A Study of Employee Stock Options and the Exercise Decision

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ABSTRACT

This paper provides an analytic approximation for finite horizon American employee stock options (ESOs) and a closed form solution for perpetual American ESOs, which take into account illiquidity and default risk. The derived formulas are simply like that of the market values with altered parameters. Using all recorded executive stock options issued between 1992 and 2004, we study the impact of factors on ESO values and the exercise decision including: the illiquidity on the stock holding, level of risk aversion, moneyness, dividend, time to maturity, total volatility and normal unsystematic volatility. We further present the sentiment estimations and analyze its effect on ESOs.

JEL: G11, G13, G32, G35

Keywords: Employee stock options, exercise boundary, jump diffusion model, sentiment

1. Introduction

The use of stock option programs for employees has attracted considerable attention both in corporate governance and finance research. In the knowledge-based economy, the most important factor in determining enterprise success may be talent. Enterprises and employees may seek a joint perspective on shared future benefits through an employee stock option (ESO) plan. Indeed, small and medium-sized enterprises often cannot attract or retain talent based on salary compensation alone, so clever applications of ESOs provide a realizable future capital gain possibility to employees that they may find attractive.[3] The question is "how to value ESOs? How factors affect the ESO values and the exercise decision?" If firms underestimate the ESO values, they spend too much to compensate employees, or if they overestimate the value of ESOs, they do not have enough incentive to attract or retain talent. Understanding the impact of factors on ESO values and the exercise decision helps firm to design the ESO program. Our paper seeks to illuminate these issues.

Standard methods for valuing options are difficult to apply in these ESOs. Unlike the traditional options, ESOs usually have a vesting period during which they cannot be exercised and employees are not permitted to sell their ESOs. We consider subjective value to be what a constrained agent would pay for the ESOs and market value to be the value perceived by an unconstrained agent. Due to the illiquidity of ESOs, many employees have undiversified portfolios with large stock options for their own firms. Hence, a risk averse employee discounts the ESO values. Lambert et al. (1991) and Hall and Murphy (2002) study how risk preferences and structures of individuals' wealth affect the valuation of ESOs. These papers and others show that subjective value is lower than market value owing to the constrained fixed holding

[3]ESOs can potentially help firms to retain talent and reduce agency costs (Jensen and Meckling, 1976) and mitigate risk-related incentive problems (Agrawal and Mandelker, 1987; Hemmer et al., 2000) as well as attract highly motivated and able potential employees (Core and Guay, 2001; Oyer and Schaefer, 2005).

in the underlying stock.

Our model extends Chang et al. (2008) which considers default jump and European ESOs in a world where an employee allocates his wealth among the company's stock, the market portfolio, and a risk-free security with constrained fixed holding in his company's stock. Different from Chang et al. (2008), our paper employs a double exponential jump diffusion model which captures the leptokurtic feature of the return distribution and the volatility smile observed in options prices and admits the jump has a recovery proportion (Kou, 2002). Besides, our option contract is American type. Hemmer et al. (1996), Huddart and Lang (1996), and Bettis et al. (2005) show that early exercise is a pervasive phenomenon owing to risk aversion and undiversification of employees. Importantly, early exercise effect is critical in valuation of ESOs, especially for employees that are more risk averse and when there are more restrictions on the stock holding. A proper calculation must recognize that the decision to exercise is endogenous. We extend the method developed in Gukhal (2001), with a modification to include that an agent faces a constrained portfolio problem, and derive the exercise policies endogenously. In fact, employee exercise decisions and American ESO values are closely related: if an employee exercises his options, he values it less than or equal to its realizable intrinsic value at the exercise date. Conversely, if an employee does not exercise his options, he deems the option value exceeds the intrinsic value he can realize by exercising. Thus, factors affecting the employees' exercise policies will directly influence the valuation of ESOs.

For simple use of the proposed model, this paper attempts to extend the analytical tractability of Black-Scholes analysis as in Ingersoll (2006). We first give an analytic approximation for finite horizon American ESOs, and then provide a closed form solution for perpetual American ESOs, which are simply like that of the market values with altered parameters. Numerical simulations are also given for illustration.

Often the manager awarded an incentive option may have different beliefs about the company's prospects than the public investor. The employee believes that he possesses private information and can benefit from it. Or he has behavioral over-confidence regarding future risk-adjusted return of his firm and believes ESOs are valuable. Hodge et al. (2009) provides survey evidence and finds that managers subjectively value stock options greater than their Black-Scholes values. Oyer and Schaefer (2005) and Bergman and Jenter (2007) posit that employees attach a sentiment premium to their stock options, and firms exploit this sentiment premium to attract and retain optimistic employees. We also study the sentiment effect on ESOs.

In order to find what risk-adjusted return is needed to compensate the ESOs risk premium, we estimate the level of sentiment from two perspectives. First, we consider the sentiment effect on ESO values, and then the estimated sentiment level can be calculated whereby the subjective value with sentiment is equal to the market value. Secondly, we estimate sentiment level from the early exercise behavior, i.e., what is the value of sentiment such that employees exercise their options at the time that unconstrained investors do. We find that the more risk averse the employee and the more restrictions on the stock holding, the higher the sentiment level is needed.

The remainder of this paper is organized as follows. Section 2 develops our model and derives the pricing formulae for finite horizon and perpetual American ESOs. Section 3 presents the exercise policies, factors effect on ESO values and the exercise decision and a comparison between perpetual and finite horizon American ESOs. Default risk analysis is also given for illustration. Section 4 studies the impact of sentiment on ESOs. Section 5 offers concluding remarks. Justifications of our formulae are deferred to the Appendix.

2. Employee Stock Option Valuation

In this section, we will study the underlying assets' models and then give pricing formulae for finite horizon and perpetual American ESOs. To this end, we consider a utility-maximizing model that the employee allocates his wealth among three assets: the company stock S, the market portfolio M, and the risk-free bond B. Due to the illiquidity of ESO, the employee is constrained to allocate a fixed fraction α of his wealth to company stock (via some form of ESO). The employee's utility function $U(\cdot)$ is set as $U(C) = \frac{C^\gamma}{\gamma}$ with a coefficient of relative risk aversion $R(C) = -\frac{CU''(C)}{U'(C)} = 1 - \gamma$. For simplicity, we assume that CAPM holds so that the efficient portfolio is the market. Define the jump-diffusion processes for the three assets as follows:

$$\begin{cases} \frac{dS}{S} = (\mu - d - \lambda k)dt + \sigma_s dW_m + \nu dW_s + d\sum_{i=0}^{N_t}(Y_i - 1), \\ \frac{dM}{M} = (\mu_m - d_m)dt + \sigma_m dW_m, \\ \frac{dB}{B} = rdt, \end{cases} \tag{1}$$

where μ, μ_m, r are instantaneous expected rates of return for the stock, market portfolio and risk-free bond, respectively. d and d_m are dividends for the stock and market portfolio, respectively. The Brownian motion process W_m represents the Normal systematic risk of the market portfolio. The Brownian motion process W_s and jump process N_t are the idiosyncratic risk of the company stock, where N_t captures the jump risk of company stock and follows a Poisson distribution with average frequency λ. $Y_i - 1$ represents the percentage of stock variation when ith jump occurs. Denote $E(Y_i - 1) = k$ and $E(Y_i - 1)^2 = k_2$ for all i. σ_s and σ_m are the Normal systematic portions of total volatility for the stock and the market portfolio, respectively, while ν is the Normal unsystematic volatility of the stock. The two Brownian motions and jump process are presumed independent.

To derive the ESO values, we find a probability measure P^* by using change of measure method for jump diffusion models. Here we give a brief summary and define necessary notations. More detailed explanation is given in the Appendix A. Let $J[W(t), t]$ and $J_W[W(t), t] = \frac{\partial J[W(t),t]}{\partial W(t)}$ be the employee's total utility and marginal utility at time t, respectively, where

$W(t)$ is the employee's wealth at time t. Assume $B(t,T)$ be the price of a zero coupon bond at time t with maturity date T and then the bond yield $r^* := \frac{-1}{T-t}\ln B(t,T)$. Define $Z(t) = e^{r^*t}J_W[W(t),t]$, hence, the marginal rate of substitution $\frac{J_W[W(T),T]}{J_W[W(t),t]} = e^{-r^*(T-t)}\frac{Z(T)}{Z(t)}$. Then the rational equilibrium value of the ESO $F(S,t)$ satisfies the Euler equation

$$F(S,t) = \frac{E_t\{J_W[W(T),T]F(S,T)\}}{J_W[W(t),t]} = e^{-r^*(T-t)}E_t^*[F(S,T)], \tag{2}$$

where $F(S,T)$ is the payoff at the maturity T, $\frac{dP^*}{dP} = \frac{Z(T)}{Z(t)}$ and E_t^* is the expectation under P^* and information at time t. By using the probability measure P^*, the derived ESO formula is simply like that of the market values with altered parameters. See Theorems 1 and 2 for details.

