5	0.4	0.4	0.3	0.2
Walmart	1.2	1.1	0.6	0.5	0.5	0.4	0.2	0.2	0.2	0.2	0.1	0.0
ExxonMobil	0.8	0.6	0.5	0.4	0.4	0.2	0.5	0.5	0.4	0.3	0.2	0.1
Average	1.0	0.9	0.6	0.5	0.4	0.2	0.4	0.4	0.3	0.3	0.2	0.1

Panel C: Sample Period 1993-2008 ($T \simeq 3900$)

Significant Level α	0.1	0.05	0.01	0.005	0.001	0.0001
Alcoa	0.6	0.6	0.5	0.4	0.3	0.2
American Express	0.7	0.6	0.4	0.3	0.3	0.2
Bank of America	0.7	0.6	0.4	0.4	0.3	0.2
Citigroup	0.7	0.7	0.5	0.4	0.3	0.2
Caterpillar	0.7	0.6	0.5	0.5	0.4	0.3
Dupont	0.7	0.6	0.3	0.3	0.2	0.1
Walt Disney	0.8	0.7	0.4	0.3	0.2	0.2
General Electric	0.7	0.6	0.5	0.3	0.1	0.1
GM	0.7	0.7	0.4	0.3	0.2	0.2
Home Depot	0.6	0.6	0.4	0.3	0.2	0.1
IBM	0.6	0.6	0.5	0.5	0.3	0.2
Intel	0.6	0.5	0.4	0.3	0.2	0.1
Johnson &Johnson	0.6	0.6	0.4	0.4	0.3	0.2
JPM	0.6	0.5	0.4	0.3	0.2	0.1
Coca Cola	0.6	0.6	0.3	0.3	0.2	0.2
McDonald's	0.7	0.6	0.5	0.3	0.3	0.2
3M	0.6	0.6	0.3	0.3	0.2	0.2
Microsoft	0.6	0.6	0.5	0.4	0.2	0.0
Pfizer	0.6	0.5	0.4	0.4	0.3	0.2
Procter &Gamble	0.7	0.6	0.5	0.4	0.3	0.2
AT &T	0.9	0.8	0.6	0.6	0.4	0.3
United Tech.Corp.	0.7	0.7	0.5	0.4	0.3	0.2
Verizon	0.7	0.6	0.5	0.4	0.3	0.2
Walmart	0.7	0.6	0.4	0.3	0.3	0.2
ExxonMobil	0.6	0.5	0.4	0.3	0.2	0.1
Average	0.7	0.6	0.4	0.4	0.3	0.2

* See notes to Table 4A.

Table 4C: Daily Realized Variation: Ratio of Truncation Jump to Total Variation, Jump Truncation Level $\gamma = 3$ *

Stock Name	*Panel A: Sample Period 1993-2000 ($T \simeq 2000$)*						*Panel B: Sample Period 2001-2008 ($T \simeq 1900$)*					
Significant Level α	0.1	0.05	0.01	0.005	0.001	0.0001	0.1	0.05	0.01	0.005	0.001	0.0001
Alcoa	0.3	0.3	0.2	0.2	0.1	0.1	0.2	0.2	0.2	0.1	0.1	0.0
American Express	0.3	0.3	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.1	0.1	0.0
Bank of America	0.5	0.5	0.3	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.1	0.1
Citigroup	0.5	0.5	0.4	0.3	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.0
Caterpillar	0.5	0.5	0.4	0.4	0.3	0.2	0.1	0.1	0.1	0.1	0.1	0.1
Dupont	0.4	0.4	0.2	0.1	0.1	0.0	0.1	0.1	0.0	0.0	0.0	0.0
Walt Disney	0.5	0.5	0.2	0.2	0.2	0.2	0.3	0.3	0.2	0.2	0.1	0.0
General Electric	0.3	0.3	0.2	0.1	0.0	0.0	0.3	0.3	0.3	0.2	0.0	0.0
GM	0.3	0.3	0.2	0.1	0.1	0.1	0.2	0.2	0.1	0.1	0.1	0.0
Home Depot	0.4	0.4	0.3	0.3	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0
IBM	0.3	0.3	0.3	0.2	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Intel	0.4	0.4	0.3	0.2	0.1	0.1	0.1	0.1	0.0	0.0	0.0	0.0
Johnson &Johnson	0.4	0.4	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.1	0.1	0.0
JPM	0.2	0.2	0.1	0.1	0.0	0.0	0.2	0.2	0.2	0.2	0.1	0.1
Coca Cola	0.5	0.5	0.2	0.2	0.1	0.1	0.2	0.2	0.1	0.1	0.1	0.1
McDonald's	0.4	0.3	0.3	0.2	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.0
3M	0.3	0.3	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.0	0.0	0.0
Microsoft	0.4	0.4	0.2	0.2	0.1	0.0	0.1	0.1	0.1	0.1	0.0	0.0
Pfizer	0.3	0.3	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.0	0.0
Procter &Gamble	0.3	0.3	0.2	0.2	0.2	0.1	0.2	0.2	0.1	0.1	0.1	0.1
AT &T	0.6	0.5	0.3	0.3	0.2	0.2	0.2	0.2	0.2	0.1	0.1	0.0
United Tech.Corp.	0.2	0.2	0.2	0.2	0.2	0.1	0.2	0.2	0.1	0.1	0.1	0.0
Verizon	0.7	0.7	0.7	0.7	0.0	0.0	0.2	0.1	0.1	0.1	0.1	0.0
Walmart	0.5	0.5	0.3	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.0	0.0
ExxonMobil	0.1	0.1	0.1	0.0	0.0	0.0	0.2	0.2	0.2	0.2	0.1	0.0
Average	0.4	0.4	0.3	0.2	0.1	0.1	0.2	0.2	0.1	0.1	0.1	0.0

