Can Time-Varying Copulas Generate Profit in Portfolio Optimization?

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Abstract

The research of modeling asset return dependence has become an indispensable element of wealth management, particularly after periods of economic downturn. In this paper, we evaluate the performance of time-varying copula-based portfolios and the variables that are associated with the disparity between conditional and unconditional correlations. Using daily data of G-7 countries, our empirical findings suggest that portfolios using time-varying copulas, particularly Clayton-dependence copula, outperform those constructed with Pearson correlations. The above results hold under different weight updating strategies and portfolio rebalancing frequencies. When equity market risk, fixed-income market risks, and currency risk are high, the copula-based dependence statistically differ from the unconditional correlations. Our findings suggest the need of copula-based models in portfolio management, especially during economic recessions.

Keywords: Time-Varying Dependence, Copulas, Portfolio Performance

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1. Introduction

How to adequately assess the comovement structures of asset returns is a key issue to consider when constructing an optimally-investing portfolio.[1] Over the past decade, copula modeling has become a popular alternative to unconditional Pearson correlation for describing data with an asymptotic dependence structure and a non-Gaussian distribution.[2] However, several critical issues related to the applications of copulas emerge: Do portfolios using time-varying copulas outperform those constructed with Pearson correlations? How does the risk-return of copula-based portfolios change throughout in a business cycle? How do market risks affect the deviation from conditional correlations to unconditional ones? Mostly recently, the estimation of parameters in constructing optimal portfolio strategies has become a particularly critical practice for finance academics and professionals in the financial crises. In this paper, we evaluate the performance of time-varying copula-based portfolios and the variables that explain the variation between conditional and unconditional correlations.

This paper extends the existing literature in three ways. First, we thoroughly analyze various copulas in portfolio optimization while considering different trading and economic scenarios. Differing from the regime-switching type in Rodriguez (2007) and Okimoto (2008) or the time-evolving type GARCH model in Patton (2006a), we estimate time-varying copulas with a rolling window based on daily data gathered from a previous year.[3] It is well accepted that the correlations between asset returns are time-varying (Kroner and Ng, 1998; Ang and Bekaert, 2002). The rolling window method allows us to generate a significant

[1] For a detailed discussion, see Bauer and Vorkink (2011); Chan, Karceski, and Lakonishok (1999); Engle and Sheppardy (2008); Jagannathan and Ma (2003).
[2] See Chollete, Heinen, and Valdesogo (2009); Dowd (2005); Patton (2006a).
[3] To capture this characteristic, a copula can be designed to vary its functional form through time, as shown with a regime switching type in Rodriguez (2007) and Okimoto (2008), or to evolve its dependence parameter through time, as shown by the GARCH model in Patton (2006a). Both of these methods use the full sample period to calibrate the dependence.

amount of observations. This is a method frequently adopted by practitioners and, therefore, is more applicable to Wall Street than the regime-switching and the time-evolving models. Furthermore, the rolling window method considers only the past year's information when forming dependencies, thus avoiding disturbances that may have existed in the distant past. To our knowledge, only Aussenegg and Cech (2009) use a setting similar to ours. However, Aussenegg and Cech (2009) only consider daily Gaussian and Student's *t*-copulas in constructing their models, and it is reasonable to consider monthly and quarterly frequencies because portfolio managers do not adjust their portfolios on a daily basis. Our research also extends Aussenegg and Cech's (2009) study by including the extreme value-based copulas, which are designed to capture tail dependence. This paper provides a robust conclusion regarding the application of copulas in risk management.

Second, our study investigates how the choice of copula functions affects portfolio performance during periods of economic expansion and recession. The expansion and recession periods that we define are based on the National Bureau of Economic Research (NBER). While the study of the use of copula functions has grown immensely, little work has been done in comparing copula dependences under different economic conditions.

Third, our paper provides insight regarding the impact of risks on the difference between the Pearson correlation and the corresponding copula estimates. The previous studies (e.g., Ang and Chen, 2002; Boyer, Gibson, and Loretan, 1999; Kolari, Moorman, and Sorescu, 2008; Longin and Solnik, 2001; and Tastan, 2006) have documented that the unconditional correlation can be biased due to the properties of non-normality such as fat-tail and excessive skewness. The bias challenges conventional portfolio optimization strategies, in which correlations are estimated using the Pearson product-moment correlation coefficient or

Gaussian-based copula methods. Yet how equity market volatility, maturity risk, default risk, and currency risk affect the difference between conditional and unconditional estimations is not clear. Such an analysis is useful for risk managers in evaluating the effect of various systemic risks on correlation and in calibrating possible bias in risk management.

A copula is a function that links marginal distributions together to form a multivariate distribution. According to Sklar's Theorem, a unique copula exists for a joint distribution with continuous marginal distribution functions. Therefore, a joint distribution can be divided into (1) the marginal distributions that describe the behavior of each asset, and (2) the copula function that reveals the interaction between the assets. The flexibility of copula modeling comes from the copulas being measured independently from marginal distributions and from their being free from non-normal or asymmetric data.

We model the time-varying dependence of an international equity portfolio using different copula functions and Pearson correlation and construct the minimum-risk portfolios based on different copula dependences. The difference in mean-variance between a copula-based portfolio and the corresponding Pearson portfolio represents the benefits to use copulas. We analyze the economic values of copula models by using Ledoit and Wolf's (2008) studentized time series bootstrap method with various balancing frequencies in different sub-periods. We next evaluate how various market risks affect the deviation between the unconditional correlation and copula-based estimations.

Using daily U.S. dollar-denominated Morgan Stanley Capital International (MSCI) indices of G-7 countries, our empirical results suggest that the copula-dependence portfolios outperform the Pearson-correlation portfolios. For most scenarios studied, the Clayton-dependence portfolios deliver the highest portfolio returns, indicating the importance of lower-tail dependence in building an international equity portfolio. Moreover, the choice of weight updating frequency matters. As we increase the weight updating frequency from quarterly to monthly, the portfolio returns for the full sample and recession periods also increase, regardless of the choice of dependence measures. Our finding supports the value of active portfolio reconstruction during periods of recession. The high departures between conditional and unconditional correlations are statistically significant associated with high risks in equity market, fixed-income market, and currency market. This suggests the need of copula-based models in portfolio management, especially during economic recessions.

This paper is organized as follows. Section 2 reviews the literature on copula applications in portfolio modeling. Section 3 describes the empirical models. Section 4 presents the data used. The main empirical results are reported in Section 5. Section 6 evaluates how the risks in financial market affect the size of departure of unconditional correlation from copula-based estimations. Section 7 concludes.

2. Literature Review

Copulas, implemented in either a static or time-varying framework, are frequently applied in options pricing, risk management, and portfolio selection. In this section, we review some areas of portfolio selection in which copulas can be used/applied.

Hu (2006) adopts a mixture of a Gaussian copula, a Gumbel copula, and a Gumbel survival copula to examine the various dependence structures of four stock indices. His results demonstrate the underestimation problem due to multivariate normality correlations as well as the importance of incorporating both the structure and the degree of dependence into the portfolio evaluation. Kole, Koedijk, and Verbeek (2007) compare the Gaussian, Student's *t*, and Gumbel copulas to illustrate the importance of selecting an appropriate copula to manage the risk of a portfolio that is composed of stocks, bonds, and real estate. Kole *et al.* (2007) empirically demonstrate that the Student's *t*-copula, which considers the dependence both in the center and the tail of the distribution, provides the best fit for the extreme negative returns of the empirical probabilities under consideration. Chollete, Peña, and Lu (2011) investigate the benefits of international diversification by using the Pearson correlation and six copula functions. Their results show that dependence of asset returns increases over time and that the intensity of the asymmetric dependence varies in different regions of the world. Patton (2006a) pioneered time-varying copulas by modifying the copula functional form in a manner that allows the copula's parameters to vary. Patton (2006a) uses conditional copulas to examine asymmetric dependence in daily Deutsche mark (DM)/US dollar (USD) and Japanese yen (Yen)/US dollar (USD) exchange rates. His empirical results suggest that the correlation between DM/USD and Yen/USD exchange rates is stronger when the DM and yen are depreciating against the dollar. Rodriguez (2007) studies financial contagions in emerging markets with switching Frank, Gumbel, Clayton, and Student's *t*-copulas. Rodriguez (2007) finds evidence that the dependence structures between assets changed during the 1998 and 2002 financial crises and that a asset allocation strategy allowing the dependence of returns to vary with time perform better than that not allowing. Chollete, Heinen, and Valdesogo (2009) model asymmetric dependence in international equity portfolios using a regime-switching, canonical vine copula approach, which is a branch of the copula family first described by Aas, Czado, Frigessi, and Bakken (2007). Chollete, Heinen, and Valdesogo (2009) documents that the canonical vine copula provides better portfolio returns and that the choice of different copula dependencies affect the value-at-risk (VaR) of the portfolio return.

