Robust Hedging Performance and Volatility Risk in Option Markets

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Abstract

We investigate daily robust hedging performance with trading costs for markets of S&P 500 Index option (SPX) and Taiwan Index option (TXO). Robust hedging refers to minimal model dependence on the risky asset price. Two hedging categories including "model-free" and "volatility-model-free," and nonparametric methods for volatility estimation are considered in our empirical study. In particular, the instantaneous volatility is estimated by a proposed nonlinear correction scheme of Fourier transform method, justified by a simulation study for a local volatility model.

An asymmetric phenomenon of hedging performances is documented. Hedging portfolios constructed from the "volatility-model-free" category induce much higher Sharpe ratios than those from the "model-free" category on SPX, while they perform comparably on TXO. Motivated from the price limit regulation in Taiwan, we further develop a time-scale change method to explain this phenomenon. Asymptotic moment estimates of differences of some hedging portfolios are consistent with our empirical findings.

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1. Introduction

It has been recognized from empirical studies that complicated asset pricing models may not have better hedging performance than the ad hoc Black-Schole model. Bakshi et al. (1997), Lam et al. (2002) and Yung et al. (2003) document that stochastic volatility models, variance gamma models, and EGARCH (GARCH) models, respectively, are superior in volatility forecast and/or option pricing, but these models perform just comparably or even worse than the *ad hoc* Black-Scholes model (Dumas et al. (1998)) in option hedging.

These observations indicate the importance of robust hedging. That is, model dependence of option hedging strategies should be minimized. We identify and classify several hedging strategies according to their level of model dependence. Since implementation of many hedging strategies essentially requires volatility as the input variable, nonparametric methods for volatility estimations are incorporated for our empirical study to keep the spirit of reducing model errors. Fourier transform method proposed by Malliavin et al. (2009) provides a new and alternative nonparametric estimation to measure the *instantaneous* volatility risk without imposing any specific volatility models. This line of investigation also differentiates our research from the current literature of merely using the implied volatility, a Black-Scholes model-dependent volatility. As a whole, option hedging performances given robust hedging strategies and nonparametric volatility estimations are comprehensively studied in this paper. Option markets including SPX in US and TXO in Taiwan are chosen for empirical studies. A surprising difference on hedging performances between these two markets is documented. The price limit effect postulated in Taiwan is attributed to such difference according to our asymptotic analysis.

This paper studies intensively on hedging performance with trading costs for index option markets of SPX and TXO. We begin with an empirical study on hedging performances of these index options by various trading strategies with transaction costs and taxes. Two categories of hedging strategies are considered. (1) “Model-Free” category includes the stop-loss strategy and an adjusted stop-loss strategy. (2) “Volatility-Model-Free” category includes the delta hedging, an adjusted delta hedging strategy, and the delta-gamma strategy. Each hedging strategy doesn’t depend on either any specific asset pricing model or any specific volatility model. Combinations of these two hedging categories with three volatility estimations[2] of the historical volatility, the instantaneous volatility and the implied volatility are selected to compare hedging performances.

A key parameter for implementing these hedging strategies except the stop loss is volatility. Besides conventional methods of volatility estimation by the historical volatility and the implied volatility, a recent progress using a nonparametric Fourier transform method (Malliavin and Mancino (2009)) to estimate volatility matrix dynamics paves a way for estimation of the instantaneous volatility. In comparison with another popular nonparametric method for volatility estimation by quadratic variation formulas (see Zhang et al. (2005) and references therein), Malliavin and Mancino (2009) claimed that Fourier transform method is more stable because it relies on the integration of Fourier coefficients of the variance process as opposed to a numerical differentiation of

[2] The historical volatility and the instantaneous volatility are estimated based on nonparametric methods. The implied volatility does depend on the Black-Scholes model. It is incorporated because of its popularity in theory and practice.

quadratic variations. However, Reno (2008) alerts that the Fourier algorithm performs badly near time boundaries of estimated volatility time series data, i.e. estimated volatility of the first and last 1% time series are not accurate enough. To avoid this "boundary effect" pitfall, Han et al. (2010) provided an effective price correction scheme based on a linear regression derived from the distribution of estimated volatility given observed price returns. They justified that stochastic volatility models calibrated to the instantaneous volatility outperform GARCH (1,1) model based on backtesting results of Value-at-Risk (Joridon (2007)). In this paper, we propose a new correction scheme based on a nonlinear regression. A Monte Carlo simulation study for a local volatility model is used to demonstrate the accuracy of volatility estimation by these two correction schemes.

We document an asymmetric phenomenon of hedging performances between SPX and TXO. For SPX, hedging portfolios associated with the "volatility-model-free" category induce much higher Sharpe ratios than those associated with the "model-free" category. (In fact, the delta hedging with the instantaneous volatility outperforms other combinations including the well-perceived delta hedging with the implied volatility.) However for TXO, Sharpe ratios of hedging portfolios associated with these two categories are comparable. That is, using "stop-loss" like strategies in daily hedge perform as good as "delta hedging" like strategies in Taiwan. We further investigate the sample mean and standard deviation of P/L differences between the stop-loss and the delta hedging portfolios. It is clear to observe that these two statistics from TXO are very small compared with those from SPX. See Figures 3 and 4 in Section 3 for graphically demonstrations. These results further motivate our additional study of moment estimates for differences of hedging strategies.

Notice that there is a strict price limit constraint in the market of Taiwan Weighted Stock Index (TAIEX). Such price limit controls the fluctuation of each stock price in daily basis. Enormous literature has been studying price limit effects, which include cooling-off, volatility spillover, delay in price discovery, trading interference, and magnet effect. See discussions from Kim and Rhee (1997), Chen (1998), Cho et al. (2003) and references therein.

The relationship between hedging performance and the price limit has received surprisingly little attention despite its highly practical relevance in emerging markets such as Taiwan. We develop a theory that qualitatively explains small values of mean and standard deviation mentioned above on TXO, whereas hedging performance of SPX is considered as a control group of no price limit. Motivated from the cooling-off effect[3] from the price limit, we apply a time-scale change method to the Black-Scholes model, and analyze differences of hedging P/L induced from the stop-loss strategy and a rescaled delta hedging strategy. We obtain an asymptotic result to show that the P/L difference between these two hedging portfolios is small when the time change variable is small. This theoretical result is consistent to empirical findings in TXO. We shall remark that the time change method has been extensively studied in probability and mathematical finance. See an overview by Geman (2005), Fouque et al. (2003) and references therein.