2.1. Finite Horizon American ESOs

Suppose that the option can be exercised at n time instants. These time instants are assumed to be regularly spaced at intervals of Δt, and denoted by t_i, $0 \le i \le n$, where $t_0 = 0$, $t_n = T$, and $t_{i+1} - t_i = \Delta t$ for all i. Denote C_A as the value of American call option, C_E as the value of European call option, K as the strike price, and $S_i = S_{t_i}$. The critical price at these time points is denoted by S_i^*, $0 \le i \le n$, and is the price at which the agent is indifferent between holding the option and exercising. Denote E_i^* as the expectation under P^* and information at time t_i.

Theorem 1 *The value of the American ESO exercisable at n time instants, when the ESO is not exercised, written on the jump-diffusion process in* (1) *is as follows*

$$\begin{aligned} & C_A(S_0,T) \\ = \; & C_E(S_0,T) + \sum_{\ell=1}^{n-1} e^{-r^*\ell\Delta t}E_0^*\{[S_\ell(1-e^{-d^*\Delta t}) - K(1-e^{-r*\Delta t})]I_{\{S_\ell \ge S_\ell^*\}}\} \\ & -\sum_{j=2}^{n} e^{-r^*j\Delta t}E_0^*\{[C_A(S_j,(n-j)\Delta t) - (S_j - K)]I_{\{S_{j-1}\ge S_{j-1}^*\}}I_{\{S_j < S_j^*\}}\}. \end{aligned} \tag{3}$$

The critical price S_i^* at time t_i for $i = 1,\cdots,n$ is defined as the solution to the following *equation*

$$
\begin{aligned}
& S_i^* - K \\
= \;& C_E(S_i^*, (n-i)\Delta t) + \sum_{\ell=1}^{n-i-1} e^{-r^*\ell\Delta t} E_i^*\{[S_{i+\ell}(1-e^{-d^*\Delta t}) - K(1-e^{-r^*\Delta t})] I_{\{S_{i+\ell}\geq S_{i+\ell}^*\}}\} \\
& - \sum_{j=2}^{n-i} e^{-r^* j\Delta t} E_i^*\{[C_A(S_{i+j}, (n-i-j)\Delta t) - (S_{i+j}-K)] I_{\{S_{i+j-1}\geq S_{i+j-1}^*\}} I_{\{S_{i+j}<S_{i+j}^*\}}\},
\end{aligned}
$$

where

$$
\begin{aligned}
C_E(S_0,T) &= \sum_{j=0}^{\infty} \frac{(\lambda^* T)^j e^{-\lambda^* T}}{j!} \left\{ S_0 e^{-d^* T} E_0^* \left[\prod_{i=0}^{j} Y_i \Phi(d_1^*) \right] - K e^{-r^* T} E_0^* \left[\Phi(d_2^*)\right] \right\}, \\
d_1^* &= \frac{\ln[S_0 \prod_{i=0}^{j} Y_i / K] + [r^* - d^* - \lambda^*(\xi^*-1) + \frac{1}{2}\sigma^2]T}{\sigma\sqrt{T}}, \; d_2^* = d_1^* - \sigma\sqrt{T}, \\
r^* &= r - (1-\gamma)(\alpha\lambda k + \frac{1}{2}\gamma\lambda k_2 \alpha^2 + \alpha^2\nu^2) - \lambda(\xi - 1), \; \sigma^2 = \sigma_s^2 + \nu^2, \\
d^* &= d - (1-\gamma)[\alpha\lambda k + \frac{1}{2}\gamma\lambda k_2\alpha^2 - (1-\alpha)\alpha\nu^2] - \lambda(\xi-1) + \lambda k - \lambda^*(\xi^*-1), \\
\xi &= E[\alpha(Y_i - 1) + 1]^{\gamma-1}, \; \lambda^* = \lambda\xi, \; \xi^* = \frac{1}{\xi} E\{Y_i[\alpha(Y_i-1)+1]^{\gamma-1}\}.
\end{aligned}
$$

The proof of Theorem 1 is in Appendix B.

The value of American call option, when exercise is allowed at any time before maturity, is obtained by taking the limit as Δt tends to zero in equation (3).

2.2. Perpetual American ESOs

Perpetual American options are interesting because they serve as simple examples to illustrate finance theory. Furthermore they have some applications in studying real options, and the solution of the infinite horizon problems can lead to an approximation for the value of finite horizon American options (Kou and Wang, 2004). In the ESO context, under *a double exponential jump diffusion model* we will derive a closed form solution for the perpetual American options. In fact, under such model, Kou (2002) shows that the rational-expectations equilibrium price of an option is given by the expectation of the discounted option payoff under a risk-neutral probability measure P^* when using a HARA type utility function for a representative agent. Under P^*, the return process of stock price S_t, $X_t := \ln(S_t/S_0)$, is given by

$$X_t = [r^* - d^* - \frac{1}{2}\sigma^2 - \lambda^*(\xi^* - 1)]t + \sigma W_t^* + \sum_{i=0}^{N_t} U_i, \;\; X_0 = 0,$$

where W_t^* is the standard Brownian motion, N_t is a Poisson process with rate λ^* and U_i are i.i.d. jumps with double exponential distribution ($U_i \sim Douexp(p, \eta_1, \eta_2)$)

$$f_U^*(u) = p\eta_1 e^{-\eta_1 u} I_{\{u \geq 0\}} + q\eta_2 e^{\eta_2 u} I_{\{u<0\}}, \quad \eta_1 > 1, \; \eta_2 > 0.$$

Denote $G(x) = x\mu^* + \frac{1}{2}x^2\sigma^2 + \lambda^*(\frac{p\eta_1}{\eta_1 - x} + \frac{q\eta_2}{\eta_2 + x} - 1)$, with $\mu^* = r^* - d^* - \frac{1}{2}\sigma^2 - \lambda^*(\xi^* - 1)$. The moment generating function of X_t is $E^*(e^{\theta X_t}) = \exp[G(\theta)t]$. Kou and Wang (2003) shows that for $a > 0$, the equation $G(x) = a$ has exactly four roots: $\beta_{1,a}$, $\beta_{2,a}$, $-\beta_{3,a}$, $-\beta_{4,a}$, where $0 < \beta_{1,a} < \eta_1 < \beta_{2,a} < \infty$ and $0 < \beta_{3,a} < \eta_2 < \beta_{4,a} < \infty$.

Theorem 2 *Assume that*

$$r^* + \lambda^* q \frac{\beta_{1,r^*}\beta_{2,r^*}(\eta_1 + \eta_2)}{\eta_1(\eta_2 + 1)(\beta_{1,r^*} + \eta_2)(\beta_{2,r^*} + \eta_2)} - d^* \frac{(\eta_1 - 1)\beta_{1,r^*}\beta_{2,r^*}}{\eta_1(\beta_{1,r^*} - 1)(\beta_{2,r^*} - 1)} < 0. \tag{4}$$

The value of the perpetual American ESO, written on the jump-diffusion process in (1)*, is given by* $V(S_t)$*, where the value function is given by*

$$V(v) = \begin{cases} v - K, & v \geq v_0, \\ Av^{\beta_{1,r^*}} + Bv^{\beta_{2,r^*}}, & v < v_0, \end{cases} \tag{5}$$

with the optimal exercise boundary

$$v_0 = K \frac{\eta_1 - 1}{\eta_1} \frac{\beta_{1,r^*}}{\beta_{1,r^*} - 1} \frac{\beta_{2,r^*}}{\beta_{2,r^*} - 1},$$

and the coefficients

$$A = {v_0}^{-\beta_{1,r^*}} \frac{\beta_{2,r^*} - 1}{\beta_{2,r^*} - \beta_{1,r^*}} (v_0 - \frac{\beta_{2,r^*}}{\beta_{2,r^*} - 1}K) > 0, \quad B = {v_0}^{-\beta_{2,r^*}} \frac{\beta_{1,r^*} - 1}{\beta_{2,r^*} - \beta_{1,r^*}} (\frac{\beta_{1,r^*}}{\beta_{1,r^*} - 1}K - v_0) > 0.$$

Furthermore, the optimal stopping time is given by $\tau^* = inf\{t \geq 0 : S_t \geq v_0\}$.

The proof of Theorem 2 is given in Appendix C.

An employee does not exercise his ESOs early when he has no constrained stock holding ($\alpha = 0$) and no dividend paying ($d = 0$). However, the assumption in Theorem 2, equation (4), ensures the possibility of early exercise. Note that equation (4) is satisfied in general parameters setting.