While some existing studies apply copulas to optimizing portfolio selection, most tend to focus on portfolio risks, i.e. value-at-risk, rather than portfolio returns. Empirically, however, investors pay at least equal attention to portfolio returns; our study is among the few that focus on equity portfolio returns using time-varying copulas.

3. Empirical Methods

3.1 Copulas

A copula C is a function that links univariate distribution functions into a multivariate distribution function. Let F be an n-dimensional joint distribution function, and let $U= (u_1, u_2, ..., u_n)^T$ be a vector of n random variables with marginal distributions $F_1, F_2, \ldots, F_n$. According to Sklar's (1959) theorem, if the marginal distributions $F_1, F_2, \ldots, F_n$ are continuous, then a copula C exists, where C is a multivariate distribution function with all uniform (0,1) marginal distributions.[4] That is,

$$F(u_1, u_2, \ldots, u_n) = C\big(F_1(u_1), F_2(u_2), \ldots, F_n(u_n)\big), \text{ for all } u_1, u_2, \ldots, u_n \in \mathbb{R}^n. \quad (1)$$

For a bivariate case, the model can be defined as

$$F(x, y) = C\big(F_X(\mathrm{x}), F_Y(\mathrm{y})\big). \quad (2)$$

3.2 Copula Specifications

In this paper, we consider four copula functions: the Gaussian, the Student's *t*, the Gumbel, and the Clayton. The Gaussian copula focuses on the center of the distribution and assumes no tail dependence. The Student's *t*-copula stresses both the center of the distribution and symmetric tail behaviors. Clayton copula emphasizes the lower-tail dependence while

[4] For detailed derivations, please refer to Cherubini *et al*. (2004), Embrechts *et al*. (2005), Franke, Härdle, and Hafner (2008), and Patton (2009).

Gumbel copula focuses on the upper tail dependence. Table 1 summarizes the characteristics of each copula in detail.

[INSERT Table 1 ABOUT HERE]

3.2.1 Gaussian Copula

The Gaussian copula is frequently used in finance literature due to its close relationship to the Pearson correlation. It represents the dependence structure of two normal marginal distributions. The bivariate Gaussian copula can be expressed as

$$C(x,y) = \int_{-\infty}^{\Phi^{-1}(x)} ds \int_{-\infty}^{\Phi^{-1}(y)} dt \frac{1}{2\pi\sqrt{1-\rho^2}} exp\left\{-\frac{s^2-2\rho st+t^2}{2(1-\rho^2)}\right\} = \Phi_\rho(\Phi^{-1}(x), \Phi^{-1}(y)), \quad (3)$$

where Φ denotes the univariate standard normal distribution function, and Φ_ρ is the joint distribution function of the bivariate standard normal distribution with a correlation coefficient $-1 \leq \rho \leq 1$. The Gaussian copula has no tail dependence unless $\rho = 1$.

3.2.2 Student's *t*-Copula

Unlike the Gaussian copula, which fails to capture tail behaviors, the Student's *t*-copula depicts the dependence in both center as well as in the tails of the distribution. The Student's *t*-copula is defined using the multivariate *t* distribution and can be written as

$$C_{v,\rho}^{t}(x,y) = \int_{-\infty}^{t_v^{-1}(x)} \int_{-\infty}^{t_v^{-1}(y)} \frac{1}{2\pi\sqrt{1-\rho^2}} \left\{1 + \frac{s^2 - 2\rho st + t^2}{v(1-\rho^2)}\right\}^{-\frac{v+2}{2}} dsdt$$

$$= t_{v,r}^{2}(t_v^{-1}(x), t_v^{-1}(y)) \quad , \quad (4)$$

where $t_{v,r}^2$ indicates the bivariate joint *t* distribution; t_v^{-1} is the inverse of the distribution of a univariate *t* distribution; and *v* is the degrees of freedom. When $v > 2$, ρ is the correlation coefficient of the bivariate *t* distribution.

3.2.3 Tail-dependence Copulas

According to Embrechts *et al*. (2005), the coefficient of the upper tail dependence (λ_u) of 2 series *X* and *Y* can be defined as:

$$\lambda_u(x,y) = \lim_{q-1^-} P\left[Y > F_y^{\leftarrow}(q) | X > F_x^{\leftarrow}(q)\right].$$

The upper-tail dependence presents the probability that *Y* exceeds its *q*-th quantile, given that *X* exceeds its *q*-th quantile, considering the limit as *q* goes to its infinity. If the limit $\lambda_u \in [0,1]$ exists, then *X* and *Y* are said to show upper tail dependence. In the same manner, the coefficient of lower tail dependence (λ_l) of *X* and *Y* is described as:

$$\lambda_l(x,y) = \lim_{q-0^+} P\left[Y \le F_y^{\leftarrow}(q) | X \le F_x^{\leftarrow}(q)\right].$$

Since both F_{U_1} and F_{U_2} are continuous density functions, the upper tail dependence can be presented as:

$$\lambda_u = \lim_{q-1^-} \frac{P\left[Y > F_y^{\leftarrow}(q) | X > F_x^{\leftarrow}(q)\right]}{P[X > F_x^{\leftarrow}(q)]} .$$

For lower tail dependence, it can be described as;

$$\lambda_l = \lim_{q-0^+} \frac{P\left[Y \le F_y^{\leftarrow}(q) | X \le F_x^{\leftarrow}(q)\right]}{P[X \le F_x^{\leftarrow}(q)]} .$$

Gumbel Copula

The Gumbel copula is a popular upper tail dependence measure. Suggested by Embrechts *et al*. (2005), Gumbel copula can be written as

$$c(x,y) = \exp\left[-\left\{(-\ln(x)^{\frac{1}{\delta}} + (-\ln(y)^{\frac{1}{\delta}}\right\}^{\delta}\right] \quad , \tag{5}$$

where $0 < \delta \leq 1$ measures the degree of dependence between *X* and *Y*. When $\delta = 1$, *X* and *Y* do not have upper tail dependence (i.e., *X* and *Y* are independent at the upper tails). When $\delta \rightarrow 0$, *X* and *Y* have perfect dependence.

Clayton Copula

The Clayton copula is used to measure lower-tail dependence. The Clayton copula is defined as

$$c(x, y) = \max\left[(x^{-\alpha} + y^{-\alpha} - 1)^{-\frac{1}{\alpha}}, 0\right] \quad , \tag{6}$$

where α describes the strength of dependence. If $\alpha \rightarrow 0$, *X* and *Y* do not have lower tail dependence. If $\alpha \rightarrow \infty$, *X* and *Y* have perfect dependence.