The organization of this paper is as follows. In Section 2, we introduce procedures of various trading strategies to hedge index options on SPX and TXO. In Section 3,

[3] The cooling-off effect means that the price limit helps dampen volatility and stabilize trading volumes particularly during turbulent trading days.

volatility estimations including the historical volatility, the instantaneous volatility, and the implied volatility are introduced. Fourier transform method with price correction schemes is used for the instantaneous volatility estimation. A local volatility model is examined as a simulation study to validate the effectiveness of our proposed nonlinear regression correction scheme. In Section 4, data sets, transaction costs and taxes of each trade, and empirical results of hedging performances are demonstrated. Comparisons of inter and intra option markets are discussed. In Section 5, a time-scale change method is developed to the Black-Scholes model in order to mimic a cooling-off effect of price limit. We analyze moments of P/L differences of two hedging strategies associated with the delta and the stop loss, and confirm that our theoretical result in a qualitative sense is consistent with empirical findings on TXO.

2. Hedging Strategies

Two categories of dynamic hedging strategies are investigated in this paper. They are “Model-Free” category and “Volatility-Model-Free” category. The first category consists of two hedging strategies including the stop loss and an adjusted stop loss. The later strategy is designed to explore the persistency and the mean-reverting property of volatility in order to improve the stop-loss strategy.

The second category consists of three dynamic hedging strategies including the delta hedging, an adjusted delta hedging, and the delta-gamma hedging. The adjusted delta hedging strategy is based on the corrected delta hedging formula derived in Fouque, Papanicolaou, and Sircar (2000). Theoretically, this strategy is helpful to improve the delta hedge by taking the smile or smirk effect of implied volatility into account. This paper provides an empirical examination for such strategy. The delta-gamma hedging incorporates an additional option into the trading portfolio in order to eliminate the volatility risk. A number of practical ways to manage the volatility risk can be found in Gatheral (2006).

2.1 Model-Free Category

Two strategies are considered: the stop loss and an adjusted stop loss. Both strategies are fully independent of any pricing model. The stop-loss strategy is even independent of the volatility.

1. *Stop Loss*: This strategy takes a hedging position as fully covered when the underlying price S_t is in the money; otherwise fully naked. It can perfectly replicate the option payout but may suffer a huge transaction cost when S_t is wandering around the strike price. See for example in Hull (2009) for a discussion.

2. *Adjusted Stop Loss*: Based on one stylized fact of volatility (Engle (2009)) – property of mean reversion, we split the *ad hoc* stop-loss threshold K (the strike price) to an upper threshold such as 1.01K and a lower threshold such as 0.99K. When the current volatility level is low enough, the index price is likely to be in the money for a call option due to the leverage effect. Hence, it might be favorable to lower the stop-loss threshold K to, say 0.99K, for an early assess into a hedging position. Analogously when the volatility is high enough, the stop-loss threshold might be changed to 1.01K for an early exit position. The volatility used to measure the depth of moneyness is chosen as the historical volatility, the instantaneous volatility or VIX.

We remark that VIX may not exist in some option markets or its leverage doesn't appear strongly. For example TAIFEX didn't announce Taiwan VIX until December 2006. Even Taiwan VIX has been calculated in TAIFEX for some time since then, the correlation between returns of TAIEX and returns of Taiwan VIX during December 2006 and May 2009 is only -0.0726. Compared with the historical correlation -0.7213 between S&P 500 Index prices and the CBOE VIX during our sample period, Taiwan VIX provides a relatively weak leverage for its index price. Hence we use the historical volatility or the instantaneous volatility as other volatility measures.

2.2 Volatility-Model-Free Category

Three dynamic strategies within this category are the delta hedging, an adjusted delta hedging, and the delta-gamma hedging. Derivations of these hedging strategies are all rooted from the Black-Scholes pricing model but no specific volatility model is actually postulated. See Fouque et al. (2000) for detailed discussions. As a result, these strategies permit straightforward implementations without a full estimation of any volatility models such as the continuous-time Heston model (Heston (1993)) or discrete-time ARCH/GARCH models (Tsay (2005)). Given the following notations: the current time t, the current index price S_t, the volatility σ, the time to maturity τ, the strike price K, the risk-free interest rate r, and the option price $P(t, S_t)$, three dynamic hedging strategies are introduced below.

1. *Delta Hedging*: This strategy is effective to reduce the risk of market price. According to the Black-Scholes theory, an option price can be approximated by the dynamic portfolio $\alpha_t S_t + \beta_t e^{rt}$, where $\alpha_t = \Delta_t$ is defined by $\Delta = \dfrac{\partial P(t, S_t)}{\partial S_t}$ and β_t denotes the net

position invested in the money market account after transaction cost and tax. In the case of call options, $\Delta_t = N(d_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d_1} e^{-x^2/2} dx$, where $d_1(\tau, S_t) = \frac{1}{\sigma\sqrt{\tau}}[\log\frac{S_t}{K} + (r + \frac{\sigma^2}{2})\tau]$.

2. *Adjusted Delta Hedging*: Theoretically, this strategy is able to reduce not only the market price risk, but also partially the volatility risk. Fouque et al. (2000) applied a singular perturbation technique to derive an option price approximation such that an adjusted delta hedge strategy $\tilde{\Delta}_t$ can be deduced as follows:

$$\tilde{\Delta}_t = \frac{\partial P(t,x)}{\partial x} - \frac{V_3 \tau}{x}(4x^2 \frac{\partial^2 P(t,x)}{\partial x^2} + 5x^3 \frac{\partial^3 P(t,x)}{\partial x^3} + x^4 \frac{\partial^4 P(t,x)}{\partial x^4}),$$

where the additional parameter V_3 can be estimated from a linear regression of implied volatilities over the logarithm of market to money ratio (LMMR). This adjusted delta is capable of taking the volatility smile or smirk into account.

3. *Delta-Gamma Hedging*: This strategy can reduce both the market price risk and the volatility risk, but such trading portfolio costs more than the previous two strategies because of extra positions in longer-dated options. In order to further reduce the volatility risk of an option price $P^{(1)}(t, S_t)$ with a shorter maturity T_1, another option $P^{(2)}(t, S_t)$ with a longer maturity T_2, $T_2 > T_1$, can be traded in the hedging portfolio. Assuming that both options have the same strike prices, we can construct a dynamic portfolio of $\alpha_t S_t + \beta_t e^{rt} + c_t P^{(2)}(t, S_t)$, where

$$c_t = \frac{\partial P^{(1)} / \partial \sigma}{\partial P^{(2)} / \partial \sigma} = \frac{\nu^{(1)}}{\nu^{(2)}} = \frac{\Gamma^{(1)}}{\Gamma^{(2)}} \times \frac{T_1 - t}{T_2 - t}$$

$$a_t = \Delta^{(1)} - c_t \times \Delta^{(2)}$$

and $\Gamma = \frac{\partial^2 P(t, S_t)}{\partial S_t^2} = \frac{\partial \Delta}{\partial S_t}$. This strategy can approximate the option payout of $P^{(1)}$ by eliminating the market price risk and the volatility risk simultaneously.