In the case of no jump part, we consider the diffusion processes for three assets as follows:

$$\begin{cases} \frac{dS}{S} = (\mu - d)dt + \sigma_s dW_m + \nu dW_s, \\ \frac{dM}{M} = (\mu_m - d_m)dt + \sigma_m dW_m, \\ \frac{dB}{B} = rdt, \end{cases} \tag{6}$$

with all parameters defined as equation (1).

Corollary 1 *The value of the perpetual American ESO with $\tilde{d} > 0$, written on the diffusion process in (6) is given by $V(S_t)$, where the value function is given by*

$$V(v) = \begin{cases} v - K, & v \geq L, \\ \tilde{A}v^h, & v < L, \end{cases} \tag{7}$$

with the optimal exercise boundary and the coefficients

$$L = \frac{h}{h-1}K; \ \tilde{A} = (L-K)L^{-h}, \ h = \frac{1}{\sigma^2}[\sqrt{\tilde{\mu}^2 + 2\tilde{r}\sigma^2} - \tilde{\mu}],$$
$$\tilde{\mu} = \tilde{r} - \tilde{d} - \frac{1}{2}\sigma^2, \ \tilde{r} = r + \alpha s - (1-\gamma)\alpha^2 v^2, \ \tilde{d} = d - (1-\alpha)s + (1-\gamma)\alpha(1-\alpha)v^2.$$

Moreover, the optimal stopping time is given by $\tilde{\tau} = inf\{t \geq 0 : S_t \geq L\}$.

Note that the value of jump-diffusion perpetual American ESO reduces to the diffusion's case by taking $\lambda^* = 0$ and $\eta_1 \to \infty$ in Theorem 2.

3. Empirical Findings

Section 2 provides a pricing model for ESOs that includes illiquidity of the options and a jump diffusion process for the stock price evolution in a world where employees balance their wealth between the company's stock, the market portfolio, and a risk-free asset. Moreover, from this ESO pricing formula, we can not only estimate the subjective values but also study the exercise policies. The exercise boundary is endogenously derived by finding the minimum stock price such that the option value equals its intrinsic value for each time. In other words, the employee exercises the option when stock price is above the exercise boundary. To illustrate our model, in this section, we discuss factors which affect ESO values and exercise decisions including: the illiquidity on the stock holding, level of risk aversion, moneyness, dividend, time to maturity, total volatility and normal unsystematic volatility. A comparison between perpetual and finite horizon American ESOs and default risk analysis are also given for illustration.

To calibrate the parameters for evaluating option prices, we collect data between 1992 and 2004 from the Compustat Executive compensation database. We use the default parameterizations according to the median values in our collected data set. Stock price S, strike price K, total volatility σ, dividend yield d, interest free rate r, time to maturity τ are 25, 25, 0.3, 2%, 5%, 10, respectively. Normal unsystematic volatility ν is two-thirds of the total volatility following calibrations applied by Bettis et al. (2005) and Ingersoll (2006). We use two jump size models: double exponential[4] and Y=0 (no residual value). Additionally, we follow Duffee (1999) and Fruhwirth and Sogner (2006), using US and German bond data, respectively, and estimate median default intensity $\lambda = 0.01$.

3.1. Exercise Behavior

Employees exercising their ESOs earlier are pervasive phenomena. Considering the exercise

[4]The parameters of double exponential are estimated by daily return data from 1992 to 2004. We define a jump if return goes beyond ±10% which relates to an approximately 3-standard-deviation daily return during this period.

policies is necessary for studying American ESOs. This is an essential departure from Chang et al. (2008) which considers European type ESOs. A number of papers link early exercise behavior to under-diversification of employees (Hemmer et al., 1996; Core and Guay, 2001; Bettis et al., 2005). The problem of valuing ESOs with early exercise is often approximated in practice by simply using the expected time until exercise in place of the actual time to maturity (Hull and White, 2004; Bettis et al., 2005). The expected time until exercise is estimated from past experience. However, Ingersoll (2006) mentions that even using an unbiased estimate of the expected time until exercise will not give a correct estimate of the option's value. And this method cannot be used to determine the subjective value since it will be smaller due to the extra discounting required to compensate the lack of diversification.

A proper calculation must recognize that the decision to exercise is endogenous. Liao and Lyuu (2009) incorporates the exercise pattern instead of using the expected time until exercise technique in valuation of ESOs, to which the exercise patterns are under Chi-square distribution assumption and not derived endogenously. Ingersoll (2006) derives the exercise boundaries endogenously, while the exercise policies are restricted constant in time. We extend the method developed in Gukhal (2001), with a modification to include that an agent faces a constrained portfolio problem, and derive the time varying exercise policies endogenously.

[Insert Figure 1 here]

Which factors cause employees to exercise their options early? Figure 1 compares the exercise boundaries for some factors. Note that exercise boundaries are decreasing function of time in all cases, which are different from the constant exercise policies in Ingersoll (2006). The more restrictions on the stock holding or the more risk averse the employee, the lower the exercise boundary. In other words, because of the impossibility of full diversification employees who are more restricted on the stock holding or more risk averse prefer early exercise their options. The employees who receive the in the money type options also tend to early exercise.

Besides, larger dividends induce employees to exercise their options sooner. Options with shorter lifetime are quicker exercised. Employees do not have much time value in these options and tend to exercise their options earlier. Employees early exercise volatile options to balance their portfolio risk especially for idiosyncratic risk increasing. Indeed, our model findings are consistent with several empirical studies. For instance, Hemmer et al. (1996), Huddart and Lang (1996), and Bettis et al. (2005) show that early exercise is a pervasive phenomenon owing to risk aversion and undiversification of employees. Huddart and Lang (1996) find that exercise is negatively related to the time to maturity and positively correlated with the market-to-strike ratio and with the stock price volatility. Hemmer et al. (1996) and Bettis et al. (2005) also find that stock price volatility has a significant effect on exercise decisions. In high volatility firms, employees exercise options much earlier than in low volatility firms.

3.2. Factors Effect on Employee Stock Options and the Exercise Decision

Understanding the factors which affect ESO values and the exercise decision is important for firm to design the stock option programs. As we mentioned before, ESO values and exercise decisions are closely related. Factors affecting the employees exercise policies will directly influence the valuation of ESOs. We discuss these issues and focus on the studying factors. The results are shown in Table 1, which presents the studying factors effect on ESO value, discount ratio, and early exercise premium, where ESO value is calculated by formula (3), discount ratio is defined as one minus the ratio of subjective to market value, and early exercise premium is the difference between American and European ESO value.

3.2.1. Illiquidity on the Stock Holding, Level of Risk Aversion, Moneyness

Unlike traditional options, ESOs usually have a vesting period during which they can not be exercised and employees are not permitted to sell their ESOs. In this situation, employees receive the ESOs in a very illiquid market. From Table 1 we find subjective values ($\alpha \neq 0$) are uniformly smaller than the market values ($\alpha = 0$). These results are consistent with Lambert et al. (1991) and Hall and Murphy (2002) that the subjective value is lower than market value due to the constrained fixed holding in the underlying stock. The more risk averse the employee (more positive $1-\gamma$) and more restrictions on the stock holding (larger α), lean to depreciate the option values and incur the higher early exercise premium. Note that early exercise effect on ESO values can not be ignored in these situations.

Because of the illiquidity of ESOs, many employees have undiversified portfolios with large stock options for their own firms. Therefore, a risk averse employee discounts the ESO values. Discount ratios increase with the illiquidity on the stock holding and the degree of risk aversion. In other words, employees who are more risk averse and more restricted on the stock holdings need to compensate more risk premium. In the money options have higher values, lower discount and higher early exercise premium. Interesting, even in the money options having less discount than out of the money, employees still more tend to early exercise in the money options to diversify their wealth portfolio risk.

[Insert Table 1 here]

3.2.2. Dividend, Time to Maturity, Volatility Risk

Larger dividends depreciate the option values and induce employees to exercise their options sooner even they have lower discount ratios. More interestingly, the early exercise premium is not zero when no dividends paid. This is a departure from traditional option theory, while it is consistent with the phenomenon that ESOs are exercised substantially before maturity date even ESOs not paying dividends because of the lack of diversification. Options with longer lifetime have more values, at the same time, they have higher discount ratios and early exercise premiums. Although not reported in the table, the lifetime of option may be negatively related to European ESO value. This is due to the longer one has to wait and then the more the risk caused by undiversification affects the ESO value. It is worth to mention that this phenomenon is different from the result of American ESO, the commonly used ESO contract.