3.3 Portfolio Constructions

The selection of optimal portfolios draws on the seminal work of Markowitz (1952). Specifically, we adopt the variance-minimization strategy with no short-selling and with no transaction cost assumptions.[5] The optimal portfolio allocation can be formed by solving the following optimization problem:

$$Min_{\{w\}} w'Vw$$

$$Subject\ to\ \ \Sigma w_i = 1, w_i \geq 0 \quad , \qquad (7),$$

where w_i is the weight of asset *i* and *V* is the covariance matrix of the asset returns. Because dependence is a time-varying parameter, the data from a subset of 250 trading days prior to the given sample date *t* is used to derive its dependence. With 1,780 daily data points in our sample, we calculate a total of 1,531 dependences for each copula method and Pearson

[5] Short-selling usually involves other service fees, which vary depending on the creditability of the investors. Because the focus of this study is on the effect of the dependence structure on portfolio performance, we assume that short-selling is not allowed to simply the comparison.

correlation. With these dependences, optimal portfolio weightings can be obtained by solving a quadratic function subject to specified constraints. The optimal weightings for time *t* are used to calculate the realized portfolio returns for *t*+1.[6]

In practice, portfolio managers periodically re-examine and update the optimal weights of their portfolios. If the asset allocation of an existing portfolio has deviated from the target allocation to a certain degree, and if the benefit of updating exceeds its costs, a portfolio will be reconstructed. In this paper, we construct a comprehensive study of portfolio returns by varying the state of the economy (i.e., expansion or recession), the dependence structure of the portfolio, and the frequency of portfolio weight updating (i.e. quarterly, monthly, and daily). Quarterly updating allows investors to rebalance the portfolio weights on the first trading days of March, June, September, and December; monthly updating allows investors to change the optimal weights on the first trading days of each month. Under daily updating, investors rebalance the optimal weights every trading day.

4. Data

The data is comprised of the U.S. dollar-denominated daily returns of the Morgan Stanley Capital International (MSCI) indices for the G7 countries which include Canada, France, Germany, Italy, Japan, the United Kingdom, and the United State. The sample period spans the first business day in June 2002 to the last business day in June 2009 for a total of 1,780 daily observations. Based on the definition provided by the National Bureau of Economic Research, we split the data into an expansion period from June 2002 to November 2007 and a

[6] For example, we use return data from t_1 to t_{250} to calculate the optimal portfolio weights with dependences estimated from the copulas and the Pearson correlation. The optimal portfolio weights are applied to the return data at t_{251} to calculate the realized portfolio returns.

recession period from December 2007 to June 2009. The data of various interest rates and exchange rate are obtained from the Federal Reserve.

Table 2 presents the summary statistics. Among the G7 countries, Canada have the highest daily returns while the US have the lowest. Germany, however, experience the most volatile returns. All return series exhibit high kurtosis, suggesting the fat tails behavior. The results of the Jarque-Bera test reject the assumption that the G7 indices have normal distributions.

[INSERT Table 2 ABOUT HERE]

5. Empirical Results

5.1 Dependence

Using 1,780 daily data points from the G7 countries, we estimate 21 dependence pairs for each dependence model, each containing a sequence of 1,531 dependences. The parameters for the Gaussian, Student's *t*, Gumbel, and Clayton copula functions are estimated using the two-stage *inference for the margins* (IFM) method proposed by Joe and Xu (1996) and Joe (1997). The bivariate joint density function can be represented as follows:

$$f(x,y) = c\left(F_X(x_i;\theta_x), F_Y\left(y_i;\theta_y\right);\Theta\right) f_X(x_i;\theta_x) f_Y\left(y_i;\theta_y\right), \tag{8}$$

where θ_x are the parameters for the marginal distribution F_X , and θ_y are the parameters for the marginal distribution F_Y, and Θ are the parameters for the copula density c. Therefore, the exact log-likelihood function of the above joint density function can be presented as

$$l(\eta) = \sum_{i=1}^{n} \ln c\left(F_X(x_i;\theta_x), F_Y\left(y_i;\theta_y\right);\Theta\right) + \sum_{i=1}^{n} \left(ln f_X(x_i;\theta_x) + ln f_Y\left(y_i;\theta_y\right)\right). \tag{9}$$

Then, by maximization, we can obtain the exact maximum likelihood estimator as

$$\hat{\eta}_{MLE} = \max_{\eta} l(\eta). \tag{10}$$

According to Joe and Xu (1996) and Joe (1997), the parameters can be estimated by an *inference for the margins* or IFM method. This method includes two steps. First, the parameters of the univariate marginal distributions are estimated as:

$$\widehat{\theta_x} = ArgMax_{\theta_x} \sum_{\mathrm{i=1}}^{\mathrm{n}} \left(ln f_X(x_i; \theta_x) \right), \tag{11}$$

and

$$\widehat{\theta_y} = ArgMax_{\theta_y} \sum_{\mathrm{i=1}}^{\mathrm{n}} \left(ln f_Y\left(y_i; \theta_y\right) \right). \tag{12}$$

At the second step, given θ_x and θ_y, the dependence parameters Θ are estimated as:

$$\widehat{\Theta} = ArgMax_{\Theta} \sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{lnc}\left(F_X\left(x_i; \widehat{\theta_x}\right), F_Y\left(y_i; \widehat{\theta_y}\right); \Theta \right). \tag{13}$$

Appendix A shows the maximum and the minimum of the 21 dependence pairs of each dependence model.

The graphs in Figure 1 show the dependences between the US and other countries that are estimated by various copulas and the Pearson correlation method. In general, the Gaussian copula estimation is similar to the corresponding Pearson correlation, but the Student's *t*-copulas show significant jumps over time. For our sample period, Japan shows a low dependence with the US market when compared to other countries' dependence on the U.S. economy.

[INSERT Figure 1 ABOUT HERE]

The differences in correlations between the time-varying copulas and unconditional model vary due to economic states. Specifically, the Pearson correlation is higher than the estimates that use fat-tail copulas, i.e., the Clayton and the Gumbel, during two sub-periods: 2003 to 2004 and 2008 to 2009. In contrast, the copula-based interdependences are higher than their corresponding unconditional estimates between 2004 and 2007. In addition, the patterns of time-variation are different across countries. Among them, Japan shows the lowest degree of comovement with the US equity return.

5.2 Average Portfolio Returns

Table 3 presents the average portfolio returns for the full sample period, the expansion period, and the recession period for the quarterly, monthly, and daily weight updating strategies. For the quarterly weight updating, the Clayton-dependence portfolios have the highest average returns at 6.07% during the expansions and -12.52% during the recessions; the Pearson-correlation portfolios have the lowest average returns at 5.48% during the expansions and -14.25% during the recessions. The order of portfolio performances, listing according to its dependence model regardless of the state of economy, is as follows: the Clayton copula, the Gumbel copula, the Student's *t*-copula, the Gaussian copula, and the Pearson correlation. The empirical results of both the Clayton and Gumbel copulas highlight the need to model the tail dependence between assets. Our finding suggests that with a quarterly weighting strategy, tail dependence, particularly lower-tail dependence, generate superior average portfolio returns across different economic conditions.

When we increase the portfolios' rebalancing frequency from quarterly to monthly, similar empirical results are observed. That is, the Clayton-copula portfolios yield the highest average returns while the Pearson-correlation portfolios provide the lowest average returns. During the expansion periods, the order of portfolio performances, listing according to dependence model, is as follows: the Clayton copula, the Student's *t*-copula, the Gaussian copula, the Gumbel copula, and the Pearson correlation. During the recession periods, the order of portfolio performances, listing according to dependence model, is as follows: the Clayton copula, the Gumbel copula, the Student's *t*-copula, the Gaussian copula, and the Pearson correlation. According to Kole *et al*. (2007), the Gaussian copula, which does not consider lower-tail dependence, tends to be too optimistic on the subject of the benefits of a portfolio's diversification, and the Gumbel copula, which focuses on the upper tail and pays no attention to the center of the distribution, tends to be too pessimistic. We verify these arguments by observing that the Gumbel-copula portfolio performs better than the Pearson correlation portfolio only during the expansion periods while the Gaussian-copula dependence portfolio performs better than the Pearson correlation portfolio only during the recession periods. Interestingly, as we increase the weight updating frequency from quarterly to monthly, the average portfolio returns for the full sample and recession periods also increase, regardless of the choice of dependence measures. Thus, the empirical results seem to support the need for active portfolio reconstruction during periods of economic recession.