In summary, the delta hedging, an adjusted delta hedging, and the delta-gamma hedging correspond to trading portfolios in the delta neutral position, the delta and partially gamma neutral position, and the delta-gamma neutral position, respectively. These positions are useful to distinguish the effectiveness of eliminating the market price risk with or without the volatility risk.

3. Volatility Estimation

Almost all hedging strategies mentioned above, except the stop loss, require an input of volatility for implementation of hedging portfolios. The study of volatility estimation either from the historical data and/or from the derivatives data has drawn tremendous attentions in past decades. See Tsay (2005), Gatheral (2006), Malliavin et al. (2009) and references therein. Its high-dimensional extension, i.e., volatility matrix estimation or correlation estimation, has been recently challenged by rapid developments in credit derivatives, credit portfolio risk management, etc. See Engle (2009) for details.

In the spirit of reducing model dependence, volatility estimation methods considered in this paper are mostly nonparametric. For example, quadratic variation and Fourier transform method have no dependence on volatility specification, and they are used for estimation of the historical volatility and the instantaneous volatility, respectively in this paper. Though the implied volatility, defined as an inversion of the Black-Scholes formula, does depend on the Black-Scholes model, it is additionally employed into our empirical study due to its popularity in theory and practice.

Next we review the Fourier transform method and its price correction scheme. A new correction scheme by a nonlinear regression method is proposed. We use a local volatility estimation problem for a simulation study to examine effectiveness of corrected Fourier transform methods.

3.1 Instantaneous Volatility Estimation by Fourier Transform Method

Malliavin and Mancino (2002, 2009) proposed a nonparametric Fourier transform method for estimation of the volatility process. The volatility time series can be reconstructed in terms of sine and cosine basis under the following continuous semi-martingale assumption. Let u_t be the log-price of a one-dimensional risky asset S at time t, i.e., which follows a diffusion process $u_t = \ln(S_t)$

$$du_t = \mu_t dt + \sigma_t dW_t, \tag{1}$$

where μ_t and W_t denote the instantaneous growth rate and a one-dimensional standard Brownian motion, respectively. When the time interval [0,T] of the data period is rescaled to $[0,2\pi]$, it is known that the underlying u_t can be reconstructed as the Fourier series expansion

$$u(t) = a_0 + \sum_{k=1}^{\infty}\left[-\frac{b_k(du)}{k}\cos(kt) + \frac{a_k(du)}{k}\sin(kt)\right],$$

in which Fourier coefficients of a's and b's are defined as follows:

$$a_0(du) = \frac{1}{2\pi}\int_0^{2\pi} du_t\ , \tag{2}$$

$$a_k(du) = \frac{1}{\pi}\int_0^{2\pi} \cos(kt)du_t \,, \qquad (3)$$

$$b_k(du) = \frac{1}{\pi}\int_0^{2\pi} \sin(kt)du_t \,, \qquad (4)$$

for any $k \geq 1$. Mallianvin and Mancino derived the Fourier coefficients of the variance σ_t^2 by

$$a_k(\sigma^2) = \lim_{N\to\infty} \frac{\pi}{2N+1} \sum_{s=-N}^{N-k} \left[a_s^*(du)a_{s+k}^*(du) + b_s^*(du)b_{s+k}^*(du)\right], \qquad (5)$$

$$b_k(\sigma^2) = \lim_{N\to\infty} \frac{\pi}{2N+1} \sum_{s=-N}^{N-k} \left[a_s^*(du)b_{s+k}^*(du) - b_s^*(du)a_{s+k}^*(du)\right], \qquad (6)$$

for $k \geq 0$, in which $a_s^*(du)$ and $b_s^*(du)$ are defined as

$$a_s^*(du) = \begin{cases} a_s(du), & if \ \mathrm{s} > 0 \\ 0, & if \ \mathrm{s} = 0 \\ a_{-s}(du), & if \ \mathrm{s} < 0 \end{cases} \quad and \quad b_s^*(du) = \begin{cases} b_s(du), & if \ \mathrm{s} > 0 \\ 0, & if \ \mathrm{s} = 0 \\ -b_{-s}(du), & if \ \mathrm{s} < 0. \end{cases}$$

A smoothing technique is conventionally applied so that the time series of variance σ_t^2 is approximated by

$$\sigma_t^2 \approx \sum_{k=0}^{N} \varphi(\delta k)[a_k(\sigma^2)\cos(kt) + b_k(\sigma^2)\sin(kt)], \qquad (7)$$

where $\phi(x) = \frac{\sin^2(x)}{x^2}$ is a smooth function with the initial condition $\phi(0) = 1$ and δ is a smooth parameter typically specified as $\delta = \frac{1}{50}$ (Reno 2008). Mattiussi et al. (2010) further investigate sensitivities of the number of Fourier series and the smoothing parameter by simulation studies.

Several advantages of this Fourier Transform method can be readily observed from Equations (2)-(6). First, the integration error of Fourier coefficients is adversely proportional to data frequency so this Fourier transform method is suitable for high-frequency data. Second, this method is easy to implement because, as shown in (5) and

(6), Fourier coefficients of the variance time series can be approximated by a finite sum of multiplications of a^* and b^*. Third, this integration method avoids the instability inherited from those traditional methods based on the differentiation of quadratic variation. See Zhang et al. (2005) for details.

3.2 Price Correction Schemes

One key drawback of this Fourier transform method is documented by the "boundary effect," i.e., a Gibbs phenomenon caused by the Fourier method. Reno (2008) noted that Fourier algorithm provides inaccurate estimate for volatility time series near the time boundary of simulated data. To remedy this boundary deficit, Han et al. (2010) took advantage of the relationship between asset returns and volatility, and proposed a price correction scheme based on a linear regression. In this paper, we propose another correction scheme based on a nonlinear regression. In our numerical simulation for estimating a local volatility time series, we find that the nonlinear regression scheme performs better.

