The trading performance of dynamic hedging models: Time varying covariance and volatility transmission effects

Michael T. Chng*and Gerard L. Gannon†

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Abstract

In this paper, we investigate the value of incorporating implied volatility from related option markets in dynamic hedging. We comprehensively model the volatility of all four S&P 500 cash, futures, index option and futures option markets simultaneously. Synchronous half-hourly observations are sampled from transaction data. Special classes of extended simultaneous volatility systems (ESVL) are estimated and used to generate out-of-sample hedge ratios. In a hypothetical dynamic hedging scheme, ESVL-based hedge ratios, which incorporate incremental information in the implied volatilities of the two S&P 500 option markets, generate profits from interim rebalancing of the futures hedging position that are incremental over competing hedge ratios. In addition, ESVL-based hedge ratios are the only hedge ratios that manage to generate sufficient profit during the hedging period to cover losses incurred by the physical portfolio .

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Key words and phrases: volatility transmission, dynamic hedging, optimal hedge ratio, S&P 500.

*Corresponding author's contacts: School of of Accounting, Economics and Finance, Deakin University, VIC 3125, Australia. Tel: 613-92446537. Email: mchng@deakin.edu.au.

†School of of Accounting, Economics and Finance, Deakin University, VIC 3125, Australia. Tel: 613-92446243. Email: gerard@deakin.edu.au. We retain full property rights to all existing errors.

1 Introduction

The size of a hedge is affected by the sensitivity of the return of the underlying asset to the return of the hedging instrument. This sensitivity measure β is also called the optimal hedge ratio (OHR). The hedge ratio is deemed optimal in terms of minimizing the variability in the value of the overall position. A dynamic hedging scheme recognizes that since the OHR varies over time i.e. $\beta = \beta_t$, the hedging outcome can be improved from interim rebalancing of the hedging position. An issue addressed by the hedging literature is the incremental performances of competing OHRs over a static hedge. If the corresponding variance and covariance terms used to calculate the OHR can be adequately modeled and forecasted, then hedging performance should improve.

The volatility literature contains a voluminous debate on the information content of implied volatility versus historical volatility. The general consensus is that some combination of both improves volatility forecasts. Our main objective in this paper is to formally blend the two literatures together. We investigate if incremental information from combining implied and historical volatilities translates into incremental hedging performance by the corresponding OHR. We consider the use of the S&P 500 index futures (henceforth FI) contract to hedge against a widely-held portfolio that tracks the S&P 500 index e.g. S&P Depository Receipts (SPDR). In this paper, we assume the S&P 500 cash index (henceforth CI) as our physical portfolio.[1] Denote N_t as the optimal number of futures contracts to short against an existing long position in the underlying portfolio at time t. This is calculated in equation (1).

$$N_t = \beta_t \times \frac{V_{CI,t}}{V_{FI,t}}$$

$$\beta_t = \frac{\sigma_{CI,FI,t}}{\sigma^2_{FI,t}} = \rho_{CI,FI,t} \times \frac{\sigma_{CI,t}}{\sigma_{FI,t}} \tag{1}$$

$V_{CI,t}$ and $V_{FI,t}$ represent the value of the physical portfolio to be hedged against and

[1]S&P Depository Receipts are yet to exist during our sample period.

the value covered by each futures contract respectively. This size-ratio $\frac{V_{CI,t}}{V_{FI,t}}$ is adjusted by β_t, which is calculated as the covariance $\sigma_{CI,FI,t}$ between the spot return $r_{CI,t}$ and futures returns $r_{FI,t}$ divided by the variance of the futures return $\sigma^2_{FI,t}$. As β_t varies during the hedging period, so does N_t, which implies a need to rebalance the hedging position. Since $\rho_{CI,FI,t} = \frac{\sigma_{CI,FI,t}}{\sigma_{CI,t} \times \sigma_{FI,t}}$, N_t can be expressed as $\rho_{CI,FI,t} \times \frac{\sigma_{CI,t}}{\sigma_{FI,t}}$, where $\sigma_{CI,t}$ is the cash index volatility. If we assume both $V_{CI,t}$ and $V_{FI,t}$ are exogenous, the OHR is the only parameter to estimate to determine the optimal N_t.[2] This demonstrates the importance of modeling $\sigma_{CI,FI,t}$ and $\sigma^2_{FI,t}$ in a dynamic hedging scheme. If we assume constant perfect positive correlation i.e. $\rho_{CI,FI,t} = 1$, modeling and forecasting the OHR is analogous to modeling and forecasting $\sigma_{CI,t}$ and $\sigma_{FI,t}$.

The preceding highlights an intimate link between the literatures on hedging and volatility modeling. Nonetheless, potential volatility transmission between spot, futures and options markets are often ignored in the OHR estimation. This is despite an incumbent literature spanning over 15 years debating the merits of implied versus historical volatility. If incremental information embedded in implied volatility exists, then incorporating such volatility transmissions should generate better volatility forecasts and improve the OHR estimation. The S&P 500 CI and FI each possesses a well-established option market. The S&P 500 futures option (FO) trading pit is located beside the futures pits on the Chicago Mercantile Exchange (CME) trading floor. The S&P 500 index option (IO) is traded on the Chicago Board Options Exchange (CBOE).

Pennings and Meulenberg (1997) provide a comprehensive review of hedging performance measures in the literature. The evaluation generally contrasts between the combined cash-futures position versus the cash position alone. Hedging performance is measured by the reduction in the variance of the combined position in Ederington (1979), the ratio of Sharpe

[2]Strictly speaking, $V_{CI,t}$ is not entirely exogenous in that a decision is required on the proportion of the underlying asset's value to hedge against. However, this is a separate issue from the OHR determination.

ratios (cash-futures position divided by cash position only) in Howard and D'Antonio (1984), difference in certainty equivalent returns between the combined versus cash position in Hsin, Kuo and Lee (1994) and expected utility maximization in Kroner and Sultan (1993). Hedging applications based on a bivariate GARCH framework include Baillie and Myers (1991) for US commodity futures, Kroner and Sultan (1993) for foreign currency futures and Park and Switzer (1995) for US stock index futures. In these studies, hedging performance is based on minimizing the variance of the overall position.

Lee, Gannon and Yeh (2000) and Yeh and Gannon (2000) evaluate out-of-sample volatility estimates generated from competing volatility models in the context of a dynamic hedging scheme. However, potential volatility transmission effects were not investigated, but transaction costs were included in the analysis in the latter paper. Chng and Gannon (2003) model contemporaneous volatility and volume effects within a framework of formalized structure of simultaneous volatility equations proposed by Gannon (1994). The order and matrix rank conditions for the simultaneous volatility and competing misspecified volatility models are documented in this latter paper. While the resultant volatility forecasts statistically dominate those from competing volatility models, an out-of-sample evaluation based on market inference was not performed.

Au-Yeung and Gannon (2004, 2005) employ a M-GARCH model modified to allow for multiple structural breaks and volatility spillovers between the HSIF cash and index futures markets and also overnight spillovers from the S&P500 index futures. Gannon (2005) repeated this analysis but employed a Full Information Maximum Likelihood (FIML) set of simultaneous volatility equations. Bhatacharya, Singh and Gannon (2007) employed the Au-Yeung and Gannon M-GARCH model to test volatility spillovers between Indian stock and share futures. Gannon and Au-Yeung (2008) repeated the early analysis of structural breaks in their M-GARCH model and showed that inclusion of volume of trade effects led to insignificant structural break parameters. Lee, Wang and Chen (2009) utilize four static and a dynamic bivariate GARCH models to find the OHR for the S&P500 and five major Asian market index futures. Lee, Lin and Chen (2010) investigate hedge ratios in international futures markets in the light of the cross-country linkage and interaction using a 3SLS estimation procedure. Gannon (2010) re-visited the simultaneous volatility class of models to test transmission and spillover effects when the intra-day sampling interval reduces. In this paper further theoretical conditions for this class of models and systems Error Correction terms are defined. In all of the above estimators, hedge ratios can be extracted but no out of sample hedging performance was undertaken.

We evaluate the hedging performances of competing OHRs base on the incremental profits from interim rebalancing that each OHR generates over a static hedge. We argue that our performance measure is more consistent with the investigation on whether incorporating incremental information from implied volatility in the OHR calculation translates into incremental profits from rebalancing the futures position. We evaluate eight competing OHRs generated from competing volatility and covariance forecasts.

Our secondary objective is to model the entire time-varying variance-covariance matrix in a formal system of simultaneous volatility (SVL) equations. This can be seen as a competing estimator to variants of the restricted class of bivariate GARCH estimators. However, we consider only the constant correlation bivariate GARCH and univariate GARCH estimators as similar applications in the published literature have report improved performance over regression-based estimators. The true unrestricted estimator of the simultaneous multivariate GARCH (MGARCH) hedge ratio is generated from the VEC-MGARCH form. This cannot be readily applied to model time varying covariance due to convergence problems.[3] As noted

[3]The unrestricted vector GARCH specification seldom leads to identifiable point estimates since doing so requires the inversion of the variance-covariance matrix at each sample point. When off-diagonal covariance terms are large relative to diagonal variance terms, the determinant of the matrix tends to zero, such that the inverse matrix can be unidentified. Such cases are very likely in the context of spot and the futures returns. This technical problem is commonly addressed by imposing a constant correlation and focus on modeling time varying spot and futures volatility, which somewhat defeats the purpose of dynamic hedging.

in the literature for the BEKK-GARCH and dynamic constant correlation forms of Engle (2002) and Tse and Tsui (2002), the covariance term from these models are functions of conditional standard deviations of equations in the system.