While options may provide incentives for employees to work harder, they can also induce suboptimal risk-taking behavior. General option pricing results show that value should increase with risk while employees need to compensate more risk premium at the same time. It is not necessarily that subjective value is positive related to risk, as is the traditional result.[5] We have usual finding that total volatility increases the option value, however, with respect to normal unsystematic volatility, we find the opposite that the subjective value decreases with it. In Black-Scholes framework, this risk is eliminated under risk-neutral measure. However, in our model, the employee has an illiquid holding and full diversification is impossible. Hence, a risk averse employee depreciates the ESO values. The discount and early exercise premium increasing with the volatility risk also can be found in Table 1. This is intuitive, since the more volatile stock price, the higher is the opportunity cost of not being able to exercise. Therefore, employees have more incentives to early exercise volatile options. All factors effect are summarized in Panel B.

3.3. Perpetual American Options

We calculate the perpetual ESO values by formula (5). Table 2 presents the perpetual option results that include values, optimal exercise boundaries and differences between perpetual and finite horizon American ESOs. Note that for values and optimal exercise boundaries, perpetual American ESOs have the same patterns as those for finite horizon American ESOs. That is, subjective values are uniformly smaller than the market values; the more risk averse the employee and more stock holding restrictions lean to depreciate the option values and decline the exercise boundaries; for moneyness, in the money options have higher option values

[5]Nohel and Todd (2005), Ryan and Wiggins (2001), and others show that option values increase with risk, however, they do not study the impact of increased idiosyncratic risk. Carpenter (2000) presents examples where convex incentive structures do not imply that the manager is more willing to take risks. The model used in Chang et al. (2008) is able to capture this result.

and lower exercise boundaries.

Interestingly, the differences between perpetual and finite horizon American ESOs are related to factors that affect exercise behavior. Specifically, the differences are reduced when employees face large restricted holding, are more risk averse and receive in the money type options. In these situations, the employees tend to exercise early. The relative difference, which is defined as the ratio of difference between perpetual and finite horizon American ESO to finite horizon American ESO, also has the same phenomenon. In other words, perpetual American ESO approximates finite horizon American ESO better when an agent with large restricted holding, more risk averse and receiving in the money type options. These phenomena can be explained as that the time values of perpetual options are reduced in these situations. Note that from our simulation studies, the same phenomenon holds when there is no jump occurs. The simulation results will be provided upon request.

[Insert Table 3 here]

3.4. Default Risk

Here, we study the impact of default risk on ESO values. In Table 3 we compare two cases: stock having no residual value if jump occurs (default jump) and stock following diffusion process (no jump). When employees face less restricted holding ($\alpha = 0, 0.25$), the values of options with default risk are lager than the options if the underlying stocks follow diffusion processes. Interestingly, unlike the traditional option theory, we have the opposite results when employees are confronted by large restricted holding of company stock ($\alpha = 0.5, 0.75$). In other words, when employees encounter large restricted holding, the option values with default risk are no longer larger than the options if the underlying stock processes are continuous. From Panel C, we find that options with default risk have higher discount ratios. Again, in this situation, employees need to compensate more risk premium. However, from Panel B, there are no obvious patterns for early exercise premiums.

[Insert Table 3 here]

4. Sentiment Analysis

Often the manager awarded an incentive option may have different beliefs about the company's prospects than the investing public does. The manager believes that he possesses private information and can benefit from it. Or he has behavioral over-confidence regarding future risk-adjusted return of his firm and believes ESOs are valuable. Now, we consider the impact of sentiment on ESO values and the exercise decision. Define the processes for the three assets as follows:

$$\begin{cases} \frac{dS}{S} = (\mu + s - d - \lambda k)dt + \sigma_s dW_m + \nu dW_s + d\sum_{i=0}^{N_t}(Y_i - 1), \\ \frac{dM}{M} = (\mu_m - d_m)dt + \sigma_m dW_m, \\ \frac{dB}{B} = rdt, \end{cases} \tag{8}$$

Here, sentiment level be denoted by s. In other words, the employee over-estimates or rationally adjusts the risk-adjusted return of the company owing to inside information by s, then the same analysis in Theorem 1 is valid with a simple adjustment in parameters. The adjusted interest rate and dividend yield used in pricing are

$$\begin{aligned} r^* &= r + \alpha s - (1-\gamma)(\alpha\lambda k + \frac{1}{2}\gamma\lambda k_2\alpha^2 + \alpha^2\nu^2) - \lambda(\xi - 1), \\ d^* &= d - (1-\alpha)s - (1-\gamma)[\alpha\lambda k + \frac{1}{2}\gamma\lambda k_2\alpha^2 - (1-\alpha)\alpha\nu^2] - \lambda(\xi - 1). \end{aligned}$$

We estimate the level of sentiment from two perspectives. First, we consider the sentiment effect on ESO value (SenV), and then the estimated sentiment level can be calculated whereby subjective value with sentiment is equal to market value. Secondly, we estimate sentiment level from the early exercise perspective (SenE), i.e., what value of sentiment such that employees exercise their options at the time that unconstrained investors do. We also calculate the sentiment level of European ESOs (SenVE). Due to the limitation of European options, they are not allowed to early exercise, we can only estimate the sentiment level from value perspective.

Sentiment results are shown in Table 4. We only list the estimated sentiment level of at the money option since there is no obvious relationship between sentiment level and moneyness. We find that the more risk averse the employee and more restricted on the stock holding, the higher the sentiment level is needed. SenVE is slight higher than SenV because of the more restrictions in European contract. Employee sentiment enhances the option value and reduces the early exercise premium. Options with high sentiment having higher discount implies the option value declining sharply when employees face undiversification problem. Employee with high sentiment will postpone the exercise timing due to the brightening prospect of the company.

[Insert Table 4 here]

5. Conclusion

In this paper, we present a model which employs a jump-diffusion methodology for ESOs that includes illiquidity of the options, a jump diffusion process for the stock price evolution, and the role of employee sentiment in a world where employees balance their wealth between the company's stock, the market portfolio, and a risk-free asset. Importantly, the option contract we considered is American type and the optimal exercise boundary is derived endogenously. From the ESO pricing formula, we can not only estimate the subjective values but also study the exercise policies. We solve for both the market value and subjective value of the ESO and find that subjective value is substantially lower than market value for reasonable parameter calibrations, and therefore the cost to the issuing firm is significantly larger than the value perceived by employees. In order to find what risk-adjusted return is needed to compensate the ESOs risk premium, we estimate the level of sentiment from two perspectives. We find that the more risk averse the employee and the more restrictions on the stock holding, the higher the sentiment level is needed. Hence, employee sentiment is a necessary consideration when issuing options and executives may be substantially over-valuing ESOs because of it.

When considering what factors cause employees to exercise their options early, it became clear that all factors need to be taken into consideration, not just a single perspective such as value, discount, or early exercise premium. As reported in the results, in the money options have higher values and idiosyncratic risk depreciates the ESO values, however, the options are exercised early in these two cases. Besides, both in the money options and long-term options have higher early exercise premium. An employee that receives in the money options will exercise soon, conversely, he will exercise late for long-term options. Hence, to study the exercise policies we need to discuss several essential factors, to which it is important for firm to design the stock option programs.

Although our model is based on the standard American ESO contract, our methodology and results can be expanded to a more generalized class of illiquid securities. For instance this model might be applied to securities with owners who are illiquid, such as Pre-IPO holders of stock who face a moratorium on selling rights. Principals at firms for whom de facto constraints might be binding for signaling reasons would likewise be appropriately subsumed here.

Appendix A. Derivation of Risk-Neutral Probability P^*

Here we solve an optimal portfolio selection problem and derive the pricing kernel to obtain the risk-neutral measure. Since the optimal portfolio weights and wealth process can be similarly derived from Chang et al. (2008), it is omitted.

By Ito's formula for jump processes, the process of employee's marginal utility or the pricing kernel can be derived as:

$$\frac{dJ_W}{J_W} = -\hat{r}dt - \hat{\sigma}dW_m - (1-\gamma)\alpha\nu dW_s + d\sum_{i=0}^{N_t}\{[\alpha(Y_i-1)+1]^{\gamma-1}-1\},$$

where $J_W = \frac{\partial J[W(t),t]}{\partial W(t)}$ is the marginal utility, $J[W(t),t]$ and $W(t)$ are the employee's total utility and wealth at time t, $\hat{r} = r-(1-\gamma)(\alpha^2\nu^2 + \frac{1}{2}\alpha^2\gamma\lambda + \alpha\lambda k)$, and $\hat{\sigma} = \frac{\mu_m - r}{\sigma_m}$.