	Panel C: Sample Period 1993-2008 ($T \simeq 3900$)					
Significant Level α	0.1	0.05	0.01	0.005	0.001	0.0001
Alcoa	0.3	0.3	0.2	0.2	0.1	0.1
American Express	0.3	0.2	0.2	0.1	0.1	0.1
Bank of America	0.3	0.3	0.2	0.2	0.1	0.1
Citigroup	0.3	0.3	0.2	0.2	0.2	0.1
Caterpillar	0.3	0.3	0.3	0.2	0.2	0.1
Dupont	0.3	0.3	0.1	0.0	0.0	0.0
Walt Disney	0.4	0.4	0.2	0.2	0.1	0.1
General Electric	0.3	0.3	0.2	0.2	0.0	0.0
GM	0.3	0.3	0.1	0.1	0.1	0.1
Home Depot	0.2	0.2	0.2	0.1	0.0	0.0
IBM	0.2	0.2	0.2	0.2	0.1	0.1
Intel	0.2	0.2	0.2	0.1	0.1	0.0
Johnson &Johnson	0.3	0.3	0.2	0.2	0.2	0.1
JPM	0.2	0.2	0.1	0.1	0.0	0.0
Coca Cola	0.3	0.3	0.1	0.1	0.1	0.1
McDonald's	0.3	0.3	0.2	0.1	0.1	0.1
3M	0.2	0.2	0.1	0.1	0.1	0.1
Microsoft	0.2	0.2	0.2	0.1	0.1	0.0
Pfizer	0.2	0.2	0.2	0.1	0.1	0.1
Procter &Gamble	0.2	0.2	0.2	0.2	0.1	0.1
AT &T	0.4	0.4	0.2	0.2	0.2	0.1
United Tech.Corp.	0.2	0.2	0.2	0.2	0.1	0.1
Verizon	0.2	0.2	0.1	0.1	0.1	0.0
Walmart	0.3	0.3	0.2	0.2	0.1	0.1
ExxonMobil	0.2	0.2	0.2	0.2	0.1	0.0
Average	0.3	0.3	0.2	0.1	0.1	0.1

* See notes to Table 4A.