As the weight updating frequency increases to daily, the Clayton copula delivers the highest average portfolio returns only during the expansion period. By contrast, the Student's *t*-copula generates the highest portfolio average returns for the full sample and recession periods. The influence from the lower-tail dependence seems to diminish under daily weight reconstruction. The Gaussian copula portfolio delivers the worst portfolio performance during both expansion and recession periods.

[INSERT Table 3 ABOUT HERE]

5.3 Testing Portfolio Performance

The results reported in the previous section show the average portfolio returns under different dependences and weight updating frequencies. One of potential difficulties with the study of average returns is the empirical results may be biased and the volatility may be high if extreme values exist over the examined period. Furthermore, previous methods used to examine the robustness of portfolio performance assume the data follow normal distribution (Jobson and Korkie,1981; Memmel, 2003). However, empirical results have indicated this assumption does not hold for financial data.

To cope with this problem, Ledoit and Wolf (2008) propose an alternative testing method using an inferential studentized time-series bootstrap. Ledoit and Wolf's (2008) method is as follows.[7] Let a and b be two investment strategies, and let r_{at} and r_{bt} be the portfolio returns for strategies a and b, respectively, at time t, where t ranges from 1 to i. The mean vector μ and the covariance matrix Σ for the return pairs are denoted by

$$\mu = \begin{pmatrix} \mu_a \\ \mu_b \end{pmatrix} \text{ and } \Sigma = \begin{pmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{pmatrix}. \tag{14}$$

The performances of strategies a and b can be examined by checking whether the difference between the Sharpe ratios for strategies a and b is statistically different from 0. That is,

$$\Delta = S_a - S_b = \frac{\mu_a}{\sigma_a} - \frac{\mu_b}{\sigma_b}, \tag{15}$$

and

$$\hat{\Delta} = \widehat{S_a} - \widehat{S_b} = \frac{\widehat{\mu_a}}{\widehat{\sigma_a}} - \frac{\widehat{\mu_b}}{\widehat{\sigma_b}}, \tag{16}$$

[7] For detailed derivations, please refer to Ledoit and Wolf (2008).

Where Δ is the difference between the two Sharpe ratios, and S_a and S_b are the Sharpe ratios for strategies *a* and *b*, respectively.

Let the second moments of the returns from strategies *a* and *b* be denoted by γ_a and γ_b. Then $\gamma_a = E(\gamma_{at}^2)$ and $\gamma_b = E(\gamma_{bt}^2)$. Let v and $\hat{v}$ be $(\mu_a, \mu_b, \gamma_a, \gamma_b)'$ and $(\widehat{\mu_a}, \widehat{\mu_b}, \widehat{\gamma_a}, \widehat{\gamma_a})'$, respectively. Then Δ and $\hat{\Delta}$ can be expressed as

$$\Delta = f(v) \text{ and } \hat{\Delta} = f(\hat{v}) \quad , \tag{17}$$

, where $f(v) = \frac{\mu_a}{\sqrt{\gamma_a - \mu_a^2}} - \frac{\mu_b}{\sqrt{\gamma_b - \mu_b^2}}$ and $\sqrt{i}(\hat{v} - v) \xrightarrow{d} N(0; \Psi)$.

For the time series data, Ledoit and Wolf (2008) argue that Ψ can be evaluated by the studentized bootstrap as $\widehat{\Psi}^* = \frac{1}{\vartheta}\sum_{j=1}^{\vartheta} \xi_j \xi_j'$, where $\xi_j = \frac{1}{\sqrt{b}}\sum_{t=1}^{b} y^*_{(j-1)b+t}$ $t = 1, \ldots, \vartheta$. ϑ is the integer part of the fraction of the total observations divided by the blocks *b*. Also,

$$y_t^* = \left(r_{ta}^* - \widehat{\mu_a^*}, r_{tb}^* - \widehat{\mu_b^*}, r_{ta}^{*2} - \widehat{\gamma_a^*}, r_{tb}^{*2} - \widehat{\gamma_b^*}\right) \; t = 1, \ldots, i. \tag{18}$$

Following Ledoit and Wolf's (2008) method, we examine the significance of 60 pairs of portfolio performances. The size of the bootstrap iteration is 10,000 to ensure a sufficient sample.[8] Table 4 presents the results from Ledoit and Wolf's (2008) portfolio performance test.

[INSERT Table 4 ABOUT HERE]

The results indicate that during the recession periods and with the use of quarterly weight updating, the Pearson correlation underperforms all the copula dependences at a confidence level of 90% or greater. During recession periods and with the adoption of

[8] Ledoit and Wolf (2008) suggest that 5,000 iterations will guarantee a sufficient sample. We adopt the higher standard of 10,000 iterations to strengthen our testing results.

monthly weight updating, the superiority of the copula dependences jumps to a 99% confidence level. Moreover, during the recession periods and with the assumption of daily updating, the Student's *t*-copula outperforms the Pearson correlation at the 99% confidence level.

Overall, Ledoit and Wolf's (2008) empirical tests illustrate the superiority of the use of copulas during recession periods, regardless of the frequency of weight updating. During the bullish market, this outperformance seems to not be as statistically significant as it is during the bearish market.

6. The Causes of Differences in Dependence Estimations

Return dependence structure is critical in determining optimal portfolios, therefore, understanding the factors that affect the variation in correlations between different methods is an indispensible element in asset management. The aforementioned empirical tests confirm the need to use the conditional estimates of dependence in portfolio management; however, the factors that may explain the variations among correlation estimations have not yet been well studied. As shown in Figure 1, the differences of correlation estimations are influenced by business cycle and market risks. In this section, we consider several factors that are widely used to characterize economic states and study their impact on estimate of return dependence structure.

Table 5 presents the variables that are used to study the disparity between the Pearson correlation and the copula-based dependences. The VIX is regarded as the "fear index" and represents the projected volatility for the equity market. High VIX values also are accompanied with a loss of equity value. We collect the data of maturity risk premium and default risk premium to evaluate the impact of risk perceptions from the point of view of a fixed-income investor. The volatility of the U.S. dollar exchange rate serves as a proxy for the U.S. dollar currency risk.

[INSERT Table 5 ABOUT HERE]

One of the key concerns surrounding the use of copulas in modeling portfolio dependence structures is in what situation the conditional correlation will differ *most significantly* from the unconditional correlation. We denote $q_{i,t}$ is the dependence computed by one of the copulas (e.g., Clayton), and $\rho_{i,t}$ is the Pearson correlation coefficient. In Table 6, we first report the percentages that copula correlation estimate is greater than the Pearson estimate ($q_{i,t} > \rho_{i,t}$) and the other ($q_{i,t} < \rho_{i,t}$). For all countries except Japan, all copulas other than the Gaussian model, especially the Clayton and the Student's t , demonstrate stronger correlations than the Pearson correlation over the sample period. This suggests that overlooking fat-tail and skewness in returns may cause an investor to underestimate the correlations among assets and overstate the *ex post* benefits of portfolio diversification.

[INSERT Table 6 ABOUT HERE]

Table 6 also presents the results testing the difference of VIX between the two sub-groups, $q_{i,t} > \rho_{i,t}$ and $q_{i,t} < \rho_{i,t}$. We report the average VIX under the two scenarios and the statistics testing the differences of the dependences between the estimates using various copulas and the Pearson correlation coefficient. Since the values of VIX are between 9 and

81 while there is no theoretical foundation to support its normality, the truncated *t*-test suggested by Bagnoli and Bergstrom (2005) and the distribution-free Mann-Whitney test are applied to minimize the possible bias. Both the truncated *t*-statistics and the Mann-Whitney (M-W) *z*-statistics suggest that the copula-based estimates are greater than the unconditional correlation when the market is less volatile. Therefore, the use of the Pearson correlation may underestimate the return comovement during bearish market, leading to an overstatement of the benefits of portfolio diversification.