To find the risk-neutral probability P^*, let $B(t,T)$ be the price of a zero coupon bond at time t with maturity date T. Then the bond yield

$$r^* := -\frac{1}{T-t}\ln B(t,T) = r - (1-\gamma)(\alpha\lambda k + \frac{1}{2}\gamma\lambda k_2\alpha^2 + \alpha^2\nu^2) - \lambda(\xi - 1),$$

where $\xi = E[\alpha(Y_i-1)+1]^{\gamma-1}$. Define $Z(t) = e^{r^*t}J_W[W(t),t]$, hence, the marginal rate of substitution $\frac{J_W[W(T),T]}{J_W[W(t),t]} = e^{-r^*(T-t)}\frac{Z(T)}{Z(t)}$. The rational equilibrium value of the ESO $F(S,t)$ satisfies the Euler equation

$$F(S,t) = \frac{E_t\{J_W[W(T),T]F(S,T)\}}{J_W[W(t),t]} = e^{-r^*(T-t)}E_t^*[F(S,T)],$$

where $\frac{dP^*}{dP} = \frac{Z(T)}{Z(t)}$, $F(S,T)$ is the payoff at the maturity T and E_t^* is the expectation under P^* and information at time t. Under P^*, the stock process can be expressed as

$$\frac{dS}{S} = [r^* - d^* - \lambda^*(\xi^*-1)]dt + \sigma dW_t^* + d\sum_{i=0}^{N_t}(Y_i - 1),$$

where

$$\begin{aligned} d^* &= d - (1-\gamma)[\alpha\lambda k + \frac{1}{2}\gamma\lambda k_2\alpha^2 - (1-\alpha)\alpha\nu^2] - \lambda(\xi-1) + \lambda k - \lambda^*(\xi^*-1), \\ \sigma^2 &= \sigma_s^2 + \nu^2,\ \lambda^* = \lambda\xi,\ \xi^* = \frac{1}{\xi}E\{Y_i[\alpha(Y_i-1)+1]^{\gamma-1}\}, \end{aligned}$$

W_t^* is the standard Brownian motion and N_t is a Poisson process with rate λ^*. $\square$

Appendix B: Proof of Theorem 1 for Finite Horizon American ESOs

We will derive the valuation formula for the American call ESO exercisable at n time instants by backward induction. At time t_{n-1}, $C_A(S_{n-1}, \Delta t) = C_E(S_{n-1}, \Delta t)$. The exercise boundary is S_{n-1}^* such that $S_{n-1}^* - K = C_A(S_{n-1}^*, \Delta t)$. At time t_{n-2},

$$\begin{aligned}
& C_A(S_{n-2}, 2\Delta t) \\
= \;& e^{-r^*\Delta t} E_{t_{n-2}}^* \{(S_{n-1} - K) I_{\{S_{n-1} \geq S_{n-1}^*\}}\} + e^{-r^*\Delta t} E_{t_{n-2}}^* \{C_A(S_{n-1}, \Delta t) I_{\{S_{n-1} < S_{n-1}^*\}}\} \\
= \;& C_E(S_{n-2}, 2\Delta t) + e^{-r^*\Delta t} E_{t_{n-2}}^* \{[S_{n-1}(1 - e^{-d^*\Delta t}) - K(1 - e^{-r^*\Delta t})] I_{\{S_{n-1} \geq S_{n-1}^*\}}\} \\
& - e^{-r^* 2\Delta t} E_{t_{n-2}}^* \{[C_A(S_T, 0) - (S_T - K)] I_{\{S_{n-1} \geq S_{n-1}^*\}} I_{\{S_T < K\}}\}.
\end{aligned}$$

The exercise boundary is S_{n-2}^* such that $S_{n-2}^* - K = C_A(S_{n-2}^*, 2\Delta t)$.

Suppose that the value of the American ESO at time t_m, for $m < n-2$, can be expressed as

$$\begin{aligned}
& C_A(S_m, (n-m)\Delta t) \\
= \;& C_E(S_m, (n-m)\Delta t) + \sum_{\ell=1}^{n-m-1} e^{-r^*\ell\Delta t} E_{t_m}^* \{[S_{m+\ell}(1 - e^{-d^*\Delta t}) - K(1 - e^{-r^*\Delta t})] I_{\{S_{m+\ell} \geq S_{m+\ell}^*\}}\} \\
& - \sum_{j=2}^{n-m} e^{-r^* j\Delta t} E_{t_m}^* \{[C_A(S_{m+j}, (n-m-j)\Delta t) - (S_{m+j} - K)] I_{\{S_{m+j-1} \geq S_{m+j-1}^*\}} I_{\{S_{m+j} < S_{m+j}^*\}}\}.
\end{aligned}$$

By induction, we consider the case at time t_{m-1},

$$\begin{aligned}
& C_A(S_{m-1}, (n-m+1)\Delta t) \\
= \;& e^{-r^*\Delta t} E_{t_{m-1}}^* \{(S_m - K) I_{\{S_m \geq S_m^*\}}\} + e^{-r^*\Delta t} E_{t_{m-1}}^* \{C_A(S_m, (n-m)\Delta t) I_{\{S_m < S_m^*\}}\}.
\end{aligned} \tag{B.1}$$

Note that the first term in (B.1)

$$\begin{aligned}
& e^{-r^*\Delta t}E^*_{t_{m-1}}\{(S_m-K)I_{\{S_m\geq S^*_m\}}\} \\
= \;& e^{-r^*(n-m+1)\Delta t}E^*_{t_{m-1}}\{(S_T-K)I_{\{S_T\geq S^*_T\}}I_{\{S_m\geq S^*_m\}}\} \\
& +\sum_{\ell=1}^{n-m} e^{-r^*\ell\Delta t}E^*_{t_{m-1}}\{[S_{m-1+\ell}(1-e^{-d^*\Delta t})-K(1-e^{-r^*\Delta t})]I_{\{S_{m-1+\ell}\geq S^*_{m-1+\ell}\}}I_{\{S_m\geq S^*_m\}}\} \\
& -\sum_{j=2}^{n-m+1} e^{-r^*j\Delta t}E^*_{t_m-1}\{[C_A(S_{m-1+j},(n-m-j+1)\Delta t)-(S_{m-1+j}-K)] \\
& \qquad \times I_{\{S_{m-1+j-1}\geq S^*_{m-1+j-1}\}}I_{\{S_{m-1+j}<S^*_{m-1+j}\}}I_{\{S_m\geq S^*_m\}}\}.
\end{aligned}$$

By induction, the second term in (B.1) is

$$\begin{aligned}
& e^{-r^*\Delta t}E^*_{t_{m-1}}\{C_A(S_m,(n-m)\Delta t)I_{\{S_m<S^*_m\}}\} \\
= \;& e^{-r^*\Delta t}E^*_{t_{m-1}}\{C_E(S_m,(n-m)\Delta t)I_{\{S_m<S^*_m\}}\} \\
& +\sum_{\ell=1}^{n-m-1} e^{-r^*(\ell+1)\Delta t}E^*_{t_{m-1}}\{[S_{m+\ell}(1-e^{-d^*\Delta t})-K(1-e^{-r^*\Delta t})]I_{\{S_{m+\ell}\geq S^*_{m+\ell}\}}I_{\{S_m<S^*_m\}}\} \\
& -\sum_{j=2}^{n-m} e^{-r^*(j+1)\Delta t}E^*_{t_{m-1}}\{[C_A(S_{m+j},(n-m-j)\Delta t)-(S_{m+j}-K)] \\
& \qquad \times I_{\{S_{m+j-1}\geq S^*_{m+j-1}\}}I_{\{S_{m+j}<S^*_{m+j}\}}I_{\{S_m<S^*_m\}}\}.
\end{aligned}$$

Hence, we prove that the result holds for $t=t_{m-1}$, and complete the whole proof. □

Appendix C: Proof of Theorem 2 for Perpetual American ESOs

To prove Theorem 2, we need the following lemma.

Lemma 1 *Suppose there exist some $x_0>\ln K$ and a non-negative C^1 function $V(x)$ such that* (1) *V is C^2 on $\mathbb{R}\backslash\{x_0\}$ and is convex with $V''(x_0-)$ and $V''(x_0+)$ existing;* (2) *$(LV)(x)-r^*V(x)=0\ \forall x<x_0$;* (3) *$(LV)(x)-r^*V(x)<0\ \forall x>x_0$;* (4) *$V(x)>(e^x-K)^+\ \forall x<x_0$;* (5) *$V(x)=(e^x-K)^+\ \forall x\geq x_0$;* (6) *there exists a random variable Z with $E^*(Z)<\infty$ such that $e^{-r(t\wedge\tau\wedge\tau^*)}V(X_{t\wedge\tau\wedge\tau^*}+x)\leq Z$, for any $t\geq 0,x$ and any stopping time τ. Then the option* *price* $\psi(S_0) = V(\ln(S_0))$ *and the optimal stopping time is given by* τ^*=*inf*$\{t \geq 0 : S_t \geq e^{x_0}\}$. *Here the infinitesimal generator* L *is defined as*

$$(LV)(x) := \frac{1}{2}\sigma^2 V''(x) + [r^* - d^* - \frac{1}{2}\sigma^2 - \lambda^*(\xi^* - 1)]V'(x) + \lambda^* \int_{-\infty}^{\infty} [V(x+u) - V(u)] f_U^*(u) du.$$

Since the proof follows an argument similar to that in Mordecki (1999) and Kou and Wang (2004), it is omitted.