The BEKK-GARCH estimator proposed by Engle and Kroner (1995) overcomes the convergence problem by employing an estimator that guarantees a positive definite variance/covariance matrix. The base BEKK estimator is still restricted as it employs a function of the product of the respective bivariate time varying standard deviations as estimators of the time varying co-variance term. Various extensions of this class of estimator allow for functions of the standard deviations to generate the covariance term. Some in sample estimators also allow for asymmetric positive and negative return effects (sign effect) in the estimation process. One drawback with employing the asymmetric versions in out of sample forecasting applications is lack of future values of the "sign effect" to generate the forecasts.

In this paper, we utilize market data from parallel option markets to capture market anomalies, including sign effects, because these are continuously observable variables rather than discrete imposed indicator variables. This can also be seen as a benchmark for the full structural volatility system estimator that incorporates market transmission effects. It could also be seen as a benchmark for the aforementioned class of BEKK-GARCH estimators within the comparison framework of this paper and an area for future research. Clearly the real focus in this paper is to compare two alternative versions of the class of simultaneous volatility estimators SVL and ESVL. Comparisons with other estimators is restricted to those reported using similar evaluation processes.

The rest of this paper proceeds as follow. Institutional features and sampling procedures are provided in Section 2. In Section 3, competing volatility models and various methods of computing the OHR are discussed. Model estimation results are reported in Section 4. Section 5 reports the out-of-sample hedging performance. Section 6 concludes.

2 Institutional background and data sampling

The analysis in our paper requires a synchronously sampled set of intraday data for all four contracts. We found it difficult to obtain a set from both options markets that was satisfactory. We decided to employ the same cash index and index futures dataset for the results reported by Miller, Muthuswamy and Whaley (1994). For this dataset, a full database of both options transactions is available[4].

The S&P 500 CI is a value-weighted broad-based market index that comprises 500 widely-held stocks.[5] It is regarded by the financial community as a barometer to gauge the performance of the US equity market. The S&P 500 FI follows a March, June, September and December contract cycle. In 1990, the contract multiplier was USD 500 per index point. The tick size is 0.05 point, or USD 25. On average, the contracts are rolled over between the 6th and 11th of the contract month. For our sample, we choose the 8th day of the delivery month as the date to switch to the next contract.

The S&P 500 FO are American options. One futures option is written on one futures contract and is quoted in index points. The tick size is 0.05[6]. These contracts follow a monthly cycle, and data is available across a range of contract months and strike prices.[7] The two nearest-to-maturity option contracts (e.g. Feb and March options written on the March futures contract) will trade at strike prices in multiples of 5. Longer maturity contracts will trade at strike prices in multiples of 10. The option is exercised at maturity if it is in

[4]The data was supplied by TICKDATA Inc but for various reasons, they stopped collection of options data streams.

[5]Individual stock prices are multiplied by the number of shares outstanding. The products are summed up and standardized by a pre-determined base value. Base values for the index are adjusted to reflect changes in capitalization due to mergers, acquisitions and rights issues etc.

[6]However, a trade may occur at a price of 0.025 index point if it is necessary to liquidate positions to allow for both parties to trade.

[7]For the March futures contract, there are option contracts expiring at the end of Jan, Feb, March etc up to May. Options expiring prior or during the March quarterly cycle are written on the March futures contracts. Else, they are written on the June futures contract.

the money. Both the S&P 500 FI and FO contracts are traded under the Index and Option Market Division on the CME. For each trading day, the average futures price of the front contract is used to determine the closest to money FO contract to include in our sample. The implied volatility from the FO will be the average of the implied volatility between the call and put. The S&P 500 IO contracts share similar contract specifications to the FO contracts. A key difference is that the IO are cash-settled and they are European options.

From 4^{th} Jan to 31^{st} December 1990, we sample near-synchronous 30-minute observations from the nearest-to-maturity contracts for each of the four markets. For all the markets, normal trading commences at 8.30am and finishes at 3.15pm. To avoid mechanically-induced opening and closing effects, we use the 8.40am and 3pm prices to compute the 9am opening return and 3pm closing return correspondingly.[8] When extracting implied volatilities, potential non-synchronous spot and option prices violate the specification of the Black-Scholes pricing model, which dilutes the validity of subsequent implied volatility measures. However, since both option markets are highly liquid, non-synchronicity between spot and option prices is not a major problem for our study. We calculate an option's term-to-maturity as $\frac{\text{No of trading days to maturity}}{\text{Trading days per annum}}$[9]. The continuously compounded 1-month Treasury bill rate is used to proxy the risk-free rate.[10] Lastly, the continuous-compounding annualized dividend yield daily time-series of the CI is used to back out implied volatility of the IO.

[8]For missing observations in one market, corresponding observations from other markets are excluded. This gives a total of 3186 out of a possible 3289 observations. There are 13 half-hourly observations per day over 253 trading days in 1990. Details of the sampling procedure can be obtained from the authors upon request.

[9]For example, an option trading on 23rd April with May as the delivery month will have 27 trading days till maturity. Following conventions, this study uses 252 days for the denominator. Thus for that option, term-to-maturity is calculated as 23/252=0.10714.

[10]This is chosen over the 30 year Treasury bond rate, which may contain term structure premium.

3 Volatility estimators and competing hedge ratios

Denote $r_{CI,t}$ and $r_{FI,t}$ as the half-hourly continuously compounded returns of CI and FI respectively. We construct CI and FI volatilities as absolute returns $\sigma_{CI,t} = |r_{CI,t}|$ and $\sigma_{FI,t} = |r_{FI,t}|$. The covariance between CI and FI is defined as $\sigma_{CI,FI,t} = |r_{CI,t}||r_{FI,t}|$.[11]

3.1 Constructing out-of-sample hedge ratios

To generate out-of-sample OHR forecasts, we divide our 1-year sample into two halves. The first half is our estimation sample and contains 1613 observations. The second half, which is our test sample, contains 1573 observations. Out-of-sample OHR projections into the test period are derived by sequentially expanding the estimation period one observation at a time to update the coefficient estimates and generate a series of 1-step ahead OHR forecasts. We evaluate the hedging performance of eight hedge ratios that are constructed from different volatility sources and models. These are summarized in Table 1.

INSERT TABLE 1

The first hedge ratio is obtained from a least square regression of $r_{CI,t}$ against $r_{FI,t}$ and a dummy variable Dum^{Close} for market closing effects.[12] Outlined in equation (2), this β_{OLS} measure is commonly used in a static hedge, where no re-balancing occurs during the hedging period. We denote the coefficient β_{OLS} as OHR(1).

$$r_{CI,t} = \beta_0 + \beta_1(Dum^{Close}) + \beta_{OLS}(r_{FI,t}) + \varepsilon_t \qquad (2)$$

OHR(2) is constructed from the conditional volatility of univariate GARCH(1,1) estimations for $\sigma_{CI,t}$ and $\sigma_{FI,t}$. In equation (3), $\varepsilon_{CI,t}$ and $\varepsilon_{FI,t}$ are residuals obtained from the CI

[11] Although negative covariance is possible and should be allowed for, the spot and futures returns are expected to move in the same direction most of the time due to cost-of-carry. An examination of the data confirms this statement.

[12] Dum^{Close}=1 for the 3.00pm closing return, and 0 otherwise.

and FI return-equations. Lastly, $\rho_{CI,FI}$ denotes the correlation between $r_{CI,t}$ and $r_{FI,t}$.

$$\sigma^2_{CI,t+1} = \alpha_{10} + \alpha_{11}(\varepsilon^2_{CI,t}) + \beta_{11}(\sigma^2_{CI,t})$$
$$\sigma^2_{FI,t+1} = \alpha_{20} + \alpha_{21}(\varepsilon^2_{FI,t}) + \beta_{21}(\sigma^2_{FI,t})$$

$$OHR(2) = \rho_{CI,FI} \times \frac{\sigma_{CI,t+1}}{\sigma_{FI,t+1}} \tag{3}$$

OHR(3) is designed to be an enhanced version of OHR(2). Here, we examine the potential incremental information provided by an array of additional variables added to the CI and FI GARCH (1,1) variance equations. These include $\sigma^2_{FI,t}$ in the $\sigma^2_{CI,t+1}$ equation, $\sigma^2_{CI,t}$ in the $\sigma^2_{FI,t+1}$ equation, $\sigma^2_{IO,t}$ and $\sigma^2_{FO,t}$. In addition, we also allow potential volume effects to enter the conditional variance equations. Denote CV_{t-1} and FV_{t-1} as the corresponding change in CI and FI tick-volume.[13] Lastly, we include Dum^{Open}[14] and Dum^{Close} for for market opening and closing effects. If there exists any incremental information in the implied volatility and/or tick volumes that can be adequately brought out through a GARCH framework, OHR(3) should outperform OHR(2).

$$\sigma^2_{CI,t+1} = \alpha_{10} + \alpha_{11}(\varepsilon^2_{CI,t}) + \beta_{11}(\sigma^2_{CI,t}) + \gamma_{11}(Dum^{Open}) + \gamma_{12}(Dum^{Close})$$
$$+ \gamma_{13}\sigma^2_{FI,t} + \gamma_{14}\sigma^2_{IO,t} + \gamma_{15}\sigma^2_{FO,t} + \gamma_{16}(CV_t)$$

$$\sigma^2_{FI,t+1} = \alpha_{20} + \alpha_{21}(\varepsilon^2_{FI,t}) + \beta_{21}(\sigma^2_{FI,t}) + \gamma_{21}(Dum^{Open}) + \gamma_{22}(Dum^{Close})$$
$$+ \gamma_{23}\sigma^2_{CI,t} + \gamma_{24}\sigma^2_{IO,t} + \gamma_{25}\sigma^2_{FO,t} + \gamma_{26}(FV_t)$$

$$OHR(3) = \rho_{CI,FI} \times \frac{\sigma_{CI,t+1}}{\sigma_{FI,t+1}} \tag{4}$$

[13] Tick volume is defined as the number of half hourly price changes. For example, CV_t is the number of half-hourly price changes between time t-1 and t.