We next evaluate how the condition of the financial market affect the departure between the unconditional correlation from copula-based estimations. The absolute value of the difference between copulas and the Pearson correlation coefficient, $\left|q_{i,t} - \rho_{i,t}\right|$, is the dependent variable in the following ordinary least squared (OLS) regression:

$$\left|r_{i,t} - \rho_{i,t}\right| = \alpha_k + \beta_k x_{k,t} + \xi_t, \tag{19}$$

where a constant α and an economic or financial variable x_k (e.g., VIX) are included in the model. A description of the explanatory variables is given in Table 5.

Table 7 reports the coefficients of the independent variables but omits α_k. For the majority of regressions, a high risk is associated with a large difference between conditional and unconditional correlations at a statistically significant level. Among the factors, high financial market implied volatility and default risk premium are the connected with a substantial disparity of dependence. The maturity risk or currency risk do not always statistically associate with the difference of correlations by using some copulas.

[INSERT Table 7 ABOUT HERE]

The empirical results indicate the importance of modeling return dependence by applying copulas in portfolio management, especially during periods of great economic risk. Most return distributions show asymmetric downside and upside movements as well as fat tails. In Tables 4 and 5, we show that the performance of portfolios formed by conditional correlation structures is higher than those using the Pearson correlation. We further show that the discrepancy between conditional and unconditional correlations is sensitive to market volatility as are the risks in fixed-income and exchange markets. All this being the case, finding an appropriate approach to modeling dependencies between asset returns has become a significant challenge in the field of risk management.

7. Conclusions

In this paper, we study whether adopting time-varying copulas can improve portfolio performance. This paper is motivated by the fact that the traditional Pearson correlation does not adequately describe most financial returns. Moreover, the robustness of copula functions has not yet been fully examined under different economic states and weight updating scenarios. We evaluate the effectiveness of various copulas in asset management while considering the impact of various portfolio rebalancing frequencies and of different stages in business cycles on the results. We use the studentized time series bootstrap method suggested by Ledoit and Wolf (2008). We also examine the financial and economic risks that affect the difference between conditional and unconditional correlations.

The main findings are as follows. First, modeling an international equity portfolio using Pearson correlations underperforms those using copula-based dependences, especially during periods of economic recession. Our findings are robust regardless of the rebalancing frequencies. Second, the importance of lower-tail behaviors in portfolio modeling is highlighted by the higher-than-average portfolio returns from the Clayton-dependence portfolios. Third, the choice of weight updating frequency affects portfolio returns. The portfolios using a monthly weight updating frequency provide better portfolio returns than those using quarterly or daily weight adjustments. Finally, when the market risks are high, the conditional dependence estimates depart from and unconditional correlations. This suggests the need of copula-based models in portfolio management, especially during periods of economic downturn.

We add to the current literature by thoroughly evaluating the effectiveness of asymmetric conditional correlations in managing portfolio risk. This paper synthesizes the major concepts and *modi operandi* of the previous research and maximizes the practicality of applying copulas under a variety of scenarios. Future research into copulas can be extended to contagion of different asset classes and interest rates.

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Table 1: The Characteristics of Different Copulas

Dependence Model	Tail Dependence	Parameter Range
Pearson Correlation	No	$\rho \in (-1, 1)$
Gaussian Copula	No	$\rho \in (-1, 1)$
Student's *t*-Copula	Yes (Symmetry)	$\rho \in (-1, 1)$, $v>2$
Gumbel Copula	Yes (Upper Tail)	$\delta \in (0, 1)$
Clayton Copula	Yes (Lower Tail)	$\alpha \in [-1, \infty)\backslash\{0\}$

Table 2: The Summary Statistics of the G7 Indices

	Canada	France	Germany	Italy	Japan	U.K.	U.S.
Mean (%)	0.0031	0.0064	0.0082	0.0003	-0.0017	-0.0039	-0.0082
Std. Dev.	0.0164	0.0171	0.0178	0.0161	0.0155	0.0159	0.0144
Skewness	-0.8781	0.0740	0.0666	0.0477	-0.1475	-0.0535	-0.1365
Kurtosis	14.1774	10.7576	8.6920	12.9310	7.4592	12.9143	12.1182
Jarque-Bera	9494	4465	2404	7315	1481	7290	6171
JB P-Value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations	1780	1780	1780	1780	1780	1780	1780

Note: The results indicate that the daily returns of the G7 indices are not normally distributed.

Table 3: Average Portfolio Returns

	Clayton	Gaussian	Gumbel	Student's *t*	Pearson
Panel A: Quarterly Adjustments					
Full Sample Returns	1.44%	0.91%	1.16%	1.11%	0.57%
	(1.1903)	(1.1749)	(1.1922)	(1.1763)	(1.0644)
Expansion Returns	6.07%	5.63%	5.70%	5.66%	5.48%
	(0.6946)	(0.0798)	(0.6962)	(0.7014)	(0.6613)
Recession Returns	-12.52%	-13.35%	-12.56%	-12.64%	-14.25%
	(2.0549)	(2.0025)	(2.0578)	(1.7922)	(2.0153)
Panel B: Monthly Adjustments					
Full Sample Returns	1.63%	1.08%	1.22%	1.22%	0.75%
	(1.1764)	(1.1812)	(1.1835)	(1.1736)	(1.0199)
Expansion Returns	6.15%	5.67%	5.66%	5.69%	5.23%
	(0.6800)	(0.6928)	(0.6870)	(0.6864)	(0.6401)
Recession Returns	-12.01%	-12.78%	-12.19%	-12.30%	-12.80%
	(2.0375)	(2.0355)	(2.0474)	(2.0247)	(1.1708)
Panel C: Daily Adjustments					
Full Sample Returns	1.34%	0.85%	1.16%	1.47%	1.03 %
	(1.1737)	(1.1651)	(1.1749)	(1.1733)	(1.0253)
Expansion Returns	5.84%	5.35%	5.59%	5.70%	5.39%
	(0.6733)	(0.6859)	(0.6839)	(0.6766)	(0.6367)
Recession Returns	-12.27%	-12.74%	-12.22%	-11.34%	-12.14%
	(2.0382)	(2.0053)	(2.0277)	(2.0346)	(1.7280)

Note: The average portfolio returns are presented in an annualized percentage format. Three weight updating frequencies are considered: quarterly, monthly, and daily. Within each frequency, we report the returns for the full sample period, the expansion period, and the recession period. The numbers in the parentheses are standard errors.

Table 4: Ledoit and Wolf (2008) Portfolio Performance Test

Panel A: Quarterly Adjustments					
	CL-GA	CL-GU	CL-PE	CL-*t*	GA-GU
Expansion	0.821	0.812	0.788	0.677	0.916
Recession	0.060*	0.987	0.054*	0.839	0.0760*
	GA-PE	GA-t	GU-PE	GU-t	PE-t
Expansion	0.892	0.930	0.766	0.912	0.778
Recession	0.0030***	0.0727*	0.0267**	0.943	0.026**
Panel B: Monthly Adjustments					
	CL-GA	CL-GU	CL-PE	CL-t	GA-GU
Expansion	0.193	0.415	0.568	0.744	0.881
Recession	0.249	0.803	0.021***	0.295	0.092*
	GA-PE	GA-t	GU-PE	GU-t	PE-t
Expansion	0.850	0.892	0.795	0.896	0.809
Recession	0.001***	0.318	0.019***	0.475	0.008***
Panel C: Daily Adjustments					
	CL-GA	CL-GU	CL-PE	CL-t	GA-GU
Expansion	0.389	0.814	0.732	0.929	0.913
Recession	0.000***	0.119	0.173	0.371	0.000***
	GA-PE	GA-t	GU-PE	GU-t	PE-t
Expansion	0.928	0.915	0.460	0.831	0.301
Recession	0.718	0.001***	0.045***	0.778	0.019***

Note: The performance tests are conducted using the approach suggested by Ledoit and Wolf (2008). The tests examine whether the returns from two portfolios are significantly different at the 95% level. CL stands for the Clayton copula, GA stands for the Gaussian copula, GU stands for the Gumbel copula, PE stands for Pearson correlation, and *t* stands for the Student's *t*-copula. * represents 90% statistical significance, ** represents 95% statistical significance, and *** represents 99% statistical significance.