To fix notations, recall that u_t defined in (1) is the natural logarithm of asset price. Based on the Euler discretization, the increment of log-price u_t can be approximated by $\sigma_t\sqrt{\delta_t}\varepsilon_t$, where δ_t denotes a small discretized time interval and ε_t denotes a sequence of i.i.d. standard normal random variables. This approximation is derived from neglecting the drift term of small order δ_t and using the increment distribution of Brownian motion $\Delta W_t = \sqrt{\delta_t}\varepsilon_t$. Let $\hat{\sigma}_t$ denote the volatility time series estimated from the original Fourier transform method. We review the linear regression correction scheme proposed by Han et al. (2010) for bias reduction of volatility estimation. A new nonlinear regression correction scheme is also devised below.

(1) *Linear Regression Correction Scheme* (Han et al. (2010)): This scheme consists of a log-linear transformation on the estimated variance process $\hat{\sigma}_t^2$ by Fourier Transform method in order to guarantee positiveness of volatility. That is, we transform $\hat{Y}_t = 2\ln\hat{\sigma}_t$ to $a + b\hat{Y}_t$ so that the corrected volatility $\sigma_t = \exp\left(\left(a + b\hat{Y}_t\right)/2\right)$ satisfies $\Delta u_t \approx \exp\left(\left(a + b\hat{Y}_t\right)/2\right)\sqrt{\delta_t}\varepsilon_t$, where $\Delta u_t = u_{t+1} - u_t$, and a and b denote the correction coefficients. Hence, one can use the maximum likelihood method to regress out these two coefficients via the relationship between the logarithm of the squared standardized return $\Delta u_t/\sqrt{\delta_t}$ and the driving volatility process $a + b\hat{Y}_t$:

$$\ln\left(\frac{\Delta u_t}{\sqrt{\delta_t}}\right)^2 = a + b\hat{Y}_t + \ln\varepsilon_t^2. \qquad (8)$$

(2) *Nonlinear Regression Correction Scheme*: By taking a direct linear transformation on estimated volatility $\hat{\sigma}_{\mathrm{t}}$ from the original Fourier transform method, we end up solving a nonlinear regression equation for estimation of correction coefficients a and b. That is, the true volatility $\sigma_t = a + b\widehat{\sigma_t}$ satisfies $\Delta u_t \approx (a + b\widehat{\sigma_t})\sqrt{\delta t}\varepsilon_t$ so that a nonlinear regression equation is obtained:

$$\ln\left(\Delta u_t/\sqrt{\delta t}\right)^2 = \ln(a + b\widehat{\sigma_t})^2 + \ln\varepsilon_t^2. \qquad (9)$$

Note these two price correction schemes (8) and (9) must be solved numerically by the maximum likelihood method due to the complex distribution of a log-Chi square

$\ln \varepsilon_t^2$. Their computational costs are the same. Though there is no guarantee that the corrected volatility estimation $\sigma_t = a + b\widehat{\sigma_t}$ based on a nonlinear regression scheme remains positive, no negative volatility has been found in either our simulation study or empirical study. In fact, this nonlinear correction scheme outperforms the linear correction scheme according to the following simulation study for a local volatility model.

3.3 A Simulation Study: Local Volatility Estimation

Since the true instantaneous volatility is not known, we test two proposed correction schemes by simulation. A local volatility model of the following form

$$dS_t = \alpha\left(m - S_t\right)dt + \beta S_t^{\gamma} dw_t$$

is considered. In Jiang (1998), those model parameters were estimated as $\alpha = 0.093$, $m = 0.079$, $\beta = 0.794$ and $\gamma = 1.474$. We employ this set of parameters, then simulate the price process S_t with its volatility process $\sigma_t = \beta S_t^{\gamma}$. The simulation is done by the Euler discretization with time step size $\delta_t = 1/250$ and the total sample number is 5000.

Based on the original Fourier transform method and those two proposed price correction schemes, three volatility time series can be estimated and used to compare with the actual volatility series. We use two criterions for error measures including Mean squared errors (MSE) and Maximum absolute errors (MAE). Comparison results are listed below:

1. MSE: 7.52E-04 (Fourier method), 1.19E-05 (Linear Regression Correction Scheme), 7.61E-06 (Nonlinear Regression Correction Scheme).

2. MAE: 0.04 (Fourier method), 0.02 (Linear Regression Correction Scheme), 0.01 (Nonlinear Regression Correction Scheme).

Noticeably, the price correction schemes (8) and (9) are able to reduce effectively both error criterions at least by half in this simulated example. Our newly proposed nonlinear regression correction scheme performs better than the linear correction scheme. We will use this nonlinear scheme for estimation of the instantaneous volatility in our empirical study of hedging performance.

4. Empirical Study of Hedging Performance: SPX and TXO

We consider the hedging performance for call options of S&P 500 Index and Taiwan Index. Profit and loss (P/L) and Sharpe ratio are used as two measures for hedging performance. Various strategies within the two hedging categories discussed in Section 2 are possibly combined with three volatility estimations, discussed in Section 3, including the historical volatility, the instantaneous volatility and the implied volatility. The historical volatility is estimated from thirty-day historical returns, the instantaneous volatility is estimated by the proposed nonlinear regression correction scheme of Fourier transform method, and the implied volatility is estimated from an inversion of the Black-Scholes formula.

4.1 Data Description

The nearest contract months of option prices with maturity times that are greater than one day but less than or equal to thirty days are selected in this empirical study. We avoid one-day option prices because some implied volatilities on TXO cannot be solved from the Black-Scholes formula. Option prices with thirty-one days to maturity and beyond are also excluded because of low trading volumes on TXO. Such selection criterions are applied to SPX for data consistency.

The sample period of S&P 500 Index prices and prices of SPX, traded in the Chicago Board Options Exchange (CBOE), is from January 2001 to June 2006. Daily data were retrieved from the Ivy Database of OptionMetrics. The total number of call options within that sample period is 105,125. One contract of SPX is on 100 US dollars (USD) times the option price. The transaction cost of trading options is set as 0.5 USD per contract. The risk-free interest rate is chosen as the three-month U. S. Treasury Bill.

Taiwan Index option (TXO) has been traded in Taiwan Futures Exchange (TAIFEX) since 2002. Its underlying is Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX). Our sample period is from July 2003 to March 2009 including the recent financial crisis. Daily prices of TAIEX and TXO are downloaded from Taiwan Stock Exchange (TWSE) and TAIFEX, respectively. The time to maturity of TXO lasts from two trading days to thirty trading days. The total number of call options within that data sample period is 43,993. One index option contract in TXO is on 50 New Taiwan Dollars (NTD) times the option price. The transaction cost in TAIFEX is 9 NTD for buying and selling each option contract with an additional 0.1% tax rate. The risk-free interest rate is chosen as the average of one-month CD rates from five large domestic banks[4] in Taiwan.