Let $x = \ln v$, $x_0 = \ln v_0$, then

$$V(x) = \begin{cases} e^x - K, & x \geq x_0, \\ Ae^{\beta_{1,r^*} x} + Be^{\beta_{2,r^*} x}, & x < x_0. \end{cases}$$

To prove Theorem 2, we only need to check conditions in Lemma 1 hold. Conditions, 1, 4, and 5 are easily to verify. Condition 6 follows from Mordecki (1999). Therefore, we only need to check conditions 2 and 3 hold. For notation simplicity, we shall write $\beta_1 = \beta_{1,r^*}$, and $\beta_2 = \beta_{2,r^*}$.

For $x < x_0$,

$$\begin{aligned}
& \int_{-\infty}^{\infty} V(x+u) dF_U^*(u) \\
= & \int_{-\infty}^{x_0 - x} [Ae^{\beta_1(x+u)} + Be^{\beta_2(x+u)}] q\eta_2 e^{\eta_2 u} du \\
& + \int_{x_0 - x}^{0} (e^{x+u} - K) q\eta_2 e^{\eta_2 u} du + \int_0^{\infty} (e^{x+u} - K) p\eta_1 e^{-\eta_1 u} du \\
= & e^x (\frac{q\eta_2}{\eta_2 + 1} + \frac{p\eta_1}{\eta_1 - 1}) + qe^{\eta_2(x_0 - x)}(K - \frac{\eta_2 e^{x_0}}{\eta_2 + 1} + \frac{A\eta_2 e^{\beta_1 x_0}}{\beta_1 + \eta_2} + \frac{B\eta_2 e^{\beta_2 x_0}}{\beta_2 + \eta_2}) - K.
\end{aligned}$$

Then

$$\begin{aligned}
& (LV)(x) - r^* V(x) \\
= & Ae^{\beta_1 x} \left\{ \frac{1}{2}\sigma^2 \beta_1^2 + \mu^* \beta_1 + \lambda^* (\frac{p\eta_1}{\eta_1 - \beta_1} + \frac{q\eta_2}{\eta_2 + \beta_1} - 1) - r^* \right\} \\
& + Be^{\beta_2 x} \left\{ \frac{1}{2}\sigma^2 \beta_2^2 + \mu^* \beta_2 + \lambda^* (\frac{p\eta_1}{\eta_1 - \beta_2} + \frac{q\eta_2}{\eta_2 + \beta_2} - 1) - r^* \right\} \\
& + \lambda^* p e^{-\eta_1 (x_0 - x)} \left\{ \frac{\eta - 1 e^{x_0}}{\eta_1 - 1} - \frac{\eta_1 A}{\eta_1 - \beta_1} e^{\beta_1 x_0} + \frac{\eta_1 B}{\beta_2 - \eta_1} e^{\beta_2 x_0} - K \right\}.
\end{aligned}$$

By using the definitions of β_1 and β_2, and

$$\begin{aligned}
&\frac{\eta-1}{\eta_1-1}v_0 - \frac{\eta_1 A}{\eta_1-\beta_1}v_0^{\beta_1} + \frac{\eta_1 B}{\beta_2-\eta_1}v_0^{\beta_2} \\
= \; & \left\{\frac{\eta_1\beta_2}{(\eta_1-\beta_1)(\beta_2-\beta_1)} + \frac{\eta_1\beta_1}{(\beta_2-\eta_1)(\beta_2-\beta_1)}\right\}K \\
&-\left\{\frac{\eta_1(\beta_2-1)}{(\eta_1-\beta_1)(\beta_2-\beta_1)} + \frac{\eta_1(\beta_1-1)}{(\beta_2-\eta_1)(\beta_2-\beta_1)} - \frac{\eta_1}{\eta_1-1}\right\}v_0 = K,
\end{aligned}$$

condition 2 follows.

For $x > x_0$,

$$\begin{aligned}
&\int_{-\infty}^{\infty} V(x+u)dF_U^*(u) \\
= \; & \int_{-\infty}^{0}[Ae^{\beta_1(x+u)} + Be^{\beta_2(x+u)}]q\eta_2 e^{\eta_2 u}du \\
&+\int_0^{x_0-x}[Ae^{\beta_1(x+u)} + Be^{\beta_2(x+u)}]p\eta_1 e^{-\eta_1 u}du + \int_{x_0-x}^{\infty}(e^{x+u}-K)p\eta_1 e^{-\eta_1 u}du \\
= \; & pe^{-\eta_1(x_0-x)}(\frac{\eta_1 e^{x_0}}{\eta_1-1} - K) + \frac{p\eta_1 A}{\eta_1-\beta_1}[e^{\beta_1 x} - e^{-(x_0-x)\eta_1+\beta_1 x_0}] \\
&+\frac{p\eta_1 B}{\beta_2-\eta_1}[e^{-\eta_1(x_0-x)+\beta_2 x_0} - e^{\beta_2 x}] + A\frac{q\eta_2 e^{\beta_1 x}}{\beta_1+\eta_2} + B\frac{q\eta_2 e^{\beta_2 x}}{\beta_2+\eta_2}.
\end{aligned}$$

Then

$$\begin{aligned}
&(LV)(x) - r^*V(x) \\
= \; & \frac{1}{2}\sigma^2 e^x + [r^* - d^* - \frac{1}{2}\sigma_2\lambda^*(\xi^*-1)]e^x - (r^*+\lambda^*)(e^x-K) \\
&+\lambda^*\left\{e^x(\frac{q\eta_2}{\eta_2+1} + \frac{p\eta_1}{\eta_1-1}) + qe^{\eta_2(x_0-x)}(K - \frac{\eta_2 e^{x_0}}{\eta_2+1} + \frac{A\eta_2 e^{\beta_1 x_0}}{\beta_1+\eta_2} + \frac{B\eta_2 e^{\beta_2 x_0}}{\beta_2+\eta_2}) - K\right\} \\
= \; & r^*K - d^*e^x + \lambda^* q e^{\eta_2(x_0-x)}(K - \frac{\eta_2 e^{x_0}}{\eta_2+1} + \frac{A\eta_2 e^{\beta_1 x_0}}{\beta_1+\eta_2} + \frac{B\eta_2 e^{\beta_2 x_0}}{\beta_2+\eta_2}) \\
= \; & r^*K - d^*e^x + \lambda^* q e^{\eta_2(x_0-x)}\frac{\eta_2\beta_1\beta_2(\eta_1+\eta_2)}{\eta_1(\eta_2+1)(\beta_1+\eta_2)(\beta_2+\eta_2)}K.
\end{aligned}$$

Since $LV(x) - r^*V(x)$ is a decreasing function, to show $LV(x) - r^*V(x) < 0$, for all $x > x_0$, it suffices to show $(LV - r^*V)(x_0+) < 0$. Under condition (6),

$$(LV - r^*V)(x_0+) = \{r^* + \lambda^* q\frac{\beta_1\beta_2(\eta_1+\eta_2)}{\eta_1(\eta_2+1)(\beta_1+\eta_2)(\beta_2+\eta_2)} - d^*\frac{(\eta_1-1)\beta_1\beta_2}{\eta_1(\beta_1-1)(\beta_2-1)}\}K < 0.$$

The proof is completed. □

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FIGURE 1

Exercise Boundaries

This figure presents the exercise boundaries according to the illiquidity on the stock holding α, level of risk aversion 1-γ, moneyness In: K=20, At: K=25, Out: K=30, where K is exercise price, dividend yield d, time to maturity τ, total volatility σ and idiosyncratic risk ν, respectively. Except where noted, the following model parameters are used in the table: S=25, K=25, α=0.5, 1-γ=2, σ=0.3, ν=0.2, d=0.02, r=0.05, τ=10, λ=0.01, p=0.62, η_1=7.897, η_2=6.529, where S, r and λ are the stock price, risk-free rate and jump frequency, respectively. p, η_1 and η_2 are the parameters of jump sizes.