Table 5: Economic Variables

Variable	Definition
VIX	CBOE S&P 500 Volatility Index.
MRP	Maturity risk premium. Difference in US 90-Day T-Bills Secondary Market and US 10-Year Government Bond Yield.
DRP	Default risk premium. Difference in Moody's Seasoned Aaa and Baa Corporate Bond Yield.
V(FX)	Exchange rate risk. The annualized daily volatility in the change of US dollar Trade Weighted Index. The standard deviation is calculated by using previous 100 daily data.
i	United States Overnight LIBOR

Table 6: VIX and the Difference between Copulas and Pearson Correlation Coefficient

This table reports the percentages and the means of VIX that the dependence computed by one of the various copulas (e.g., Clayton), $q_{i,t}$, is greater (or less) than Pearson correlation coefficient, $\rho_{i,t}$. The truncated *t*- statistics and the Mann-Whitney (M-W) *z* statistics testing the difference of VIX between the two groups, $q_{i,t} > \rho_{i,t}$ and $q_{i,t} < \rho_{i,t}$, are also presented.

	$q_{i,t} > \rho_{i,t}$				$q_{i,t} < \rho_{i,t}$			
Dependence		Copula Model				Copula Model		
	Clayton	Gaussian	Gumbel	Student's *t*	Clayton	Gaussian	Gumbel	Student's *t*
US-Canada								
%	64.90	10.07	52.16	75.82	35.10	89.93	47.84	24.18
Mean of VIX	14.36	19.65	13.65	15.59	30.36	23.68	26.87	33.73
t (VIX Difference)	-26.41	-6.66	-26.35	-22.47				
M-W *z* (VIX Difference)	-29.66	-13.31	-7.73	-66.01				
US-France								
%	72.68	43.73	63.53	73.92	27.32	56.27	36.47	26.08
Mean of VIX	15.50	16.63	14.81	15.93	31.88	22.58	28.98	31.43
t (VIX Difference)	-21.52	-11.40	-22.80	-18.81				
M-W *z* (VIX Difference)	-55.61	-18.13	-29.89	-63.10				
US-Germany								
%	65.88	34.90	62.61	71.70	34.12	65.10	37.39	28.30
Mean of VIX	15.16	17.56	14.99	16.00	29.27	21.27	28.33	30.05
t (VIX Difference)	-21.34	-7.52	-21.44	-17.79				
M-W *z* (VIX Difference)	-36.73	-15.16	-29.32	-56.82				
US-Italy								
%	68.50	25.82	65.75	75.03	31.50	74.18	34.25	24.97
Mean of VIX	15.10	18.81	15.01	15.89	30.58	20.38	29.50	32.24
t (VIX Difference)	-22.69	-3.26	-22.22	-19.51				
M-W *z* (VIX Difference)	-41.91	-13.41	-35.53	-66.28				
US-Japan								
%	73.27	64.58	90.46	87.97	26.73	35.42	9.54	12.03
Mean of VIX	19.31	17.93	18.19	16.96	21.81	23.71	20.96	42.03
t (VIX Difference)	-3.50	-8.02	-6.08	-24.53				
M-W *z* (VIX Difference)	-73.78	-56.42	-184.46	-140.92				
US-UK								
%	70.33	40.26	61.63	74.18	29.67	59.74	38.37	25.82
Mean of VIX	15.07	17.47	14.55	15.88	31.61	21.66	28.68	31.75
t (VIX Difference)	-23.73	-8.16	-23.80	-19.27				
M-W *z* (VIX Difference)	-46.11	-19.45	-24.87	-63.57				

Table 7: Difference between Copulas and the Pearson Correlation Coefficient

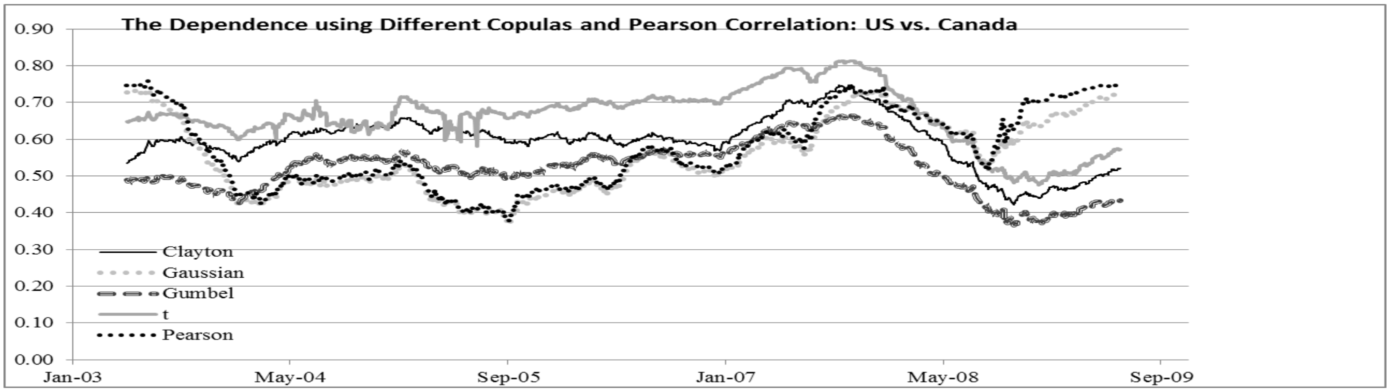
Table 7 presents the coefficients of the ordinary least squared (OLS) on the absolute value of the difference between copulas and the Pearson correlation coefficient ($|q_{i,t} - \rho_{i,t}|$) with the economic and financial variables. The independent variables are described in Table 5. The model is $|q_{i,t} - \rho_{i,t}| = \alpha_k + \beta_k x_t + \xi_t$, where $q_{i,t}$ is the dependence computed by one of the copulas (e.g., Clayton), and $\rho_{i,t}$ is the Pearson correlation coefficient. A constant α and an economic variable (e.g., VIX) are included in the regression. α_k is not reported. Panel B reports the regressions of the independent variables that are statistically significant in Panel A. A constant and one-period lags have been added but are not reported.