[14] Dum^{Open}=1 for the 9.00am opening return, and 0 otherwise.

For the actual estimation, we begin with a comprehensive GARCH (1,1) specification outlined in equation (4). This includes all volatility and tick-volume variables from related markets. First, we exclude non-estimable variables from the weighting series of the variance equations. Next, we use both the Wald test statistics and Akaike Information Criterion (AIC) to guide us as to which variables to exclude in an effort to systematically exclude variables from the comprehensive model. The trimmed down GARCH(1,1) specification is then used to generate out-of-sample projections for OHR(3).

Implied volatility is regarded by option traders as a forward-looking estimate. Accordingly, a hedge ratio can be constructed simply with $\sigma_{IO,t}$ and $\sigma_{FO,t}$. We term this as the implied hedge ratio (IHR), which is a novel alternative to compute hedge ratios by substituting $\sigma_{CI,t}$ with $\sigma_{IO,t}$ and $\sigma_{FI,t}$ with $\sigma_{FO,t}$. Since both $\sigma_{IO,t}$ and $\sigma_{FO,t}$ vary over time, IHR is applicable to a dynamic hedging scheme. In equation (5), we label IHR as OHR(4).

$$OHR(4) = IHR = \rho_{CI,FI} \times (\frac{\sigma_{IO,t}}{\sigma_{FO,t}}) \tag{5}$$

Since we consider only nearest to money put and call options of the front contract, and since both option markets are liquid, non-synchronicity between spot and option prices should not be a major concern. The Black-Scholes model adjusted for continuously compounded dividend yield is used to compute $\sigma_{IO,t}$ from the observed premium of the European index options, assuming dividends are non-stochastic. However, Black's (1976) model for pricing futures options cannot be readily applied to back out $\sigma_{FO,t}$, as these are American options. If the premium for early exercise is non-trivial, then this should be acknowledged.[15] However, Whaley (1986) finds that early exercise premium exists only for in-the-money S&P 500 futures put options. Since $\sigma_{FO,t}$ is computed as the average of nearest-to-maturity call and put options, we postulate that any potential upward biases in $\sigma_{FO,t}$ from not explicitly

[15]While it can be shown that early exercise is never optimal for a futures option if the premium is subjected to futures-style margining, the premiums for FO are paid up front. With positive interest rates, early exercise remains a possibility.

adjusting for early exercise premium should be trivial.

OHR(5) is generated from a 4-equation extended simultaneous volatility (ESVL) framework that incorporates cross-market volatility transmissions across all four S&P 500 markets. The ESVL system is based on the 3-equation SVL model proposed in Gannon (1994). Since OHR(5) requires forecasts for both $\sigma_{CI,t}$ and $\sigma_{FI,t}$, two separate ESVL systems are considered, where $\sigma_{CI,t}$ and $\sigma_{FI,t}$ is each specified as the variable to be forecasted. We refer to these as ESVL(CI) and ESVL(FI) respectively. In specifying the structure of both ESVL systems, we assume that the FO possesses the greatest informational efficiency, followed concurrently by IO and FI. The cash index ranks last in terms of efficiency ranking. This is confirmed from pairwise Granger-causality tests on the volatility time series. ESVL(CI) is presented in equation (6). Diagnostic tests indicate a significant opening effect for the cash index. As such, an opening dummy Dum^{Open} is included in the estimation of ESVL(CI).

$$\begin{aligned}
\sigma_{CI,t} &= \alpha_{10} + \alpha_{11} Dum^{Open} + \beta_{11}(\sigma_{FI,t}) + \beta_{12}(\sigma_{IO,t}) + \gamma_{11}(\sigma_{CI,t-1}) + \gamma_{12}(\sigma_{FI,t-1}) \\
&\qquad + \gamma_{13}(CV_{t-1}) + \varepsilon_{CI,t} \\
\sigma_{FI,t} &= \alpha_{20} + \alpha_{21} Dum^{Open} + \beta_{21}(\sigma_{CI,t}) + \beta_{22}(\sigma_{FO,t}) + \gamma_{21}(\sigma_{CI,t-1}) + \gamma_{22}(\sigma_{FI,t-1}) \\
&\qquad + \gamma_{23}(\sigma_{FO,t-1}) + \varepsilon_{FI,t} \\
\sigma_{IO,t} &= \alpha_{30} + \alpha_{31} Dum^{Open} + \beta_{31}(\sigma_{CI,t}) + \beta_{32}(\sigma_{FO,t}) + \gamma_{31}(\sigma_{CI,t-1}) + \gamma_{32}(\sigma_{IO,t-1}) \\
&\qquad + \gamma_{33}(\sigma_{FO,t-1}) + \varepsilon_{IO,t} \\
\sigma_{FO,t} &= \alpha_{40} + \alpha_{41} Dum^{Open} + \beta_{41}(\sigma_{CI,t}) + \beta_{42}(\sigma_{FI,t}) + \gamma_{41}(\sigma_{CI,t-1}) + \gamma_{42}(\sigma_{FI,t-1}) \\
&\qquad + \gamma_{43}(\sigma_{FO,t-1}) + \varepsilon_{FO,t} \qquad (6)
\end{aligned}$$

Since $\sigma_{CI,t}$ is the variable to be forecasted, it is present in all 4 equations of ESVL(CI) to ensure that a reduced-form can be obtained for generating out-of-sample forecasts. Lagged tick-volume CV_{t-1} is included in the $\sigma_{CI,t}$ equation to overcome a singularity estimation problem. The futures market leads the spot market due to lower execution costs, higher liquidity and more informed trading. As such, $\sigma_{FI,t}$ is specified to enter the $\sigma_{CI,t}$ equation. Similarly, $\sigma_{FO,t}$ is specified to enter both the $\sigma_{FI,t}$ and $\sigma_{IO,t}$ equations. $\sigma_{CI,t-1}$ and $\sigma_{FI,t-1}$ are included in the $\sigma_{IO,t}$ and $\sigma_{FO,t}$ equations to account for possible volatility feedback from the underlying assets back to the corresponding option market.

The derivation of the reduced-form ESVL(CI) is provided in the appendix. The coefficients of the reduced-form $\sigma_{CI,t}$ equation are functions of the structural coefficients from ESVL(CI). These are estimated using the estimation sample. Together with time t volatility variables, we can generate a 1-step ahead forecast $\sigma_{CI,t+1}$ according to the specification of the reduced-form $\sigma_{CI,t}$ equation.[16] Conducting 1-step ahead forecasts based on the reduced-form $\sigma_{CI,t}$ equation allows inherent volatility transmission effects inherent among all four S&P 500 markets to be incorporated into $\sigma_{CI,t+1}$. After each forecast, the coefficients are sequentially updated 1 observation at a time. This recursive process generates a time-series of $\sigma_{CI,t+1}$.[17]

ESVL(FI) is presented in equation (7). A closing dummy Dum^{Close} is included as the FI market displays a significant closing effect. The series of 1-step ahead forecasts $\sigma_{FI,t+1}$ from

[16] An alternative way to view the mapping of reduced form parameters back to the structural form parameters is to think about the systems in terms of the normalized matrix rank condition rather than the unnormalized matrix rank condition. In the former, there is an implied unity restriction imposed on endogenous variables in own structural equations. Then substitution between the structural and reduced forms is straightforward and in an identifiable system all structural parameters in every structural equation are identified. As such, the reduced-form equations provide unique projections of endogenous variables in the systems. However, there can be cases where alternative structural parameterizations provide non-nested competing sets of structural systems. In this case, artificial nested testing procedures need be employed to select between competing systems. In our paper, there are competing identifiable 3 and 4 equation systems. These are compared in terms of out of sample hedge ratio estimation and subsequently in terms of trade re-balancing performance.

[17] To note, utilizing ESVL-based hedge ratios may seem computationally tedious. However, as the forecasts are made 1-step at a time, the majority of coefficients are stable when we sequentially expand the estimation period. Once the initial coefficient estimates are recorded, subsequent updating is computationally easy.