Table 5A: R^2 of Linear Models for Individual Stocks*

Symbol	h =1 (Daily forecast)			h=5 (Weekly forecast)			h=22 (Monthly forecast)		
	Model 1 RV	Model 2 RV-J	Model 3 RV-CJ	Model 1 RV	Model 2 RV-J	Model 3 RV-CJ	Model 1 RV	Model 2 RV-J	Model 3 RV-CJ
Alcoa	0.0000	0.0000	0.0001	0.0000	0.0000	0.0001	0.0000	0.0000	0.0001
American Express	0.0115	0.0133	0.0436	0.0100	0.0100	0.0457	0.0043	0.0044	0.0070
Bank of America	0.1085	0.1241	0.1375	0.0799	0.0905	0.0999	0.0262	0.0282	0.0312
Citigroup	0.0006	0.0025	0.0039	0.0012	0.0017	0.0027	0.0022	0.0022	0.0040
Caterpillar	0.0012	0.0012	0.0018	0.0011	0.0012	0.0016	0.0099	0.0099	0.0144
Dupont	0.0033	0.0033	0.0045	0.0025	0.0025	0.0045	0.0014	0.0014	0.0039
Walt Disney	0.0072	0.0073	0.0088	0.0049	0.0049	0.0071	0.0070	0.0070	0.0079
General Electric	0.0249	0.0292	0.0292	0.0194	0.0197	0.0202	0.0419	0.0419	0.0432
GM	0.0203	0.0203	0.0203	0.0144	0.0144	0.0146	0.0024	0.0028	0.0033
Home Depot	0.0412	0.0493	0.0559	0.0338	0.0352	0.0425	0.0186	0.0196	0.0236
IBM	0.0432	0.0439	0.0447	0.0320	0.0323	0.0333	0.0311	0.0312	0.0314
Intel	0.0716	0.0766	0.0833	0.0603	0.0612	0.0674	0.0430	0.0438	0.0470
Johnson &Johnson	0.0006	0.0007	0.0024	0.0005	0.0010	0.0025	0.0003	0.0004	0.0025
JPM	**0.3740**	**0.3766**	**0.3799**	**0.1898**	**0.1960**	**0.2024**	**0.0977**	**0.0994**	**0.1018**
Coca Cola	0.0381	0.0387	0.0496	0.0315	0.0366	0.0449	0.0201	0.0206	0.0221
McDonald's	0.0046	0.0055	0.0097	0.0039	0.0041	0.0075	0.0031	0.0032	0.0051
3M	0.0152	0.0153	0.0197	0.0126	0.0127	0.0168	0.0108	0.0108	0.0120
Microsoft	0.0484	0.0486	0.0520	0.0373	0.0373	0.0425	0.0231	0.0246	0.0293
Pfizer	0.0972	0.1069	0.1123	0.0733	0.0759	0.0792	0.0436	0.0436	0.0441
Proctor &Gamble	**0.2345**	**0.2469**	**0.2516**	**0.1358**	**0.1381**	**0.1446**	**0.0975**	**0.0996**	**0.1040**
AT &T	0.0054	0.0055	0.0061	0.0056	0.0063	0.0064	0.0018	0.0019	0.0019
United Tech.Corp.	0.0857	0.0857	0.0864	0.0649	0.0650	0.0670	0.0450	0.0451	0.0467
Verizon	**0.4585**	**0.4687**	**0.4710**	**0.3749**	**0.3784**	**0.3826**	**0.2243**	**0.2254**	**0.2284**
Walmart	0.0857	0.0868	0.0944	0.0797	0.0801	0.0872	0.0726	0.0732	0.0767
ExxonMobil	**0.5117**	**0.5118**	**0.5123**	**0.3398**	**0.3399**	**0.3445**	**0.1403**	**0.1405**	**0.1430**

Table 5B: R^2 of Square Root Models for Individual Stocks*

	h =1 (Daily forecast)			h=5 (Weekly forecast)			h=22 (Monthly forecast)		
Symbol	Model 1 RV	Model 2 RV-J	Model 3 RV-CJ	Model 1 RV	Model 2 RV-J	Model 3 RV-CJ	Model 1 RV	Model 2 RV-J	Model 3 RV-CJ
Alcoa	0.0000	0.0000	0.0003	0.0000	0.0000	0.0001	0.0000	0.0000	0.0004
American Express	**0.4287**	**0.4291**	**0.4388**	**0.3722**	**0.3723**	**0.3837**	**0.2625**	**0.2625**	**0.2658**
Bank of America	**0.4691**	**0.4807**	**0.5049**	**0.3851**	**0.3905**	**0.4101**	**0.2328**	**0.2328**	**0.2456**
Citigroup	0.1527	0.1958	0.2345	0.1264	0.1531	0.1839	0.0993	0.1101	0.1298
Caterpillar	0.1230	0.1260	0.1583	0.0990	0.1028	0.1289	0.0871	0.0872	0.1054
Dupont	0.1711	0.1711	0.1770	0.1352	0.1353	0.1433	0.0978	0.0996	0.1115
Walt Disney	0.2195	0.2195	0.2274	0.1723	0.1732	0.1799	0.1337	0.1342	0.1387
General Electric	0.2865	0.2873	0.2875	0.2142	0.2143	0.2165	0.1809	0.1809	0.1862
GM	0.2191	0.2192	0.2225	0.1638	0.1643	0.1645	0.0709	0.0709	0.0719
HomeDepot	**0.3495**	**0.3541**	**0.3738**	**0.2796**	**0.2812**	**0.2966**	**0.1924**	**0.1931**	**0.2026**
IBM	**0.4042**	**0.4069**	**0.4129**	**0.3072**	**0.3082**	**0.3128**	**0.2389**	**0.2393**	**0.2413**
Intel	**0.4028**	**0.4060**	**0.4212**	**0.3338**	**0.3344**	**0.3452**	**0.2576**	**0.2578**	**0.2670**
Johnson &Johnson	0.2496	0.2505	0.2731	0.2167	0.2223	0.2440	0.1684	0.1705	0.1983
JPM	**0.6033**	**0.6039**	**0.6052**	**0.4612**	**0.4633**	**0.4636**	**0.3101**	**0.3103**	**0.3111**
Coca Cola	0.3909	0.3912	0.4026	0.3396	0.3441	0.3533	0.2580	0.2591	0.2684
McDonald's	0.2284	0.2360	0.2712	0.1896	0.1910	0.2209	0.1573	0.1577	0.1782
3M	0.3380	0.3382	0.3432	0.2619	0.2624	0.2670	0.1936	0.1945	0.1970
Microsoft	**0.4275**	**0.4276**	**0.4304**	**0.3491**	**0.3491**	**0.3527**	**0.2651**	**0.2659**	**0.2693**
Pfizer	0.3679	0.3698	0.3838	0.2875	0.2875	0.2966	0.1984	0.1988	0.2015
Proctor &Gamble	0.5761	0.5780	0.5826	0.4563	0.4565	0.4615	0.3509	0.3509	0.3553
AT &T	0.1635	0.1656	0.1787	0.1405	0.1443	0.1545	0.0867	0.0870	0.0946
United Tech.Corp.	**0.4108**	**0.4110**	**0.4160**	**0.3317**	**0.3317**	**0.3381**	**0.2416**	**0.2416**	**0.2467**
Verizon	**0.6301**	**0.6325**	**0.6424**	**0.5475**	**0.5488**	**0.5583**	**0.4042**	**0.4043**	**0.4060**
Walmart	**0.5339**	**0.5356**	**0.5484**	**0.4958**	**0.4964**	**0.5086**	**0.4452**	**0.4461**	**0.4542**
ExxonMobil	**0.6094**	**0.6095**	**0.6120**	**0.4543**	**0.4544**	**0.4580**	**0.2186**	**0.2186**	**0.2203**