	Clayton		Gaussian		Gumbel		Student's *t*	
US - Canada								
VIX	0.0025	***	0.0009	***	0.0062	***	0.0006	***
MRP	1.7607	***	0.2642	***	3.9434	***	1.7585	***
DRP	5.5363	***	1.5274	***	12.1609	***	-0.0709	
V(FX)	7.1199	***	1.2497	***	11.0285	***	0.9703	**
US - France								
VIX	0.0017	***	0.0012	***	0.0017	***	0.0005	***
MRP	1.6814	***	0.8199	***	-0.1195		1.7402	***
DRP	4.6254	***	2.1720	***	4.8017	***	1.2931	***
V(FX)	4.3864	***	2.1972	***	4.7370	***	-0.5699	
US - Germany								
VIX	0.0034	***	0.0023	***	0.0031	***	0.0021	***
MRP	1.2575	***	1.2786	***	0.9335	***	1.1063	***
DRP	8.1215	***	4.1589	***	7.1726	***	4.3048	***
V(FX)	6.8203	***	4.0097	***	7.0686	***	2.1772	***
US - Italy								
VIX	0.0016	***	0.0012	***	0.0012	***	0.0014	***
MRP	1.3895	***	0.6565	***	-0.2226		2.1252	***
DRP	4.4813	***	2.0798	***	3.3713	***	2.8906	***
V(FX)	3.9733	***	1.9343	***	3.1867	***	-0.2563	
US - Japan								
VIX	0.0023	***	0.0003	***	0.0017	***	0.0013	***
MRP	2.5678	***	0.2333	***	4.0340	***	1.5898	***
DRP	3.8739	***	0.3664	***	2.8251	***	2.2431	***
V(FX)	4.0778	***	0.2582	***	5.1887	***	3.1814	***
US - UK								
VIX	0.0009	***	0.0012	***	0.0012	***	0.0011	***
MRP	2.0686	***	0.8385	***	0.7006	***	2.3186	***
DRP	3.6487	***	2.3059	***	4.1042	***	2.5386	***
V(FX)	3.7611	***	2.2670	***	4.1853	***	-0.1559	

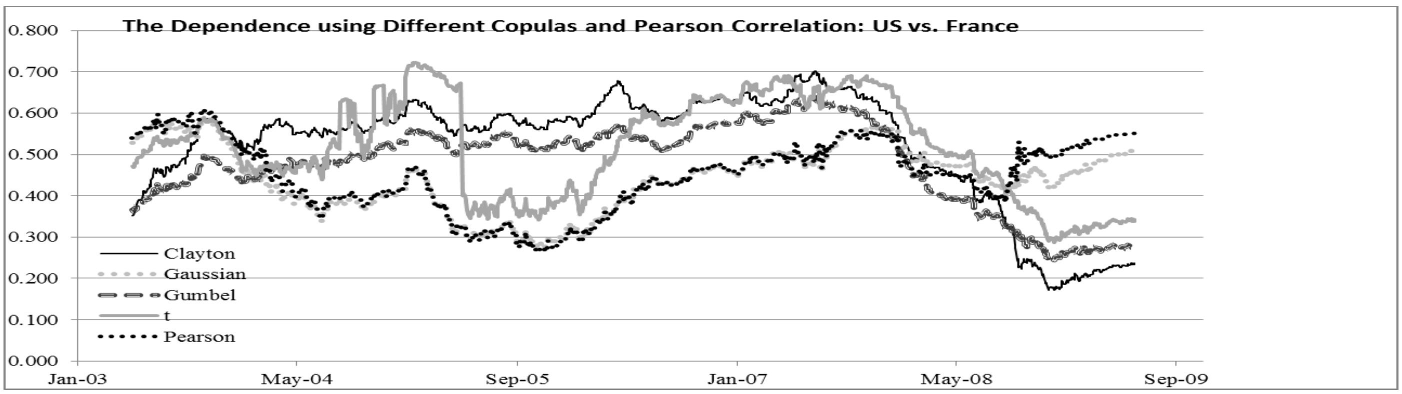
* represents 90% statistical significance, ** represents 95% statistical significance, and *** represents 99% statistical significance.