ESVL(FI) are obtained in a similar fashion to $\sigma_{CI,t+1}$. OHR(5) is presented in equation (8).

$$\begin{aligned}
\sigma_{FI,t} = \alpha_{10} + \alpha_{11} Dum^{Close} + \beta_{11}(\sigma_{CI,t}) + \beta_{12}(\sigma_{IO,t}) + \gamma_{11}(\sigma_{FI,t-1}) + \gamma_{12}(\sigma_{FO,t-1}) \\
+ \gamma_{13}(FV_{t-1}) + \varepsilon_{CI,t} \\
\sigma_{CI,t} = \alpha_{20} + \alpha_{21} Dum^{Close} + \beta_{21}(\sigma_{FI,t}) + \beta_{22}(\sigma_{IO,t}) + \gamma_{21}(\sigma_{FI,t-1}) + \gamma_{22}(\sigma_{CI,t-1}) \\
+ \gamma_{23}(\sigma_{IO,t-1}) + \varepsilon_{FI,t} \\
\sigma_{IO,t} = \alpha_{30} + \alpha_{31} Dum^{Close} + \beta_{31}(\sigma_{FI,t}) + \beta_{32}(\sigma_{FO,t}) + \gamma_{31}(\sigma_{FI,t-1}) + \gamma_{32}(\sigma_{IO,t-1}) \\
+ \gamma_{33}(\sigma_{FO,t-1}) + \varepsilon_{IO,t} \\
\sigma_{FO,t} = \alpha_{40} + \alpha_{41} Dum^{Close} + \beta_{41}(\sigma_{FI,t}) + \beta_{42}(\sigma_{CI,t}) + \gamma_{41}(\sigma_{FI,t-1}) + \gamma_{42}(\sigma_{CI,t-1}) \\
+ \gamma_{43}(\sigma_{FO,t-1}) + \varepsilon_{FO,t} \qquad (7)
\end{aligned}$$

$$OHR(5) = \rho_{CI,FI} \times (\frac{\sigma_{ESVL(CI),t+1}}{\sigma_{ESLV(FI),t+1}}) \qquad (8)$$

OHR(6) is based on the modeling of a time-varying variance-covariance matrix of the spot and futures markets based on the SVL system in equation (9). Two separate reduced-form equations with $\sigma_{FI,t}$ and $\sigma_{CI,FI,t}$ as the variable to be forecasted are derived from (9). These are used to generate a series of 1-step ahead forecasts for $\sigma_{FI,t}$ and $\sigma_{CI,FI,t}$.[18] The presence of $\sigma_{CI,FI,t}$ in the $\sigma_{FI,t}$ equation is driven by the fact that during non-volatile trading periods, the basis is expected to be small, reflecting cost-of-carry. But during volatile trading periods, the futures market is expected to experience a greater increase in trading activity relative to the spot market, causing a transitory deviation between the two. This implies both a

[18]The derivations are available upon request.

decrease in $\sigma_{CI,FI,t}$ and an increase in $\sigma_{FI,t}$.

$$\sigma_{FI,t} = \alpha_{10} + \alpha_{11} Dum^{Close} + \beta_{11}(\sigma_{CI,FI,t}) + \gamma_{11}(\sigma_{FI,t-1}) + \gamma_{12}(\sigma_{CI,FI,t-1}) + \gamma_{13}(FV_{t-1}) + \varepsilon_{FI,t}$$

$$\sigma_{CI,t} = \alpha_{20} + \alpha_{21} Dum^{Close} + \beta_{21}(\sigma_{FI,t}) + \gamma_{21}(\sigma_{FI,t-1}) + \gamma_{22}(\sigma_{CI,t-1}) + \gamma_{23}(\sigma_{FI,CI,t-1}) + \varepsilon_{CI,t}$$

$$\sigma_{CI,FI,t} = \alpha_{30} + \alpha_{31} Dum^{Close} + \beta_{31}(\sigma_{FI,t}) + \gamma_{31}(\sigma_{FI,t-1}) + \gamma_{32}(\sigma_{CI,t-1}) + \gamma_{23}(\sigma_{FI,CI,t-1}) + \varepsilon_{FI,CI,t} \quad (9)$$

$$OHR(6) = \frac{\sigma_{CI,FI,t+1}}{\sigma^2_{FI,t+1}} \quad (10)$$

As our preceding argument does not imply anything about the direction of causality, we allow $\sigma_{FI,t}$ to enter the $\sigma_{CI,FI,t}$ equation. OHR(6) is presented in equation (10). Compared to other hedge ratios, OHR(6) does not incorporate any volatility transmissions from related options markets. However, it does consider time-varying covariance.

$$OHR(7) = \frac{\sigma_{CI,FI,t+1}}{\sigma^2_{FO,t+1}} \quad (11)$$

$$OHR(8) = \frac{\sigma_{CI,FI,t+1}}{\sigma^2_{ESVL,FI,t+1}} \quad (12)$$

The last two hedge ratios are based on hybrids projections from ESVL and/or SVL. Outlined in equation (11), OHR(7) combines time-varying covariance forecasts generated from equation (9) with potential incremental information from the implied variance of the FO market. OHR(8) is presented in equation (12). Here, we combine $\sigma_{CI,FI,t}$ forecasts generated from the SVL with $\sigma_{FI,t}$ forecasts generated from ESVL(FI). As such, OHR(8) is the only hedge ratio in this paper that incorporates both spot-futures time-varying covariance and volatility transmission from related option markets.

4 Empirical results

4.1 Descriptive statistics and preliminary results

In Table 2, descriptive statistic for $r_{CI,t}$ and $r_{FI,t}$ are presented in Panel 1, the autocorrelation of key variables in Panel 2, the correlation matrix of volatility and volume variables in Panel 3 and stationarity test statistics in Panel 4.

INSERT TABLE 2

To note, $r_{FI,t}$ is highly significant at the 13^{th} lag. Since our sample consists of 13 half-hourly return observations per trading day, the significant 13^{th} lag could reflect time of the day effect that is evident in most index futures markets. The strong correlation between $\sigma_{CI,t}$ and $\sigma_{FI,t}$ is expected. Both $\sigma_{CI,t}$ and $\sigma_{FI,t}$ are also correlated with CV_t and FV_t respectively. In contrast, the correlations between $\sigma_{CI,t}$ and $\sigma_{IO,t}$ as well as between $\sigma_{FI,t}$ and $\sigma_{FO,t}$ are comparatively weaker. Lastly, $\sigma_{IO,t}$ and $\sigma_{FO,t}$ are not strongly correlated with each other, suggesting that the two option markets are not necessarily close substitutes. Augmented Dickey Fuller (ADF) test statistics indicate that cash index, futures prices and their tick volumes are integrated of order one I(1), such that $r_{CI,t}$, $r_{FI,t}$, CV_t and FV_t are all stationary. The volatility time series of the four S&P markets are also tested and found to be stationary.

4.2 Results from model estimates

First, we report GARCH estimation results. Potential higher-order GARCH-effects are investigated using nested tests, but additional parameters are found to be insignificant for both $\sigma_{CI,t}$ and $\sigma_{FI,t}$. The final GARCH specification for the CI and FI are determined from a 'top-down' approach based on Wald tests statistics and the AIC to systematically exclude variables from the weighting series in the GARCH variance equation. As such, the specification of $\sigma_{CI,t}$ is slightly different from $\sigma_{FI,t}$. Both are presented below with the coefficient estimates and significance levels.

$$\begin{aligned}\sigma^2_{CI,t} = 0.000^{a**} &+ 0.1065^{**}(\varepsilon^2_{CI,t-1}) + 0.7989^{**}(\sigma^2_{CI,t-1}) + 0.000^{**}(Dum^{Open}) \\ &+ 0.0001^{**}(CV_{t-1}) + 0.0001^{**}(FV_{t-1})\end{aligned}$$

$$\begin{aligned}\sigma^2_{FI,t} = 0.000 &+ 0.0389(\varepsilon^2_{FI,t-1}) + 0.8857^{**}(\sigma^2_{FI,t-1}) + 0.000^{*}(Dum^{Close}) \\ &+ 0.021(\sigma^2_{IO,t-1}) + 0.0001(CV_{t-1}) + 0.0001^{**}(FV_{t-1})\end{aligned}$$

*a: * indicate significance at 5% level; ** indicate significance at 1% level.*

Next, we discuss results from ESVL and SVL estimations. We perform two diagnostic tests to improve the validity of the ESVL specifications. First, we test for significant autocorrelations in the residuals, which is indicative of omission of other relevant variables. Our preliminary analysis reveals significant auto-correlation in the residuals up to the second order, which generates inefficient coefficient estimates. Accordingly, the estimation of ESVL(CI) involves a two-step procedure. First, estimate ESVL(CI) according to equation (6). This generates a series of residuals $u_{i,t}$ for each of the four equations. Next, include $u_{i,t-1}$ and $u_{i,t-2}$ in their corresponding equations as additional variables to proxy for other variables that are not explicitly considered. Table 3 reports an improvement in model fitting from an increase in the log-likelihood function.

INSERT TABLE 3

Second, we test where the order in which volatility variables enter the various equations of the system based on our assumed informational pecking-order is appropriate. We benchmark each of the two ESVL models against two corresponding ESVL systems whether the order of efficiency is intentionally specified to be in stark contrast with ESLV(CI) and ESVL(FI). Table 4 reveals a huge difference in the log-likehood functions e.g. the log-likehood function for ESVL(CI) is 2138.01, whereas for its mis-specified counterpart, the value dropped to 1209.78. This suggests the importance of specifying the ESVL according to an appropriate order of volatility transmission.

INSERT TABLE 4

ESVL(CI) and ESVL(FI) estimation results are reported in Table 5 and Table 6 respectively. For ESVL(CI), $\sigma_{FI,t}$ is significant in the $\sigma_{CI,t}$ equation. Dum^{Open} is significant in all except the $\sigma_{FO,t}$ equation. Lastly, all lag-1 volatility variables are significant in their corresponding equations.