Notations of hedging strategies are shown in the following: Δ-H, Δ-F and Δ-Imp denote the delta hedging strategy combined with the historical volatility, the instantaneous volatility and the implied volatility, respectively. We denote by adΔ and Δ-Γ the adjusted delta hedging and the delta-gamma hedging, respectively. Both use the historical volatility. SL denotes the stop-loss strategy, and adSL-H, adSL-F and adSL-V denote the adjusted stop-loss strategy using the historical volatility, the instantaneous volatility and VIX, respectively.

4.2 Hedging Performance of SPX

Results of two measures (P/L and Sharpe ratio) for hedging call options with time to maturity T=20 days are reported in Table 1. On each row of that table, the year period, the total number N of hedged call options and their hedging performances are reported. The best measure of hedging performances within each row is highlighted in bold face with an underline. The sample mean of P/L, i.e. averaged P/L, with a parenthesis means a loss; otherwise it means a profit. For example, Panel A-(1) illustrates that there are 441 call options hedged in 2001. The best hedging performance is obtained by the stop-loss strategy, which makes a profit of USD 296 on average per contract. The last row of this subpanel records the hedging performance for the whole sample period from year 2001 to 2007.

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[4] Bank of Taiwan, Taiwan Cooperative Bank, Firstbank Commercial Bank, Hua Nan Bank, and Chang Hwa Bank

Given such fixed hedging period T=20, the P/L average of each hedging strategy is positive. In general, hedging strategies within the model-free category generate larger profits than those in the volatility model-free category (see Panel A-(1)), so do their standard deviations (see Panel A-(2)). This implies hedging performance of the model-free category is less stable than the volatility-model-free category. In terms of Sharpe ratio as a measure of hedging performance, the volatility model-free category outperforms the model-free category in general. Within these two categories, adjusted strategies including the adjusted stop loss and the adjusted delta hedging perform roughly the same as their ordinary strategies including the delta hedging and stop loss, respectively. Next, we demonstrate the time evolution of hedging performance of Sharpe ratios from T=2 to 30 in Figure 1.

This figure shows dynamic behaviors of Sharpe ratios associated with all hedging strategies within the volatility-model-free category and the model-free category. It is observed that first, the delta hedging using the instantaneous volatility (Del-F) performs best within the volatility-model-free category. This result is a new finding in hedging literature to our knowledge and consistent with the use of the instantaneous volatility in Value-at-Risk estimation (Han et al. (2010)). However, it is not so clear to determine the best strategy within the model-free category. Second, all these Sharpe ratios increase with maturity time. That is, longer the time period a hedging position is formed, higher Sharpe ratio is obtained. Third, the volatility-model-free category outperforms in general than the model-free category except when the time to maturity is short.

The last observation, i.e., a separation of hedging performances between the two hedging categories on SPX will have a contradict result on TXO.

4.3 Hedging Performance of TXO

Table 2 records results of two measures including P/L and Sharpe ratio for the performance of hedging call options on TXO. The time to maturity T is chosen as 20 trading days. Several observations can be made below. First, aggregate hedging performances, shown in the last lines of each panel, of the volatility-model-free category and the model-free category are roughly of the same numeric order. This means that hedging performances of all strategies are comparable in TXO. This phenomenon is significantly contradictory to what observed a well separation of hedging performances on SPX. Second, in the volatility-model-free category the delta hedging using the implied volatility performs worse than the other three hedging strategies using the historical volatility. This is also different from what observed in Table 1 on SPX.

Figure 2 demonstrates dynamic behaviors of several hedging performances. We observe that first all hedging strategies perform rather comparable except the delta hedging using the implied volatility. Second, Sharpe ratios decrease with maturity time. This means that shorter the time period of the hedging position is formed, higher the Sharpe ratio is gained. It is worth noting that these two phenomena are significantly different from what observed on SPX.

4.4 Comparisons of Hedging Performances for SPX and TXO

A summary of hedging performances on SPX and TXO is listed below.

1. Dynamics of Sharpe ratios for hedging call options are different. Sharpe rations of SPX tend to increase with maturity time while they tend to decrease on TXO.

2. In both measures of P/L and Sharpe ratios, the volatility-model-free category dominates the model-free category on SPX, while these two categories perform comparably on TXO.

We further investigate the P/L difference between the delta hedging strategy and the stop-loss strategy for a comparison. Each strategy is considered as the delegate of the volatility-model-free category and the model-free category. The empirical hedging differences of P/L are demonstrated in Figures 3 and 4 for their means and standard deviations, respectively. Maturity times span from two to thirty trading days. In each figure, HE1(2,3) represents the averaged hedging difference between the stop loss and the delta hedging in use of the historical volatility (the instantaneous volatility, the implied volatility, respectively). The dollar unit is USD.

Hedging differences on TXO are all close to zero rather uniformly for both mean and standard deviation. In contrast, these two statistics are relatively large on SPX. The next section is managed to provide a theoretical justification about small hedging differences between the delta and the stop loss on TXO.

5. A Moment Analysis for Hedging Differences

We have seen from Table 2 that hedging performances between model-free category and volatility-model-free category are comparable on TXO. More specifically Figures 3 demonstrates that the hedging differences between the delta hedging and the stop loss are relatively small compared with those differences induced from SPX. This section is devoted to justify these small hedging differences on TXO by a mathematical moment analysis.

Notice that there is a strict price limit constraint on TAIEX while S&P 500 Index doesn't. Motivated from the volatility cooling-off effect of the price limit (Kim and Rhee (1997)), we develop a time-scale change method for the classical Black-Scholes model, and analyze the hedging difference between the stop-loss strategy and a rescaled delta hedge strategy.

The Black-Scholes-Merton's option pricing theory assumes that the dynamic of the underlying risky asset price S_t follow a geometric Brownian motion. That is, under the physical probability measure, the asset price satisfies

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t,$$

where μ is the return rate, σ is the volatility, and W_t is the Brownian motion. The solution of this stochastic differential equation is $S_T = S_t \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)(T-t) + \sigma W_{T-t}\right)$ given that $T \geq t$ and $S_t \geq 0$. The time-scale change method postulates a variable change in time as the follows:

$$\begin{aligned}\frac{dS_t}{S_t} &= \mu d\delta t + \sigma dW_{\delta t} \\ &= \mu \delta dt + \sigma\sqrt{\delta} dW_t \qquad (1)\end{aligned}$$

where $\delta > 0$ is a small time scale, which controls the speed of the new time variable δt. The time scale δ can be either deterministic or random, assuming independent of the Brownian motion $(B_t)_{t\geq 0}$. In this study we assume a deterministic δ for ease of explanation. Thus, the solution of Eq. (1) is $S_T = S_t \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)\delta(T-t) + \sigma\sqrt{\delta} W_{T-t}\right)$.