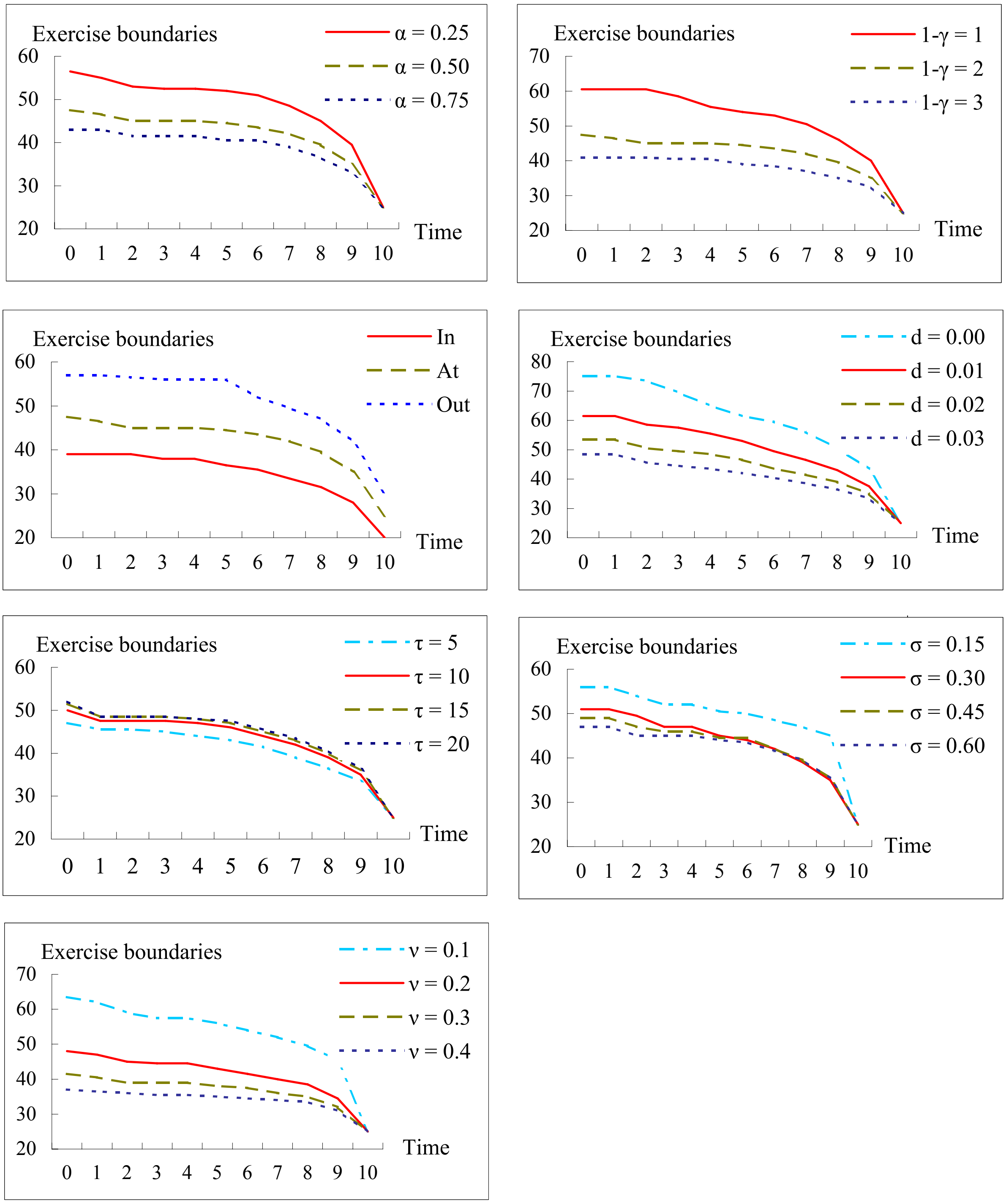

TABLE 1

Factors Effect on Employee Stock Options and the Exercise Decision

This table presents the impact of factors on employee stock options (ESOs) and the exercise decision. The results of ESO values, early exercise premiums, and discount ratios are shown in Panels A, B and C, respectively. α, K, and 1-γ represent the illiquidity on the stock holding, exercise price and level of risk aversion. The following model parameters are used in the table: S=25, σ=0.3, ν=0.2, d=0.02, r=0.05, λ=0.01, p=0.62, η_1=7.897, η_2=6.529, where S, σ, ν, d, r, τ and λ are the stock price, total volatility, normal unsystematic volatility, dividend yield, risk-free rate, time to maturity and jump frequency, respectively. p, η_1 and η_2 are the parameters of jump sizes.

Panel A: ESO Values

	In the money (K=20)			At the money (K=25)			Out of the money (K=30)		
	1-γ = 1	1-γ = 2	1-γ = 3	1-γ = 1	1-γ = 2	1-γ = 3	1-γ = 1	1-γ = 2	1-γ = 3
α = 0.00	11.2694	11.2694	11.2694	9.6487	9.6487	9.6487	8.3902	8.3902	8.3902
α = 0.25	10.2323	9.3061	8.6979	8.5661	7.5859	6.9702	7.3532	6.4228	5.7020
α = 0.50	9.6120	8.2193	7.3964	7.8852	6.4628	5.5200	6.6944	5.3040	4.2714
α = 0.75	9.2649	7.6015	6.6195	7.4822	5.7706	4.6858	6.2934	4.6129	3.4367

Panel B: Early Exercise Premiums

	In the money (K=20)			At the money (K=25)			Out of the money (K=30)		
	1-γ = 1	1-γ = 2	1-γ = 3	1-γ = 1	1-γ = 2	1-γ = 3	1-γ = 1	1-γ = 2	1-γ = 3
α = 0.00	0.3440	0.3440	0.3440	0.1839	0.1839	0.1839	0.1405	0.1405	0.1405
α = 0.25	0.6990	1.0268	1.5489	0.4010	0.5796	0.9958	0.3089	0.4413	0.6562
α = 0.50	0.9080	1.4265	2.2173	0.5191	0.8592	1.3642	0.4070	0.6271	0.8844
α = 0.75	0.9520	1.5777	2.4918	0.5345	0.9366	1.5192	0.4285	0.6747	0.9545

Panel C: Discount Ratios

	In the money (K=20)			At the money (K=25)			Out of the money (K=30)		
	1-γ = 1	1-γ = 2	1-γ = 3	1-γ = 1	1-γ = 2	1-γ = 3	1-γ = 1	1-γ = 2	1-γ = 3
α = 0.25	0.0920	0.1733	0.2302	0.1122	0.2110	0.2811	0.1236	0.2321	0.3209
α = 0.50	0.1471	0.2698	0.3454	0.1828	0.3278	0.4306	0.2021	0.3658	0.4913
α = 0.75	0.1779	0.3247	0.4142	0.2245	0.3998	0.5167	0.2499	0.4484	0.5907

TABLE 1 (Conti.)

Factors Effect on Employee Stock Options and the Exercise Decision

This table presents the impact of factors on employee stock options (ESOs) and the exercise decision. Except where noted, the following model parameters are used in the table: S=25, K=25, α=0.5, 1-γ=2, σ=0.3, ν=0.2, d=0.02, r=0.05, τ=10, λ=0.01, p=0.62, η_1=7.897, η_2=6.529, where S, K, α, 1-γ, σ, ν, d, r, τ and λ are the stock price, exercise price, the illiquidity on the stock holding, level of risk aversion, total volatility, normal unsystematic volatility, dividend yield, risk-free rate, time to maturity and jump frequency, respectively. p, η_1 and η_2 are the parameters of jump sizes. In Panel A, CA, CD and Premium are the ESO value, discount ratio (1-subjective/market) and early exercise premium, respectively. Panel B shows the relationship between the factor and the item listed in the left column. The last item Exercise means early exercise and sok is the ratio of stock price to exercise price. Symbols "+" and "-" represent positive and negative relationship.