INSERT TABLES 5 and 6

Cross-market volatility transmission effects are more evident in ESVL(FI). Both $\sigma_{CI,t}$ and $\sigma_{FO,t}$ are significant in the $\sigma_{FI,t}$ equation. Both $\sigma_{FI,t}$ and $\sigma_{IO,t}$ are significant in the $\sigma_{CI,t}$ equation. Contemporaneous and lagged volatility from both CI and FI are significant in the $\sigma_{FO,t}$ equation. This may be indicative of volatility feedback from the underlying asset to the futures index, and then from there back to the futures option market. Lagged residuals are all significant in their corresponding equations.

INSERT TABLE 7

Lastly, we report SVL estimation results in Table 7. All own-market lag-1 volatilities are significant. All lagged residuals are significant as well. Lagged futures tick volume is also significant in the $\sigma_{FI,t}$ equation. To note, $\sigma_{FI,t}$ is significant and negative in the $\sigma_{CI,FI,t}$ equation. This result is consistent with our argument of an inverse relation between $\sigma_{FI,t}$ and $\sigma_{CI,FI,t}$ since the futures market experiences more trading activity than the underlying asset market in response to (say) unexpected macroeconomics news.

5 Out-of-sample hedging performances

In this section, we evaluate the incremental profits from rebalancing the futures position that competing OHRs generate over a static hedge. Indeed, given that true volatility is unobservable, an economic evaluation in the context of an out-of-sample dynamic hedging scheme is more meaningful than a statistical evaluation. To reiterate, our focus on incremental profits from rebalancing the futures position rather than the standard risk minimization as our hedging performance criterion stems from the focus of our paper. We are interested in whether incremental information in implied volatility translates into incremental profits from rebalancing the futures position

5.1 Details of the hedging scheme

We consider hedging against the exposure of a long position in a widely-held index-tracking portfolio with short positions in index futures contracts. We refer to the combined cash and futures positions as our net position. A static hedge involves no interim re-balancing during the hedging period. In contrast, a dynamic hedge involves interim re-balancing of the futures position during the hedging period, thereby accumulating interim profits or losses. In our dynamic hedging scheme, we consider the following scenario. An institutional trader owns a USD 5 million (m) widely-held equity portfolio that is very highly correlated with the S&P 500 index.[19] The objective is to protect the portfolio against market downside risk, and S&P 500 futures contracts is the only hedging instrument that will be considered.

The size of the futures position is also affected by $V_{CI,t}$ and $V_{FI,t}$. The latter is calculated as $V_{FI,t} = p_{FI,t} \times 500$ i.e. the time t futures price times the contract multiplier of USD 500 per index point. Note that $V_{CI,t=0}$= USD 5m. Let $V_{CI,t=1} = 5m \times (1 + r_{CI,t=1})$ and $V_{CI,t=2} = V_{CI,t=1} \times (1 + r_{CI,t=2})$ etc. Accordingly, the half-hourly profit/loss from the physical

[19] As mentioned in the introduction, we assume the S&P 500 index as our physical portfolio.

portfolio at time t is $V_{CI,t} - V_{CI,t-1}$. The change in value of a futures contract at time t is $(p_{FI,t} - p_{FI,t-1}) \times 500$. Since these are short positions, a profit arises when $(p_{FI,t} - p_{FI,t-1}) < 0$. The interim profit from re-balancing the futures position at time t is calculated as $(V_{FI,t-1} - V_{FI,t}) \times (N_{t-1} - N_{t-2})$, where $(N_{t-1} - N_{t-2})$ is the 'Number of contracts rebalanced at time t-1'.[20] To elaborate, if $(N_{t-1} - N_{t-2}) = +4$, this implies that an additional 4 contracts were shorted at time $t-1$. Subsequently, if the value of the futures contract decreases between time $t-1$ and time t e.g. $V_{FI,t-1} - V_{FI,t}$=+1000, then the interim profit at time t generated from the rebalancing at time $t-1$ is $4,000. If $N_{t-1} - N_{t-2} = 0$, this implies no rebalancing is required at time $t-1$, such that the interim profit for time t is zero.

Time series plots reveal that ESVL-based OHRs exhibit more intraday variability relative to both GARCH-based OHR and IHR. Accordingly, in the hedging exercise, we allow interim rebalancing as frequently as every half-hour, which implies that rebalancing will occur more often for ESVL-based OHRs. However, this does not necessarily imply that that ESVL-based OHRs will be more profitable given the presence of transaction costs and the fact that some rebalancing transactions will generate interim losses. Intraday rebalancing does occur for certain OHRs on some days during the test period. However, consecutive half-hourly rebalancing seldom occurs. As our test sample covers the second half of 1990, we conduct our hedging exercise separately on the September and December contracts. We assume that the institutional investor rolled over from the September to December contract after the 8th trading day in September 1990, which is consistent with our sampling procedure.

5.2 Incremental profit results

The incremental profits generated by competing OHRs are reported in Table 8. In the September quarter of 1990, the US equity market experienced a sharp downturn. This is

[20]Note that 'N', which represents the number of futures contracts, is always positive. The 'No. of contracts rebalanced at time t-1' is defined as $(N_{t-1} - N_{t-2})$. As such, if $N_{t-1} = 5$ and $N_{t-2} = 3$, then the 'No. of contracts rebalanced at time t-1' is +2, which implies shorting an additional two futures contracts.

followed by a moderate recovery in the December quarter. The value of the physical portfolio decreased by $771,716 or 15.43% in the September quarter. While it gained back $150,458 by the end of the December quarter, what transpired is an overall half-year loss of 12.43% on the initial investment value of $5m. The static hedge in OHR(1), which is ranked third overall, performed surprisingly well. By construction, OHR(1) is the only hedge ratio that counter-balances any profit (loss) from the the physical portfolio. However, OHR(1) is unable to completely preserve the value of the physical portfolio, and resulted in an overall loss of $165,858. However, this is still better than having an unhedged position .

INSERT TABLE 8

Next, the table shows that OHR(5) and OHR(8), which are generated from ESVL(CI) and ESVL(FI), clearly outperform all other OHRs considered in this paper. In addition to being the only two hedge ratios that outperform the static hedge OHR(1), these ESVL-based OHRs are also the only OHRs that manage to generate sufficient profits from interim rebalancing to cover the overall loss of $621,258 incurred by the physical portfolio during the test period. The enhanced GARCH-based OHR(3) is ranked fourth overall. It performs well in that it is able to generate interim profits of $52,325 and $15,775 from both the September and December contracts, although its overall profit of $68,100 is lower than that of the static hedge. Despite incorporating additional volatility variables in the weighting series, OHR(3) is overshadowed by the ESVL-based OHRs. Similar comments apply to OHR(4) and OHR(7), the two OHRs based on implied volatilities from the IO and FO markets.

OHR(6), which focuses on modeling time-varying covariance between the spot and futures prices, performed poorly. It improves the unhedged position by a modest $20,125. The latter consists of a loss of $39,175 in the September contract and a profit of $59,300 in the December contract. To note, the gain and loss in the futures position generated by OHR(6) are in the same direction as that of the underlying portfolio. This implies that OHR(6) exacerbates the variability of the overall position, rather than reducing it. OHR(2), which is based on the standard GARCH(1,1), is ranked last. It generates losses from both contracts, with an accumulated loss of $313,050 in the futures position alone.

We draw two implications from the preceding results. First, the findings support our proposition that incorporating incremental information from implied volatilities to generate the OHR projections translates into incremental profits from interim balancing the futures position. OHRs based solely on cash-futures data or options data, have all under-perform OHRs derived from a combination of cash, futures and options data, with the exception of the static hedge. The enhanced GARCH-based OHR(3) is superior to OHR(4), which is based solely on implied volatilities. It also outperforms OHR(6) and OHR(2), although OHR(3) itself failed to outperform the static hedge. Second, the profit dominance of ESVL-based OHR(5) and OHR(8) over OHR(3) suggests that the ESVL framework is more suitable than GARCH (1,1) at modeling intraday volatility transmission across the four S&P markets for the purpose of making out-of-sample OHR projections.

In general, the consideration of transaction costs offers a realistic balance of the incremental costs and benefits from interim rebalancing when bench-marked against a static hedge. However, we argue that transaction cost is a moot consideration for our paper. This is because most of the OHRs we consider, including GARCH, SVL and implied volatility, are already ranked below the static hedge OHR(1) based on gross profit. On the other hand, the ESVL-based OHR(5) and OHR(8) do instigate very frequent re-balancing of large numbers of futures contracts throughout the test period. It is obvious that any interim rebalancing based on either OHR(5) or OHR(8) will accumulate the most transaction costs. However, note the extravagant incremental profits from OHR(5) and OHR(8) of $1,591,700 and $1,545,225 correspondingly from rebalancing the September contract, and $145,450 and $150,100 from rebalancing the December contract. For the combined September and December futures positions, each of OHR(5) and OHR(8) generated interim profits that is approximately 2.5

times the loss incurred by the physical portfolio. The presence of transaction cost is unlikely to alter the extreme dominance of the ESVL-based OHRs over the static OHR(1).