Under the risk-neutral probability measure, the market price risk or risk premium is chosen as $\frac{r-\mu\delta}{\sigma\sqrt{\delta}}$. Let $p^{\delta}(t,S_t)$ denotes the European option price under the scaled dynamic (1) with the payoff $h(S_T)$. By applications of Ito's lemma used in the option pricing theory, it is straightforward to obtain the following results. The scaled pricing partial differential equation is

$$L^{\delta}P^{\delta}(t,x)=0$$

with the terminal condition $P^{\delta}(T,x)=h(x)$, where the partial differential operator $L^{\delta}=\frac{\partial}{\partial t}+\frac{\sigma^2\delta x^2}{2}\frac{\partial^2}{\partial x^2}+rx\frac{\partial}{\partial x}-r$. Hence at the current time t, the vanilla call option price with the strike price K and the maturity T is

$$p^{\delta}(t,x)=xN\left(d_1^{\delta}(t,x)\right)-e^{-r(T-t)}KN\left(d_2^{\delta}(t,x)\right),$$

where $d_1^{\delta}(t,x)=\frac{\ln(z/K)+\left(r+\sigma^2\delta/2\right)(T-t)}{\sigma\sqrt{\delta}\sqrt{T-t}}$ and $d_2^{\delta}(t,x)=d_1^{\delta}-\sigma\sqrt{\delta}\sqrt{T-t}$, and its delta is

$$\frac{\partial P^{\delta}(t,x)}{\partial x}=N\left(d_1^{\delta}(t,x)\right).$$

We remark that the time-scale change model proposed can be possibly extended to random time change. For example in the case of the variance gamma model (see Geman (2005)), the scaled time δt has a gamma distribution, so that the option pricing formula can be carried out by Fourier transformation. We leave this theoretical issue for a future research topic.

Suppose one hedges the option using the trading strategy (α_t, β_t), where this strategy invests α_t units of the index at time t and $\beta_t = \left(P^\delta(t, S_t) - \alpha_t S_t\right) / e^{rt}$. We define the instantaneous hedging error by

$$\alpha_t dS_t + \beta_t r e^{rt} dt - dP^\delta(t, S_t).$$

The accumulative hedging errors from time 0 to time T, denoted by HE_T, is defined by

$$HE_T = \int_0^T \alpha_t dS_t + \int_0^T \beta_t r e^{rt} dt - \left(P^\delta(T, S_T) - P^\delta(0, S_0)\right)$$

$$= P^\delta(0, S_0) + \int_0^T \alpha_t dS_t + \int_0^T \beta_t r e^{rt} dt - P^\delta(T, S_T)$$

Assume two different hedging strategies $\alpha_t^{(1)}$ and $\alpha_t^{(2)}$ are used for hedge. Let $HE_T^{(1)}$ and $HE_T^{(2)}$ denotes their accumulative hedging errors respectively. Their difference equals to

$$HE_T^{(1)} - HE_T^{(2)} = \int_0^T \left(\alpha_t^{(1)} - \alpha_t^{(2)}\right) dS_t - \int_0^T \left(\alpha_t^{(1)} - \alpha_t^{(2)}\right) r S_t dt$$

$$= \int_0^T \left(\alpha_t^{(1)} - \alpha_t^{(2)}\right)(\mu\delta - r) S_t dt + \sigma\sqrt{\delta} \int_0^T \left(\alpha_t^{(1)} - \alpha_t^{(2)}\right) S_t dW_t.$$

In cases of the stop-loss strategy $\alpha_t^{(1)} = I\left(S_t > e^{-r(T-t)} K\right)$ and the delta hedging strategy $\alpha_t^{(2)} = N\left(d_1^\delta(t, x)\right)$, we will prove that any moment of the accumulative hedging difference $HE_T^{(1)} - HE_T^{(2)}$ converges to zero when the time scale δ approaches to zero. To obtain this result, we first prove the following lemma. Its proof is showed in the Appendix.

Lemma 1. $E\left\{\left(\alpha_t^{(1)} - \alpha_t^{(2)}\right)^2\right\} \le \frac{C}{\sqrt{\delta t}} e^{-\frac{1}{\delta}}$ for some constant C independent of time and the scale δ. It implies that $E\left\{\left(\alpha_t^{(1)} - \alpha_t^{(2)}\right)^2\right\}$ converges to zero as δ goes to zero.

By applications of the Cauchy-Schwartz inequality on the hedging error equation, any moment of the accumulative hedging errors is bounded by

$$C_1 \int_0^T E\left\{\left(\alpha_t^{(1)} - \alpha_t^{(2)}\right)^2\right\} dt$$

for some constant C_1 independent of δ. By Lemma 1, it is easy to obtain the following theorem.

Theorem 2 (Moments Bound) For any positive integer n, $\left| E\left\{\left(HE_T^{(1)} - HE_T^{(2)}\right)^n\right\} \right| \leq \frac{C}{\sqrt{\delta}} e^{-1/\delta}$ for some constant C independent of δ.

We obtain an asymptotic result to show that the difference between two hedging strategies is small when the time change variable is small. This theoretical result is consistent to observed hedging performance in Taiwan.

6. Conclusion

This paper extends previous empirical studies on option hedging performance. Robust hedging strategies and nonparametric volatility estimations are comprehensively studied. It shows that the instantaneous volatility estimated from a corrected Fourier transform method may play an important role in hedging on SPX. An asymmetric phenomenon arising from our empirical study is also observed as follows. The "volatility-model-free" hedging category generally outperforms the "model-free" hedging category on SPX; while these two categories perform roughly the same on TXO.

The second part of this paper aims to explain this documented phenomenon by a detailed comparison between the delta hedging and the stop-loss strategy as delegates of two hedging categories. We propose a time-scale change method to account for the price limit, which is typically regulated in emerging markets such as Taiwan. The SPX market serves as a control group of no price limit. An asymptotic analysis confirms estimated moments of hedging portfolio differences with our empirical finding.