Figure 1: The Dependence Using Different Copulas and Pearson Correlations

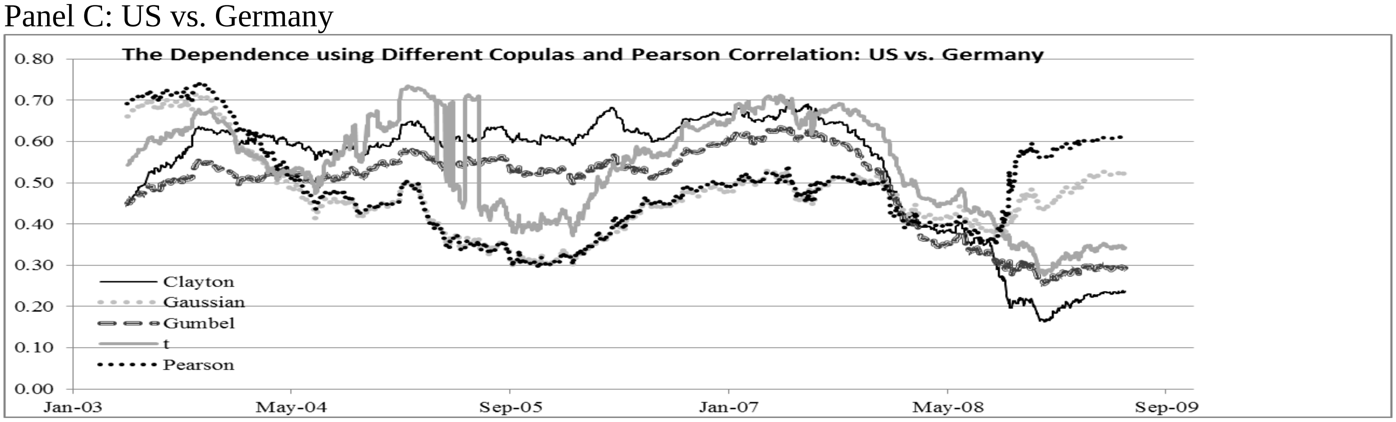
Panel A: US vs. Canada



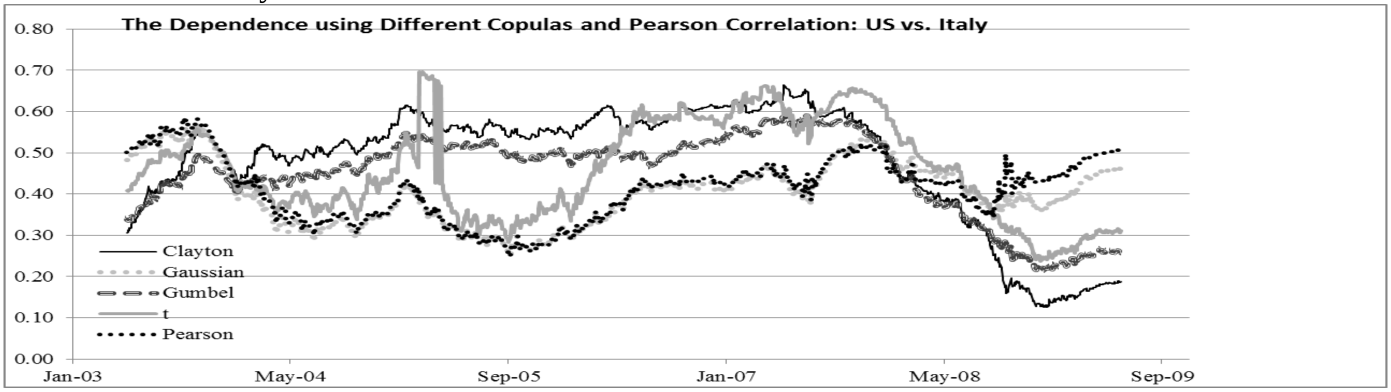
Panel B: US vs. France



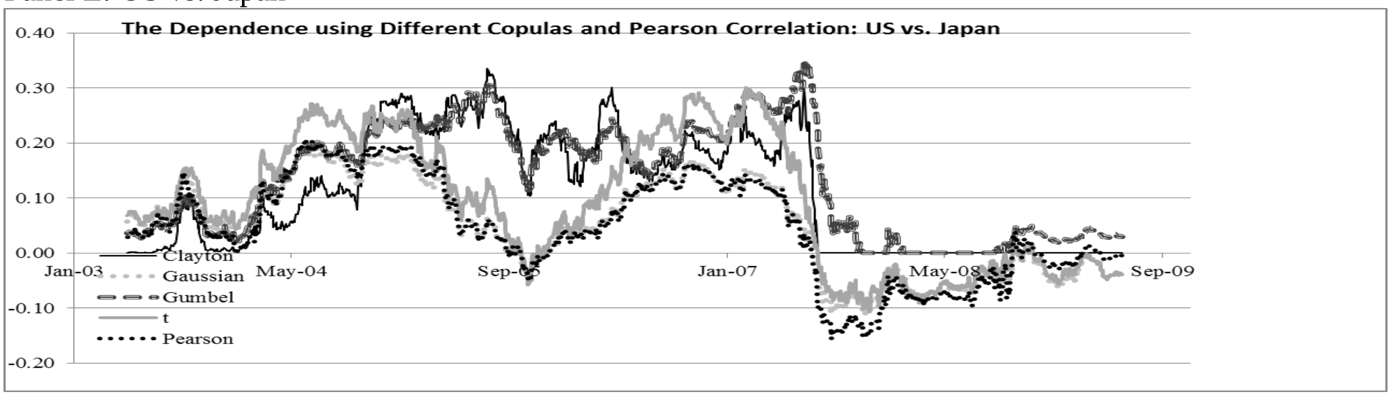
Panel C: US vs. Germany



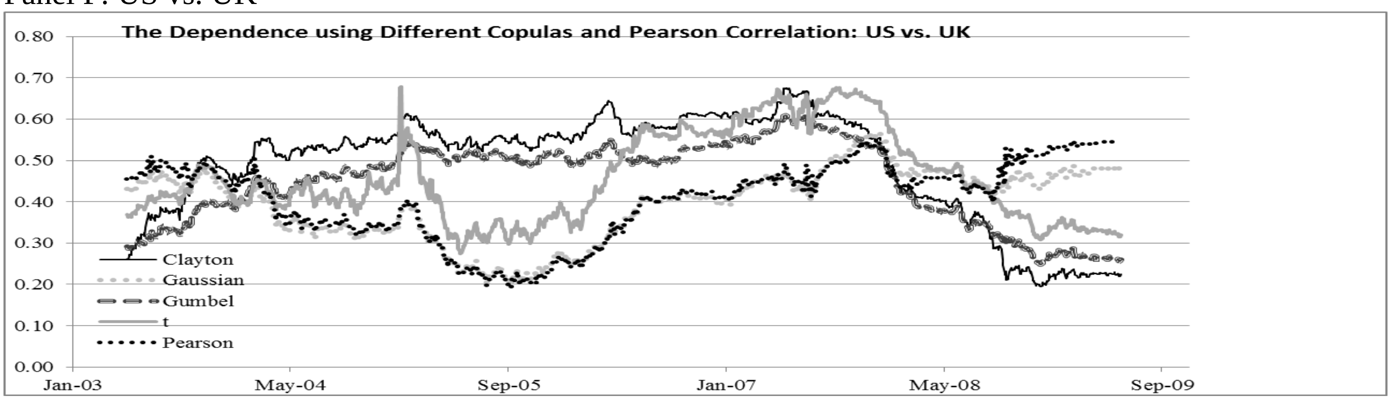
Panel D: US vs. Italy



Panel E: US vs. Japan



Panel F: US vs. UK



Appendix A

Appendix A illustrates the dependence of the G7 countries from different dependence models. Note that to ease the comparison between dependences, we transform the Gumbel dependences by (1- δ). Therefore, the range for the Clayton and the Gumbel copulas is between 0 and 1, with 0 meaning no dependence and 1 standing for perfect dependence. The range for the Gaussian copula, the Student's *t*-copula, and the Pearson correlation is -1 to 1, with 0 meaning no dependence and 1 or -1 standing for complete dependence.

	Panel A: Gaussian Dependence						
	CA	FR	DE	IT	JP	U.K.	U.S.
CA							
Max							
Min							
FR							
Max	0.7064						
Min	0.3914						
DE							
Max	0.6487	0.9686					
Min	-0.2016	0.7856					
IT							
Max	0.6320	0.9407	0.9274				
Min	-0.2143	-0.2303	0.7001				
JP							
Max	0.1814	0.2761	0.2478	0.4023			
Min	-0.2115	-0.2238	-0.2229	0.0119			
U.K.							
Max	0.6393	0.9100	0.8665	0.8575	0.4664		
Min	-0.1945	-0.2384	-0.2008	-0.2050	0.0329		
U.S.							
Max	0.7221	0.5031	0.5231	0.4661	0.5831	0.5671	
Min	-0.1864	-0.2127	-0.2087	-0.2414	-0.2241	0.1997	
	Panel B: Student's t Dependence						
	CA	FR	DE	IT	JP	U.K.	U.S.
CA							
Max							
Min							
FR							
Max	0.7509						
Min	0.3921						
DE							
Max	0.4578	0.9810					
Min	-0.1070	0.7476					
IT							
Max	0.4367	0.9810	0.9586				
Min	-0.1089	0.7476	0.6937				
JP							
Max	0.1093	0.1679	0.1522	0.6846			
Min	-0.1284	-0.1511	-0.1574	0.0093			
U.K.							
Max	0.4478	0.8055	0.7380	0.7316	0.7335		
Min	-0.1094	-0.1546	-0.1359	-0.1230	0.0284		
U.S.							
Max	0.5733	0.8055	0.3457	0.3176	0.4375	0.6614	
Min	-0.1107	-0.1546	-0.1343	-0.1360	-0.1519	0.2687	

Panel C: Gumbel Dependence							
	CA	FR	DE	IT	JP	U.K.	U.S.
CA							
Max							
Min							
FR							
Max	0.5947						
Min	0.3220						
DE							
Max	0.3744	0.9063					
Min	0.0000	0.5928					
IT							
Max	0.3666	0.7286	0.8544				
Min	0.0000	0.0000	0.5516				
JP							
Max	0.0961	0.1384	0.1222	0.5356			
Min	0.0000	0.0000	0.0000	0.0200			
U.K.							
Max	0.3650	0.6946	0.6156	0.6086	0.5855		
Min	0.0000	0.0000	0.0000	0.0000	0.0234		
U.S.							
Max	0.4340	0.2816	0.2952	0.2649	0.3261	0.5967	
Min	0.0000	0.0000	0.0000	0.0000	0.0000	0.2493	

Panel D: Clayton Dependence							
	CA	FR	DE	IT	JP	U.K.	U.S.
CA							
Max							
Min							
FR							
Max	0.6763						
Min	0.2899						
DE							
Max	0.3585	0.9327					
Min	0.0000	0.6696					
IT							
Max	0.3463	0.7635	0.9004				
Min	0.0000	0.0000	0.6006				
JP							
Max	0.0038	0.0408	0.0287	0.6476			
Min	0.0000	0.0000	0.0000	0.0000			
U.K.							
Max	0.3657	0.7193	0.6567	0.6506	0.6794		
Min	0.0000	0.0000	0.0000	0.0000	0.0000		
U.S.							
Max	0.5248	0.2450	0.2476	0.1987	0.3472	0.6622	
Min	0.0000	0.0000	0.0000	0.0000	0.0000	0.1982	

Panel E: Pearson Correlation							
	CA	FR	DE	IT	JP	U.K.	U.S.
CA							
Max							
Min							
FR							
Max	0.7002						
Min	0.3966						
DE							
Max	0.6944	0.9726					
Min	0.3645	0.7890					
IT							
Max	0.6833	0.9596	0.9477				
Min	0.3877	0.8204	0.6965				
JP							
Max	0.3549	0.4594	0.4702	0.4073			
Min	-0.0411	0.0251	0.0170	-0.0072			
U.K.							
Max	0.7080	0.9573	0.9298	0.9181	0.4612		
Min	0.3676	0.7791	0.6572	0.6990	0.0154		
U.S.							
Max	0.7586	0.6096	0.7443	0.5871	0.2078	0.5480	
Min	0.3764	0.2647	0.2921	0.2481	-0.1562	0.1913	