Panel A: ESO Values & Discount Ratios & Early Exercise Premiums

	CA	CD	Premium		CA	CD	Premium
d = 0.00	8.4788	0.3557	0.3372	τ = 5	5.4222	0.2545	0.4084
d = 0.01	7.5724	0.3260	0.8031	τ = 10	6.7636	0.3018	1.1608
d = 0.02	6.7704	0.3054	1.1729	τ = 15	7.4605	0.3257	1.9932
d = 0.03	6.0673	0.2941	1.4674	τ = 20	7.8992	0.3363	2.8481
σ = 0.15	5.4571	0.1800	0.0677	ν = 0.1	7.1077	0.2607	0.2400
σ = 0.30	6.5440	0.3216	0.9413	ν = 0.2	6.2785	0.3470	1.0146
σ = 0.45	7.0701	0.4378	2.1005	ν = 0.3	5.6321	0.4142	2.1113
σ = 0.60	7.2776	0.5184	3.4397	ν = 0.4	5.1440	0.4650	3.0637

Panel B: Summary of Factors Effect

	α	1-γ	sok	d	τ	σ	ν
CA	-	-	+	-	+	+	-
CD	+	+	-	-	+	+	+
Premium	+	+	+	+	+	+	+
Exercise	+	+	+	+	-	+	+

TABLE 2

Perpetual Employee Stock Options

This table studies perpetual employee stock options (ESOs). The results of perpetual ESO values and optimal exercise boundaries are shown in Panels A and B, respectively. Panel C and D exhibit the absolute and relative differences between perpetual and finite horizon American ESO values. α, K, and 1-γ represent the illiquidity on the stock holding, exercise price and level of risk aversion. The following model parameters are used in the table: S=25, σ=0.3, ν=0.2, d=0.02, r=0.05, λ=0.01, p=0.62, η_1=7.897, η_2=6.529, where S, σ, ν, d, r, τ and λ are the stock price, total volatility, normal unsystematic volatility, dividend yield, risk-free rate, time to maturity and jump frequency, respectively. p, η_1 and η_2 are the parameters of jump sizes.

Panel A: Perpetual ESO Values

	In the money (K=20)			At the money (K=25)			Out of the money (K=30)		
	1-γ = 1	1-γ = 2	1-γ = 3	1-γ = 1	1-γ = 2	1-γ = 3	1-γ = 1	1-γ = 2	1-γ = 3
α = 0.00	14.5067	14.5067	14.5067	13.7747	13.7747	13.7747	13.2042	13.2042	13.2042
α = 0.25	12.6407	11.1791	10.0119	11.7198	10.0940	8.7818	11.0174	9.2861	7.8898
α = 0.50	11.7755	9.7389	8.2450	10.7594	8.4725	6.7572	9.9947	7.5611	5.7433
α = 0.75	11.5666	9.1089	7.3460	10.5268	7.7546	5.6954	9.7469	6.7989	4.6262

Panel B: Optimal Exercise Boundaries

	In the money (K=20)			At the money (K=25)			Out of the money (K=30)		
	1-γ = 1	1-γ = 2	1-γ = 3	1-γ = 1	1-γ = 2	1-γ = 3	1-γ = 1	1-γ = 2	1-γ = 3
α = 0.00	106.165	106.165	106.165	132.706	132.706	132.706	159.248	159.248	159.248
α = 0.25	78.973	63.687	54.023	98.716	79.609	67.529	118.459	95.531	81.035
α = 0.50	69.432	52.019	42.411	86.790	65.023	53.014	104.148	78.028	63.616
α = 0.75	67.350	47.706	37.520	84.187	59.632	46.900	101.024	71.559	56.281

Panel C: Absolute Differences Between Perpetual and Finite Horizon American ESO Values

	In the money (K=20)			At the money (K=25)			Out of the money (K=30)		
	1-γ = 1	1-γ = 2	1-γ = 3	1-γ = 1	1-γ = 2	1-γ = 3	1-γ = 1	1-γ = 2	1-γ = 3
α = 0.00	3.2373	3.2373	3.2373	4.1261	4.1261	4.1261	4.8140	4.8140	4.8140
α = 0.25	2.4084	1.8730	1.3140	3.1536	2.5081	1.8116	3.6642	2.8634	2.1877
α = 0.50	2.1634	1.5196	0.8486	2.8742	2.0098	1.2372	3.3002	2.2571	1.4719
α = 0.75	2.3017	1.5074	0.7265	3.0446	1.9840	1.0096	3.4536	2.1860	1.1895

Panel D: Relative Differences Between Perpetual and Finite Horizon American ESO Values

	In the money (K=20)			At the money (K=25)			Out of the money (K=30)		
	1-γ = 1	1-γ = 2	1-γ = 3	1-γ = 1	1-γ = 2	1-γ = 3	1-γ = 1	1-γ = 2	1-γ = 3
α = 0.00	0.2873	0.2873	0.2873	0.4276	0.4276	0.4276	0.5738	0.5738	0.5738
α = 0.25	0.2354	0.2013	0.1511	0.3682	0.3306	0.2599	0.4983	0.4458	0.3837
α = 0.50	0.2251	0.1849	0.1147	0.3645	0.3110	0.2241	0.4930	0.4255	0.3446
α = 0.75	0.2484	0.1983	0.1098	0.4069	0.3438	0.2155	0.5488	0.4739	0.3461

TABLE 3
Default Risk

This table compares the results for stock having no residual value if jump occurs (default jump) with stock following diffusion process (no jump). The results of ESO values, early exercise premiums, and discount ratios are shown in Panels A, B and C, respectively. α and K represent the illiquidity on the stock holding and exercise price. The following model parameters are used in the table: S=25, 1-γ=2, σ=0.3, ν=0.2, d=0.02, r=0.05, λ=0.01, p=0.62, η_1=7.897, η_2=6.529, where S, 1-γ, σ, ν, d, r, τ and λ are the stock price, level of risk aversion, total volatility, normal unsystematic volatility, dividend yield, risk-free rate, time to maturity and jump frequency, respectively. p, η_1 and η_2 are the parameters of jump sizes.

Panel A: ESO Values

	Default Jump			No Jump		
	K = 20	K = 25	K = 30	K = 20	K = 25	K = 30
α = 0.00	11.8111	10.3038	9.0617	11.2495	9.7201	8.3836
α = 0.25	9.4634	8.0239	6.7112	9.3837	7.8436	6.4794
α = 0.50	8.2630	6.5258	5.0974	8.3920	6.7768	5.3815
α = 0.75	7.9499	5.3414	4.4200	7.8234	6.1118	4.6976

Panel B: Early Exercise Premiums

	Default Jump			No Jump		
	K = 20	K = 25	K = 30	K = 20	K = 25	K = 30
α = 0.00	0.2659	0.1967	0.1614	0.3406	0.2734	0.1460
α = 0.25	1.1163	0.8778	0.5432	1.1209	0.8539	0.5104
α = 0.50	1.8060	1.1317	0.5417	1.6181	1.1912	0.7180
α = 0.75	2.6161	1.0025	0.8403	1.8243	1.2999	0.7760

Panel C: Discount Ratios

	Default Jump			No Jump		
	K = 20	K = 25	K = 30	K = 20	K = 25	K = 30
α = 0.25	0.1988	0.2213	0.2594	0.1659	0.1931	0.2271
α = 0.50	0.3004	0.3527	0.4375	0.2540	0.3028	0.3581
α = 0.75	0.3269	0.4957	0.5122	0.3046	0.3712	0.4397

TABLE 4

Sentiment Analysis

This table presents the sentiment levels necessary to offset the employee stock option (ESO) risk premium and the impact of sentiment on ESO values and the exercise decision. The estimated sentiment levels are listed in Panel A. Sentiment levels SenV and SenVE are calculated while the subjective value with sentiment is equal to market value for American and European options, respectively. SenE is the value of sentiment such that an employee exercises his options at the time that unconstrained investors do. The results of sentiment effect are shown in Panel B. s, CA, CD and Premium are the sentiment level, ESO value, discount ratio and early exercise premium, respectively. Except where noted, the following model parameters are used in the table: S=25, K=25, α=0.5, 1-γ=2, σ=0.3, ν=0.2, d=0.02, r=0.05, τ=10, λ=0.01, p=0.62, η_1=7.897, η_2=6.529, where S, K, α, 1-γ, σ, ν, d, r, τ and λ are the stock price, exercise price, the illiquidity on the stock holding, level of risk aversion, total volatility, normal unsystematic volatility, dividend yield, risk-free rate, time to maturity and jump frequency, respectively. p, η_1 and η_2 are the parameters of jump sizes.

Panel A: Sentiment Level

	1-γ = 1			1-γ = 2			1-γ = 3		
	SenV	SenE	SenVE	SenV	SenE	SenVE	SenV	SenE	SenVE
α = 0.25	0.0100	0.0097	0.0100	0.0200	0.0180	0.0200	0.0300	0.0291	0.0300
α = 0.50	0.0199	0.0194	0.0200	0.0399	0.0369	0.0400	0.0599	0.0585	0.0599
α = 0.75	0.0298	0.0289	0.0299	0.0598	0.0579	0.0598	0.0896	0.0866	0.0897

Panel B: Sentiment Effect on ESO Values and the Exercise Decision

	CA	CD	Premium
s=-0.005	6.1495	0.3120	0.9385
s= 0.000	6.4611	0.3279	0.8592
s= 0.005	6.7632	0.3469	0.7504
s= 0.010	7.1101	0.3655	0.6661