Appendix A: Proof of Lemma 1

Recall that the solution of (1) is $S_t = S_0 \exp\left(\left(r + \sigma^2\delta/2\right)t + \sigma\sqrt{\delta}\sqrt{t}Z\right)$, where Z denotes the standard normal random variable. Substituting this into $\alpha_t^{(1)}$ and $\alpha_t^{(2)}$, we deduce

$$\alpha_t^{(1)} = \mathrm{I}\left(\ln\frac{S_0}{K} + rT + \frac{\sigma^2\delta}{2}t + \sigma\sqrt{\delta}\sqrt{t}Z > 0 \right)$$

$$\alpha_t^{(2)} = \mathrm{N}\left(\frac{\ln\frac{S_0}{K} + \left(r + \frac{\sigma^2\delta}{2}\right)T + \sigma\sqrt{\delta}\sqrt{t}Z}{\sigma\sqrt{\delta}\sqrt{T-t}} \right).$$

Let $z^* = -\left(\frac{\ln\frac{S_0}{K} + rT + \frac{\sigma^2\delta}{2}t}{\sigma\sqrt{\delta}\sqrt{t}} \right)$, then we deduce

$$\begin{aligned} &E\left\{\left(\alpha_t^{(1)} - \alpha_t^{(2)}\right)^2\right\} \\ &= \int_{z^*}^{\infty} \left(\alpha_t^{(1)} - 1\right)^2 \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz + \int_{-\infty}^{z^*} \left(\alpha_t^{(1)}\right)^2 \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \end{aligned} \tag{2}$$

We first consider the convergence result in the first term. Assuming that $\ln\frac{S_0}{K} + rT > 0$, $z^* \to -\infty$ as $\delta \downarrow 0$. To analyze the first term in (2), we divide the integration domain $\left(z^*, \infty\right)$ into three regions $\left(z^*, -\varepsilon\right), \left(-\varepsilon, \varepsilon\right), \left(\varepsilon, \infty\right)$ for some $\varepsilon > 0$, then study the convergent result for each corresponding sub-integral.

Because $\alpha_t^{(1)} - 1 = \mathrm{N}\left(-\frac{\ln\frac{S_0}{K} + \left(r + \frac{\sigma^2\delta}{2}\right)T + \sigma\sqrt{\delta}\sqrt{t}z}{\sigma\sqrt{\delta}\sqrt{T-t}} \right)$, the third sub-integral equals to

$$\int_{\varepsilon>0}^{\infty}\left(\mathrm{N}\left(-\frac{\ln\frac{S_0}{K}+\left(r+\frac{\sigma^2\delta}{2}\right)T+\sigma\sqrt{\delta}\sqrt{t}z}{\sigma\sqrt{\delta}\sqrt{T-t}}\right)\right)^2\frac{1}{\sqrt{2\pi}}e^{-z^2/2}dz$$

$$\leq\left(\mathrm{N}\left(-\frac{\ln\frac{S_0}{K}+\left(r+\frac{\sigma^2\delta}{2}\right)T+\sigma\sqrt{\delta}\sqrt{t}\varepsilon}{\sigma\sqrt{\delta}\sqrt{T-t}}\right)\right)^2\int_{\varepsilon}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-z^2/2}dz$$

$$\leq\frac{\sigma\sqrt{\delta}\sqrt{T-t}}{\ln\frac{S_0}{K}+\left(r+\frac{\sigma^2\delta}{2}\right)T+\sigma\sqrt{\delta}\sqrt{t}\varepsilon}\exp\left\{-\left(\frac{\ln\frac{S_0}{K}+\left(r+\frac{\sigma^2\delta}{2}\right)T+\sigma\sqrt{\delta}\sqrt{t}\varepsilon}{\sigma\sqrt{\delta}\sqrt{T-t}}\right)^2/2\right\}\frac{1}{2}$$

If one choose $\varepsilon=e^{-1/\delta}$, then this term is bounded above by $C\sqrt{\delta}e^{-1/\delta}$ for some constant C, independent of δ and t. Next we consider the second sub-integral

$$\int_{-\varepsilon}^{\varepsilon}\left(\mathrm{N}\left(-\frac{\ln\frac{S_0}{K}+\left(r+\frac{\sigma^2\delta}{2}\right)T+\sigma\sqrt{\delta}\sqrt{t}z}{\sigma\sqrt{\delta}\sqrt{T-t}}\right)\right)^2\frac{1}{\sqrt{2\pi}}e^{-z^2/2}dz\leq 2\varepsilon \quad ,$$

where we use the bound of the normal integral function. Next we proceed to the first sub-integral by a further division as $\left(z^*,-\varepsilon\right)=\left(z^*,z^*+1/\sqrt{\delta}\right)\cup\left(z^*+1/\sqrt{\delta},-\varepsilon\right)$.

$$\int_{z^*}^{z^*+\frac{1}{\sqrt{\delta}}}\left(\mathrm{N}\left(-\frac{\ln\frac{S_0}{K}+\left(r+\frac{\sigma^2\delta}{2}\right)T+\sigma\sqrt{\delta}\sqrt{t}z}{\sigma\sqrt{\delta}\sqrt{T-t}}\right)\right)^2\frac{1}{\sqrt{2\pi}}e^{-z^2/2}dz$$

$$\leq\int_{z^*}^{z^*+\frac{1}{\sqrt{\delta}}}e^{-z^2/2}dz$$

$$\leq\frac{1}{\sqrt{\delta}}e^{-\left(z^*+\frac{1}{\sqrt{\delta}}\right)^2/2}$$

$$\approx\frac{1}{\sqrt{\delta}}e^{-\left(\frac{\ln\frac{S_0}{K}+rT}{\sigma\sqrt{\delta}\sqrt{t}}\right)^2/2}$$

Note that $z^* + \frac{1}{\sqrt{\delta}} = -\left(\frac{\ln\frac{S_0}{K} + rT + \frac{\sigma^2\delta}{2}t - \sigma\delta\sqrt{t}}{\sigma\sqrt{\delta}\sqrt{t}} \right) \approx -\left(\frac{\ln\frac{S_0}{K} + rT}{\sigma\sqrt{\delta}\sqrt{t}} \right)$ when δ is small.