* Entries in the table are adjusted R^2 statistics from linear and square root regression models, as discussed in Section 3 and Andersen, Bollerslev and Diebold (2007). The regressions are designed to forecast realized volatility for 25 individual log stock returns at different forecasting horizons h=1 (daily), h=5 (weekly) and h=22 (monthly), for the sample period 1993-2008. Further details are given in Section 4.

Table 5C: R^2 of Log Models for Individual Stocks*

	h =1 (Daily forecast)			h=5 (Weekly forecast)			h=22 (Monthly forecast)		
Symbol	Model 1 RV	Model 2 RV-J	Model 3 RV-CJ	Model 1 RV	Model 2 RV-J	Model 3 RV-CJ	Model 1 RV	Model 2 RV-J	Model 3 RV-CJ
Citigroup	0.7003	0.7109	0.7052	0.6217	0.6312	0.6321	0.5087	0.5133	0.5116
Dupont	0.5531	0.5541	0.5426	0.4631	0.4635	0.4640	0.3620	0.3620	0.3637
Home Depot	0.6028	0.6062	0.5850	0.5170	0.5192	0.5089	0.3932	0.3938	0.3892
Intel	0.6265	0.6301	0.6202	0.5362	0.5377	0.5329	0.4463	0.4473	0.4438
Microsoft	0.6982	0.6991	0.6921	0.6124	0.6133	0.6087	0.5076	0.5077	0.5077
Verizon	0.6863	0.6894	0.6912	0.5984	0.6033	0.6059	0.4635	0.4639	0.4630
ExxonMobil	0.6348	0.6354	0.6340	0.4964	0.4966	0.4960	0.2632	0.2634	0.2645

Table 5D : R^2 of Linear, Square Root and Log Models- S&P Futures Market Index

Panel A: Linear Model								
h=1 (daily forcast)			h=5 days (weekly forcast)			h=22 days (monthly forcast)		
Model 1	Model 2	Model 3	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
RV	RV-J	RV-CJ	RV	RV-J	RV-CJ	RV	RV-J	RV-CJ
0.3787	0.3790	0.3800	0.3652	0.3654	0.3732	0.3219	0.3222	0.3331
Panel B: Square Root Model								
h=1 (daily forcast)			h=5 days (weekly forcast)			h=22 days (monthly forcast)		
RV	RV-J	RV-CJ	RV	RV-J	RV-CJ	RV	RV-J	RV-CJ
0.4726	0.4728	0.4741	0.4569	0.4574	0.4582	0.4198	0.4202	0.4213
Panel C: Log Model								
h=1 (daily forcast)			h=5 days (weekly forcast)			h=22 days (monthly forcast)		
RV	RV-J	RV-CJ	RV	RV-J	RV-CJ	RV	RV-J	RV-CJ
0.4673	0.4675	0.4592	0.4462	0.4462	0.4418	0.3886	0.3886	0.3890

* See notes in Table 5A. Results for all 25 stocks are reported for log realized volatility regression models in Table 5C, and for log returns based on the S&P Futures market index in Table 5D.