6 Conclusion

If option markets contribute incremental information to their underlying asset markets, then incorporating implied volatility into modeling and forecasting dynamic hedge ratios should translate into incremental profits from interim rebalancing. Our results support this proposition. In addition, we show that such cross-market volatility transmission effects are captured by a system of simultaneous volatility equations. Hedge ratios generated from extended volatility systems ESVL, which incorporate intraday volatility transmissions across all four S&P 500 cash, futures, index option and futures option markets, significantly outperform all other competing OHRs considered in this paper.

Furthermore, the ESVL-based OHRs are the only hedge ratios that manage to generate profit from rebalancing the futures positions across both the September and December contracts in excess of the losses incurred by the physical portfolio. In fact, the extravagant profit results reported in Table 8 should attract considerable attention not only from hedgers. Since institutional speculators do not hold the physical portfolio, their focus is solely on the profit/loss generated from actively rebalancing the futures position based on a time-varying measure of the cash-futures sensitivity. The out-of-sample OHR projections can be easily applied in a trading strategy using index futures.

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Appendix: Deriving the reduced-form of ESVL(CI)

The appendix provides details on a time-series of $\sigma_{CI,t+1}$ is generated from equation (6) for constructing hedge ratios. To note, a similar methodology applies for generating a time-series of $\sigma_{FI,t+1}$. The key step involves deriving the reduced-form $\sigma_{CI,t}$ equation corresponding to equation (6). We show that the reduced-form $\sigma_{CI,t}$ equation contains only lagged volatility variables, which is required to be able to generate 1-step ahead forecasts of $\sigma_{CI,t+1}$. The coefficients of the reduced-form $\sigma_{CI,t}$ are functions of coefficients from the structural system. These coefficients are derived by fitting the structural system with sample over the estimation period. These, together with time t volatility variables, allows us to obtain $\sigma_{CI,t+1}$. After each forecast, the estimation period is expanded by 1 observation; the coefficients are updated and $\sigma_{CI,t+1}$ is calculated as per normal. This recursive process generates a time-series of $\sigma_{CI,t+1}$. To note, while generating such hedge ratios may seem computationally tedious, but since the projections are made 1-step at a time, the coefficients are stable even as we sequentially move across the test period. Thus updating the coefficient estimates is fairly easy.

The following describes the substitution process based on equation (6) to obtain the reduced-form $\sigma_{CI,t}$. The discussion emphasizes only on the contemporaneous volatility variables. Denote the individual equations in the system below as A1, A2, A3 and A4.

$$\sigma_{CI,t} = \alpha_{10} + \alpha_{11} Dum^{Open} + \beta_{11}(\sigma_{FI,t}) + \beta_{12}(\sigma_{IO,t}) + \gamma_{11}(\sigma_{CI,t-1}) + \gamma_{12}(\sigma_{FI,t-1}) + \gamma_{13}(CV_{t-1}) + \varepsilon_{CI,t}$$

$$\sigma_{FI,t} = \alpha_{20} + \alpha_{21} Dum^{Open} + \beta_{21}(\sigma_{CI,t}) + \beta_{22}(\sigma_{FO,t}) + \gamma_{21}(\sigma_{CI,t-1}) + \gamma_{22}(\sigma_{FI,t-1}) + \gamma_{23}(\sigma_{FO,t-1}) + \varepsilon_{FI,t}$$

$$\sigma_{IO,t} = \alpha_{30} + \alpha_{31} Dum^{Open} + \beta_{31}(\sigma_{CI,t}) + \beta_{32}(\sigma_{FO,t}) + \gamma_{31}(\sigma_{CI,t-1}) + \gamma_{32}(\sigma_{IO,t-1}) + \gamma_{33}(\sigma_{FO,t-1}) + \varepsilon_{IO,t}$$

$$\sigma_{FO,t} = \alpha_{40} + \alpha_{41} Dum^{Open} + \beta_{41}(\sigma_{CI,t}) + \beta_{42}(\sigma_{FI,t}) + \gamma_{41}(\sigma_{CI,t-1}) + \gamma_{42}(\sigma_{FI,t-1}) + \gamma_{43}(\sigma_{FO,t-1}) + \varepsilon_{FO,t}$$

First, note that the $\sigma_{FO,t}$ equation contains $(\sigma_{CI,t}, \sigma_{FI,t})$ and the $\sigma_{FI,t}$ equation contains $(\sigma_{CI,t}, \sigma_{FO,t})$. Accordingly, substituting $\sigma_{FO,t}$ into $\sigma_{FI,t}$ allows us to express $\sigma_{FI,t}$ in terms of $\sigma_{CI,t}$. Second, substitute $\sigma_{FI,t}$ in terms of $\sigma_{CI,t}$ into $\sigma_{FO,t}$, such that $\sigma_{FO,t}$ can also be expressed entirely in terms of $\sigma_{CI,t}$. Third, substitute $\sigma_{FO,t}$ in terms of $\sigma_{CI,t}$ into $\sigma_{IO,t}$, such that $\sigma_{IO,t}$ can also be expressed entirely in terms of $\sigma_{CI,t}$. Lastly, substitute $\sigma_{FI,t}$ and $\sigma_{IO,t}$ in terms of $\sigma_{CI,t}$ into $\sigma_{CI,t}$, thereby obtaining the reduced-form $\sigma_{CI,t}$, which is expressed entirely in terms of lagged volatility variables.

From equation A4:

$$\beta_{22}\sigma_{FO,t} = \beta_{22}\beta_{41}\sigma_{CI,t} + \beta_{22}\beta_{42}\sigma_{FI,t} \\ + \beta_{22}[\alpha_{40} + \alpha_{41}Dum^{Open} + \gamma_{41}(\sigma_{CI,t-1}) + \gamma_{42}(\sigma_{FI,t-1}) + \gamma_{43}(\sigma_{FO,t-1}) + \varepsilon_{FO,t}]$$

Substitute preceding into equation A2 and label as equation A5:

$$\sigma_{FI,t} = \alpha_{20} + \alpha_{21}Dum^{Open} + \beta_{21}(\sigma_{CI,t}) + \beta_{22}\beta_{41}\sigma_{CI,t} + \beta_{22}\beta_{42}\sigma_{FI,t} \\ + \beta_{22}[\alpha_{40} + \alpha_{41}Dum^{Open} + \gamma_{41}(\sigma_{CI,t-1}) + \gamma_{42}(\sigma_{FI,t-1}) + \gamma_{43}(\sigma_{FO,t-1}) + \varepsilon_{FO,t}] \\ + \gamma_{21}(\sigma_{CI,t-1}) + \gamma_{22}(\sigma_{FI,t-1}) + \gamma_{23}(\sigma_{FO,t-1}) + \varepsilon_{FI,t}$$

$$\sigma_{FI,t} = \frac{\beta_{21} + \beta_{22}\beta_{41}}{1 - \beta_{22}\beta_{42}}\sigma_{CI,t} + \frac{1}{1 - \beta_{22}\beta_{42}}[\alpha_{20} + \alpha_{21}Dum^{Open} + (\gamma_{21} + \beta_{22}\gamma_{41})\sigma_{CI,t-1} \\ + (\gamma_{22} + \beta_{22}\gamma_{41})\sigma_{FI,t-1} + (\gamma_{23} + \beta_{22}\gamma_{43})\sigma_{FO,t-1} + \beta_{22}\varepsilon_{FO,t} + \varepsilon_{FI,t}]$$

Substitute equation A5 into A4 and label as equation A6:

$$\sigma_{FO,t} = \beta_{41}\sigma_{CI,t} + \frac{\beta_{42}}{1 - \beta_{22}\beta_{42}}[(\beta_{21} + \beta_{22}\beta_{42})\sigma_{CI,t} + (\alpha_{20} + \beta_{22}\alpha_{40}) + (\alpha_{21} + \beta_{22}\alpha_{41})Dum^{Open} \\ + (\gamma_{21} + \beta_{22}\gamma_{41})\sigma_{CI,t-1} + (\gamma_{22} + \beta_{22}\gamma_{42})\sigma_{FI,t-1} + (\gamma_{23} + \beta_{22}\gamma_{43})\sigma_{FO,t-1} + \beta_{22}\varepsilon_{FO,t} + \varepsilon_{FI,t}] \\ + \alpha_{40} + \alpha_{41}Dum^{Open} + \gamma_{41}\sigma_{CI,t-1} + \gamma_{42}\sigma_{FI,t-1} + \gamma_{43}\sigma_{FO,t-1} + \varepsilon_{FO,t}$$

$$\sigma_{FO,t} = \frac{\beta_{41} + \beta_{21}\beta_{22}}{1 - \beta_{22}\beta_{42}}\sigma_{CI,t} + \frac{1}{1 - \beta_{22}\beta_{42}}[(\alpha_{40} + \alpha_{20}\beta_{42})Dum^{Open} \\ + (\gamma_{41} + \gamma_{21}\beta_{42})\sigma_{CI,t-1} + (\gamma_{42} + \gamma_{22}\beta_{42})\sigma_{FI,t-1} + (\gamma_{43} + \gamma_{23}\beta_{42})\sigma_{FO,t-1} + \beta_{42}\varepsilon_{FI,t} + \varepsilon_{FO,t}$$

Substitute equation A6 into A3 and label as equation A7:

$$\sigma_{IO,t} = \beta_{31}\sigma_{CI,t} + \beta_{32}\frac{\beta_{41} + \beta_{21}\beta_{22}}{1 - \beta_{22}\beta_{42}}\sigma_{CI,t} + \frac{\beta_{32}}{1 - \beta_{22}\beta_{42}}[(\alpha_{40} + \alpha_{20}\beta_{42})Dum^{Open} \\ + (\gamma_{41} + \gamma_{21}\beta_{42})\sigma_{CI,t-1} + (\gamma_{42} + \gamma_{22}\beta_{42})\sigma_{FI,t-1} + (\gamma_{43} + \gamma_{23}\beta_{42})\sigma_{FO,t-1} + \beta_{42}\varepsilon_{FI,t} + \varepsilon_{FO,t}] \\ + \alpha_{30} + \alpha_{31}Dum^{Open} + \gamma_{31}(\sigma_{CI,t-1}) + \gamma_{32}(\sigma_{FI,t-1}) + \gamma_{33}(\sigma_{FO,t-1}) + \varepsilon_{IO,t}$$

$$\sigma_{IO,t} = \frac{\beta_{31} + \beta_{32}\beta_{41} + \beta_{21}\beta_{22}\beta_{32} - \beta_{22}\beta_{31}\beta_{42}}{1 - \beta_{22}}\sigma_{CI,t} + \frac{1}{1 - \beta_{22}\beta_{42}}[(\beta_{30} + \beta_{40}\beta_{32} + \alpha_{20}\beta_{32}\beta_{42} - \alpha_{30}\beta_{22}\beta_{42}) \\ + (\alpha_{31} + \alpha_{41}\beta_{32} + \alpha_{21}\beta_{32}\beta_{42} - \gamma_{31}\beta_{22}\beta_{42})(Dum^{Open} + \sigma_{CI,t-1}) \\ + (\alpha_{32} + \gamma_{42}\beta_{32} + \gamma_{22}\beta_{32}\beta_{42} - \gamma_{32}\beta_{22}\beta_{42})\sigma_{FI,t-1} + (\gamma_{33} + \gamma_{43}\beta_{32} + \gamma_{23}\beta_{32}\beta_{42} - \gamma_{33}\beta_{22}\beta_{42})(\sigma_{FO,t-1}) \\ + \beta_{32}(\varepsilon_{FO,t-1} + \beta_{42}\varepsilon_{FI-t}) + \varepsilon_{IO,t}]$$

Substitute equations A5 & A7 into A1 to obttain the reduced-form.

$$\begin{aligned}
\sigma_{CI,t} =& \beta_{11}\frac{\beta_{21}+\beta_{22}\beta_{41}}{1-\beta_{22}\beta_{42}}\sigma_{CI,t} + \frac{\beta_{11}}{1-\beta_{22}\beta_{42}}[(\alpha_{20}+\alpha_{40}\beta_{22}) + (\alpha_{21}+\alpha_{41}\beta_{22})Dum^{Open} \\
&+(\gamma_{21}+\beta_{22}\gamma_{41})\sigma_{CI,t-1} + (\gamma_{22}+\beta_{22}\gamma_{42})\sigma_{FI,t-1} + (\gamma_{23}+\beta_{22}\gamma_{43})\sigma_{FO,t-1} + \beta_{22}\varepsilon_{FO,t} + \varepsilon_{FI,t}] \\
&+\beta_{12}\frac{\beta_{31}+\beta_{32}\beta_{41}+\beta_{21}\beta_{22}\beta_{32}-\beta_{22}\beta_{31}\beta_{42}}{1-\beta_{22}\beta_{42}}\sigma_{CI,t} \\
&+\frac{\beta_{12}}{1-\beta_{22}\beta_{42}}[(\alpha_{30}+\alpha_{40}\beta_{32}+\alpha_{20}\beta_{32}\beta_{42}-\alpha_{30}\beta_{22}\beta_{42}) + (\alpha_{31}+\alpha_{41}\beta_{32} \\
&+\alpha_{21}\beta_{32}\beta_{42}-\gamma_{31}\beta_{22}\beta_{42})Dum^{Open} \\
&+(\alpha_{31}+\alpha_{40}\beta_{32}+\alpha_{21}\beta_{32}\beta_{42}-\gamma_{31}\beta_{22}\beta_{42})\sigma_{CI,t-1} + (\alpha_{32}+\gamma_{42}\beta_{32}+\gamma_{22}\beta_{32}\beta_{42}-\gamma_{32}\beta_{22}\beta_{42})\sigma_{FI,t-1} \\
&+(\gamma_{33}+\gamma_{43}\beta_{32}+\gamma_{23}\beta_{32}\beta_{42}-\gamma_{33}\beta_{22}\beta_{42})\sigma_{FO,t-1} \\
&+\beta_{32}(\varepsilon_{FO,t}+\beta_{42}\varepsilon_{FI,t}) + \varepsilon_{IO,t}]
\end{aligned}$$

$$\begin{aligned}
\sigma_{CI,t} =& \frac{\beta_{11}}{\mathbf{A}}[(\alpha_{20}+\alpha_{40}\beta_{21}) + (\alpha_{21}+\alpha_{41}\beta_{22})Dum^{Open} \\
&+(\gamma_{21}+\gamma_{41}\beta_{22})\sigma_{CI,t-1} + (\gamma_{22}+\gamma_{42}\beta_{22})\sigma_{FI,t-1} + (\gamma_{23}+\gamma_{43}\beta_{22})\sigma_{FO,t-1} + \varepsilon_{FI,t} \\
&+\frac{\beta_{12}}{\mathbf{A}}[(\alpha_{30}+\alpha_{40}\beta_{32}+\alpha_{20}\beta_{32}\beta_{42}-\alpha_{30}\beta_{22}\beta_{42}) + (\alpha_{31}+\alpha_{41}\beta_{32}+\alpha_{21}\beta_{32}\beta_{42}-\alpha_{31}\beta_{22}\beta_{42})Dum^{Open} \\
&+(\alpha_{31}+\alpha_{41}\beta_{32}+\alpha_{21}\beta_{32}\beta_{42}-\alpha_{31}\beta_{22}\beta_{42})\sigma_{CI,t-1} + (\alpha_{32}+\gamma_{42}\beta_{32}+\gamma_{22}\beta_{32}\beta_{42}-\gamma_{32}\beta_{22}\beta_{42})\sigma_{FI,t-1} \\
&+(\gamma_{33}+\gamma_{43}\beta_{32}+\gamma_{23}\beta_{32}\beta_{42}-\gamma_{33}\beta_{22}\beta_{42})\sigma_{FO,t-1} + \beta_{32}(\varepsilon_{FO,t}+\beta_{42}\varepsilon_{FI,t}) + \varepsilon_{IO,t}]
\end{aligned}$$

where $\mathbf{A} = (1-\beta_{22}\beta_{42}+\beta_{11}\beta_{21}+\beta_{11}\beta_{22}\beta_{41}+\beta_{12}\beta_{31}+\beta_{12}\beta_{32}\beta_{41}+\beta_{12}\beta_{21}\beta_{22}\beta_{32}-\beta_{12}\beta_{22}\beta_{31}\beta_{42})$

The preceding demonstrates that equation (6) can be 'collapsed' to express $\sigma_{CI,t}$ in terms of lag-1 volatility variables.

Table 1: The list of competing hedge ratios

OHR (1)	Naïve Hedge
OHR (2)	Constant Covariance Bivariate GARCH(1,1)
OHR (3)	Constant Covariance Bivariate (Enhanced) GARCH(1,1)
OHR (4)	Constant Covariance IHR
OHR (5)	Constant Covariance ESVL(CI) and ESVL(FI)
OHR (6)	SVL
OHR (7)	Time Varying Covariance IHR
OHR (8)	Time Varying Covariance SVL, ESVL(CI) and ESVL(FI)

Table 3: Log-likelihood function of the ESVL models and their augmented counterparts

	ESVL(CI) with lagged residuals	ESVL(CI)	ESVL(FI) with lagged residuals	ESVL(FI)
Log-likelihood function	2138.01	2094.51	1872.68	1647.87

Note: Strictly speaking, evaluation on improved model fitting where competing models are of different dimension has to be made based of other information criteria like the Schwartz-Bayesian Information Criteria (SBIC), or the Akaike Information Criterion (AIC). However, the vast improvement in parameter significance and the removal of auto-correlation in the disturbance terms of the augmented ESVL models are adequate justification to favor applying the augmented versions of the ESVL models.

Table 4: Log-likelihood of ESVL models and mis-specified-ESVL models

	ESVL (CI)	Mis-specified ESVL(CI)	ESVL (FI)	Mis-specified ESVL(FI)
Log-likelihood function	2138.01	1209.78	1872.68	1650.41

Note: Here, model evaluation can be made based solely on the log-likelihood function since the corresponding competing models are of identical dimensions.

Table 2: Descriptive statistics of return, volatility and volume

Panel A: Descriptive statistics

	Mean	Min	Max	Median	Std dev	Skewness	Excess kurtosis
$r_{CI,t}$	0.0000058	-0.02057	0.030119	0.000058	0.002156	-0.29727	15.27724
$r_{FI,t}$	0.0019101	-0.037979	0.10251	0.000148	0.009579	5.31734	33.55719

Panel B: Autocorrelation features

Lag	PACF $r_{CI,t}$	Q(k) of DX[b]	PACF $r_{FI,t}$	Q(k) of DX^2	PACF $\sigma_{CI,t}$	Q(k) of DX	Q(k) of DX	PACF $\sigma_{FI,t}$
1	0.0978*	60.2*	-0.0476	14.2*	0.184*	213*	-0.0132	1.09
2	0.0171	64.6*	-0.0277	18.3*	0.113*	342*	-0.0153	2.54
5	0.0197	67.2*	-0.0500*	48.3*	0.0277*	508*	-0.0421*	25.9*
10	0.0362*	83.3*	-0.0589*	96.7*	0.0395*	615*	0.0415*	74.9*
13	0.0117	85.8*	0.929*	5580*	0.0644*	827*	0.959*	5890*
26	0.0086	105*	0.437*	11100*	0.1011*	1380*	0.386*	11700*

Panel C: Correlation matrix

	$\sigma^2_{CI,t}$	$\sigma^2_{FI,t}$	$\sigma^2_{FO,t}$	$\sigma^2_{IO,t}$	CV_t	FV_t	dCV_t	dFV_t
$\sigma^2_{CI,t}$	**1**							
$\sigma^2_{FI,t}$	0.2280	**1**						
$\sigma^2_{FO,t}$	-0.0233	-0.0054	**1**					
$\sigma^2_{IO,t}$	0.0722	-0.0009	-0.0414	**1**				
CV_t	0.3113	0.1738	-0.0490	0.0682	**1**			
FV_t	0.3803	0.2938	-0.0451	0.0287	0.3944	**1**		
dCV_t	0.1665	0.0313	-0.0023	0.0042	0.4905	0.1236	**1**	
dFV_t	0.2379	0.2451	-0.0112	0.0023	0.1084	0.3912	0.2685	**1**

Panel D: ADF statistics

ADF Test 2	$p_{CI,t}$	$p_{FI,t}$	$\sigma^2_{CI,t}$	$\sigma^2_{FI,t}$	$\sigma^2_{FO,t}$	$\sigma^2_{IO,t}$	CV_t	FV_t
Test statistics	-0.2464	-1.9394	-13.53*	-3.973*	-12.195*	-10.14*	-1.49	-1.89
Max lag	1	18	13	13	15	9	12	13

* indicates significant at 5% level.

[a] If the Autocorrelation coefficient (Partial Autocorrelation Function or PACF) lies within the 95% confidence interval range, it is not significant. The 95% confidence interval for the PACF is (-0.0247, 0.0247).

[b] Q(k) is the Ljung-Box test-statistics of joint significance for first to k^{th} order Autocorrelation, and is χ^2-distributed.

Table 5: Results from the FIML estimation on the ESVL(CI)

	$\sigma_{CI,t}$	$\sigma_{FI,t}$	$\sigma_{IO,t}$	$\sigma_{FO,t}$
Constant	-0.0751 (0.150)[a]	0.0581 (0.003)**	0.0157 (0.340)	-2.9534 (0.809)
Dum^{open}	-2.3323 (0.007)**	1.3919 (0.000)**	-0.0162 (0.008)**	-102.173 (0.754)
$\sigma_{CI,t}$	~	0.3875 (0.159)	0.0418 (0.8580)	-48.1824 (0.739)
$\sigma_{FI,t}$	1.6797 (0.007)**	~	~	73.5818 (0.754)
$\sigma_{IO,t}$	0.01124 (0.535)	~	~	~
$\sigma_{FO,t}$	~	-0.0002 (0.966)	-0.0013 (0.874)	~
$\sigma_{CI,t-1}$	-0.0691 (0.378)	0.0582 (0.061)	-0.0104 (0.780)	-1.1531 (0.948)
$\sigma_{FI,t-1}$	-0.0012 (0.909)	0.0034 (0.587)	~	-0.0627 (0.969)
$\sigma_{IO,t-1}$	~	~	0.8584 (0.000)**	~
$\sigma_{FO,t-1}$	~	0.0001 (0.983)	0.0011 (0.886)	0.9183 (0.000)**
CV_{t-1}	-0.0001 (0.310)	~	~	~
$u_{CI,t-1}$	-0.0069 (0.880)	~	~	~
$u_{CI,t-2}$	0.0484 (0.228)	~	~	~
$u_{FI,t-1}$	~	0.0157 (0.707)	~	~
$u_{FI,t-2}$	~	0.0380 (0.247)	~	~
$u_{IO,t-1}$	~	~	-0.0238 (0.054)	~
$u_{IO,t-2}$	~	~	0.0364 (0.043)*	~
$u_{FO,t-1}$	~	~	~	-0.2283 (0.000)**
$u_{FO,t-2}$	~	~	~	-0.0196 (0.702)

[a] p-values in parentheses
** significant at 1% level
* significant at 5% level

Table 6: Results from the FIML estimation on the ESVL(FI)

	$\sigma_{CI,t}$	$\sigma_{FI,t}$	$\sigma_{IO,t}$	$\sigma_{FO,t}$
Constant	0.4352 (0.000)[a]**	-162.222 (0.027)*	-0.0114 (0.995)	-133.240 (0.000)**
Dum^{close}	0.3031 (0.000)**	-50.0409 (0.287)	0.1591 (0.982)	-82.7965 (0.000)**
$\sigma_{CI,t}$	~	129.865 (0.023)*	-0.0610 (0.875)	4.1211 (0.038)*
$\sigma_{FI,t}$	-0.6375 (0.000)**	~	~	1740.69 (0.000)**
$\sigma_{IO,t}$	~	10646.3 (0.006)**	~	~
$\sigma_{FO,t}$	0.1646 (0.019)*	~	0.3556 (0.088)	~
$\sigma_{CI,t-1}$	0.1908 (0.001)**	156.525 (0.053)	-0.1647 (0.840)	-40.9111 (0.001)**
$\sigma_{FI,t-1}$	~	63.9502 (0.842)	~	-118.136 (0.000)**
$\sigma_{IO,t-1}$	~	-9762.23 (0.004)**	0.9682 (0.000)**	~
$\sigma_{FO,t-1}$	-0.1659 (0.127)	~	-0.2979 (0.102)	1.5648 (0.644)
FV_{t-1}	-4029 (0.567)	~	~	~
$u_{CI,t-1}$	378.8470 (0.000)**	~	~	~
$u_{CI,t-2}$	-203.605 (0.000)**	~	~	~
$u_{FI,t-1}$	~	1.2902 (0.003)**	~	~
$u_{FI,t-2}$	~	0.2994 (0.022)*	~	~
$u_{IO,t-1}$	~	~	-0.2888 (0.000)**	~
$u_{IO,t-2}$	~	~	-0.1597 (0.002)**	~
$u_{FO,t-1}$	~	~	~	-0.9268 (0.000)**
$u_{FO,t-2}$	~	~	~	0.3531 (0.000)**

[a] p-values in parentheses
** significant at 1% level
* significant at 5% level

Table 7: Results from the FIML estimation on SVL

	$\sigma_{FI,t}$	$\sigma_{CI,t}$	$\sigma_{CI,FI,t}$
Constant	-7.5256 (0.000)**	-1.0675 (0.573)	-46.3706 (0.000)**
Dum^{close}	0.3108 (0.093)	-1.1309 (0.001)**	-0.7649 (0.000)**
$\sigma_{FI,t}$	~	-0.0414 (0.669)	-0.2503 (0.049)*
$\sigma_{CI,t}$	~	~	~
$\sigma_{FI,t-1}$	-1.3060 (0.000)**	~	~
$\sigma_{CI,t-1}$	~	1.0742 (0.000)**	0.3628 (0.000)**
$\sigma_{t-1}^{CI,FI}$	0.0667 (0.197)	0.2191 (0.153)	5.6177 (0.000)**
FV_{t-1}	0.0117 (0.005)**	~	~
$u_{FI,t-1}$	-0.4869 (0.001)**	~	~
$u_{FI,t-2}$	-0.0794 (0.000)**	~	~
$u_{CI,t-1}$	~	-1.1272 (0.000)**	~
$u_{CI,t-2}$	~	-0.1377 (0.000)**	~
$u_{CI,FI,t-1}$	~	~	-5.6253 (0.000)**
$u_{CI,FI,t-2}$	~	~	-0.8211 (0.000)**

[a] p-values in parentheses
** significant at 1% level
* significant at 5% level

Table 8: Out-of-sample incremental profits generated by competing hedge ratios

OHR ranking		Profit from Sep contract	Profit from Dec contract	Profit from both contracts	Combined cash & futures position
1st place OHR (5)	Constant covariance ESVL(CI) & ESVL(FI)	$1,591,700	$145450	$1,737,150	$1,115,892
2nd place OHR (8)	Time varying covariance SVL, ESVL(CI) & ESVL(FI)	$1,545,225	$150,100	$1,695,325	$1,074,067
3rd place OHR (1)	Static hedge	$478,800	$(23,400)	$455,400	$(165,858)
4th place OHR (3)	Enhanced GARCH (1,1)	$52,325	$15,775	$68,100	$(553,158)
5th place OHR (7)	Time varying covariance IHR	$13,025	$19,225	$32,250	$(589,008)
6th place OHR (4)	Constant covariance IHR	$12,400	$18,300	$30,700	$(590,558)
7th place OHR (6)	SVL	$(39,175)	$59,300	$20,125	$(601,133)
8th place OHR (2)	GARCH (1,1)	$(239,425)	$(73,625)	$(313,050)	$(934,308)
Profit/Loss from the physical portfolio		$(771,716)	$150,458	$(621,258)	$(621,258)