Next we consider the other sub-integral and obtain an upper bound

$$\int_{z^*+\frac{1}{\sqrt{\delta}}}^{-\varepsilon} \left(\mathrm{N}\left(-\frac{\ln\frac{S_0}{K} + \left(r + \frac{\sigma^2\delta}{2} \right)T + \sigma\sqrt{\delta}\sqrt{t}z}{\sigma\sqrt{\delta}\sqrt{T-t}} \right) \right)^2 \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \le \sqrt{\delta}\sqrt{\frac{T-t}{t}} e^{-\frac{t}{2\delta(T-t)}} .$$

Note that we have used the following result: when $z^* + \frac{1}{\sqrt{\delta}} < z < -\varepsilon$,

$$\frac{\ln\frac{S_0}{K} + \left(r + \frac{\sigma^2\delta}{2} \right)T + \sigma\sqrt{\delta}\sqrt{t}z}{\sigma\sqrt{\delta}\sqrt{T-t}} > \frac{\ln\frac{S_0}{K} + \left(r + \frac{\sigma^2\delta}{2} \right)T + \sigma\sqrt{\delta}\sqrt{t}\left(z^a st + \frac{1}{\sqrt{\delta}} \right)}{\sigma\sqrt{\delta}\sqrt{T-t}} = \frac{\frac{\sigma^2\delta}{2}(T-t) + \sigma\sqrt{t}}{\sigma\sqrt{\delta}\sqrt{T-t}}$$

which is approximately $\frac{\sqrt{t}}{\sqrt{\delta}\sqrt{T-t}}$ when δ is small enough, so that

$$\mathrm{N}\left(-\frac{\ln\frac{S_0}{K} + \left(r + \frac{\sigma^2\delta}{2} \right)T + \sigma\sqrt{\delta}\sqrt{t}z}{\sigma\sqrt{\delta}\sqrt{T-t}} \right) \approx \mathrm{N}\left(-\frac{1}{\sqrt{\delta}}\sqrt{\frac{t}{T-t}} \right)$$

$$\approx \sqrt{\delta}\sqrt{\frac{T-t}{t}} e^{-\frac{t}{2\delta(T-t)}} .$$

The procedure to prove the other case $\ln\frac{S_0}{K} + rT < 0$, is similar, so we skip the proof here. We conclude this lemma by $E\left\{ \left(\alpha_t^{(1)} - \alpha_t^{(2)} \right)^2 \right\} \le \frac{C}{\sqrt{\delta t}} e^{-\frac{1}{\delta}}$ for some constant C independent of time and the scale δ.

Table 1: P/L and Sharpe ratio of hedging performance on SPX (time to maturity T=20 trading days)

Panel A : P/L of Hedging Strategies

(1) Mean (Average) – US Dollars

Mean (T=20)		Volatility-Model-Free					Model-Free			
Year	N	Δ-H	Δ-F	Δ-Imp	adΔ	Δ-Γ	SL	adSL-H	adSL-F	adSL-V
2001	441	224	236	189	218	159	**296**	265	276	267
2002	451	181	2	251	197	84	**270**	252	232	247
2003	454	142	120	**175**	135	61	157	141	150	106
2004	494	120	158	136	118	45	119	**177**	70	138
2005	591	166	166	154	166	112	168	**182**	176	158
2006	783	267	304	274	268	200	**422**	262	228	244
2007	411	77	156	99	79	78	95	**460**	442	374
2001 ~ 07	3625	177	176	190	178	114	235	**244**	219	216

(2) Standard Deviation

S.D. (T=20)		Volatility-Model-Free					Model-Free			
Year	N	Δ-H	Δ-F	Δ-Imp	adΔ	Δ-Γ	SL	adSL-H	adSL-F	adSL-V
2001	441	467	513	469	484	354	802	980	917	946
2002	451	888	488	983	916	592	1296	1331	1375	1362
2003	454	351	404	398	365	259	676	740	697	689
2004	494	402	396	407	398	323	545	663	634	608
2005	591	212	246	196	203	159	428	570	542	548
2006	783	418	295	371	414	370	4598	801	791	820
2007	411	843	577	524	841	756	1077	7285	7304	7290
2001 ~ 07	3625	540	422	510	547	428	2264	2585	2587	2584

Panel B : Sharpe Ratio of Hedging Strategies

S.R. (T=20)		Volatility-Model-Free					Model-Free			
Year	N	Δ-H	Δ-F	Δ-Imp	adΔ	Δ-Γ	SL	adSL-H	adSL-F	adSL-V
2001	441	**0.4805**	0.4612	0.4023	0.4504	0.4488	0.3687	0.2701	0.3004	0.2820
2002	451	0.2034	0.0033	**0.2558**	0.2148	0.1411	0.2086	0.1895	0.1690	0.1812
2003	454	0.4056	0.2976	**0.4411**	0.3697	0.2337	0.2331	0.1912	0.2144	0.1533
2004	494	0.2991	**0.3989**	0.3345	0.2956	0.1399	0.2190	0.2663	0.1096	0.2271
2005	591	0.7802	0.6723	0.7826	**0.8190**	0.7037	0.3924	0.3199	0.3253	0.2878
2006	783	0.6392	**1.0306**	0.7392	0.6489	0.5396	0.0917	0.3277	0.2882	0.2973
2007	411	0.0919	**0.2702**	0.1884	0.0934	0.1029	0.0886	0.0631	0.0605	0.0513
2001 ~ 07	3625	0.3287	**0.4168**	0.3731	0.3254	0.2656	0.1038	0.0943	0.0845	0.0836

Table 2: P/L and Sharpe ratio of hedging performance in TXO (time to maturity T=20 trading days)

Panel A : P/L of Hedging Strategies

(1) Mean (Average) – New Taiwan Dollars

Mean (T=20)		Volatility-Model-Free					Model-Free			
Year	N	Δ-H	Δ-F	Δ-Imp	adΔ	Δ-Γ	SL	adSL-H	adSL-F	adSL-V[5]
2003	93	200	107	(19)	200	(269)	153	76	**266**	(63)
2004	218	66	246	97	66	(400)	**319**	87	(210)	(135)
2005	191	(352)	(401)	(437)	(352)	(675)	**(207)**	(423)	(299)	(500)
2006	223	(178)	(304)	(366)	(178)	(536)	(98)	(73)	(241)	**(47)**
2007	326	(108)	(142)	(295)	(108)	**157**	(131)	(521)	(266)	(365)
2008	393	53166	53289	48609	53166	**54062**	53630	53409	53393	53389
2009	73	(1362)	(1385)	(1707)	(1362)	**(944)**	(1212)	(1434)	(1246)	(1434)
2003 ~ 09	1517	13636	13655	12351	13636	13756	**13822**	13609	13628	13591

(2) Standard Deviation

S.D. (T=20)		Volatility-Model-Free					Model-Free			
Year	N	Δ-H	Δ-F	Δ-Imp	adΔ	Δ-Γ	SL	adSL-H	adSL-F	adSL-V
2003	93	989	1197	1436	989	770	1961	2714	2496	2598
2004	218	1918	1859	2158	1918	1406	3549	4065	4946	5016
2005	191	991	1134	981	991	969	1885	2606	2357	2579
2006	223	1350	2181	1379	1350	1077	2680	3371	3171	3332
2007	326	3268	3244	3190	3267	4287	5102	5733	5865	5727
2008	393	79641	79560	81188	79640	79379	79719	79949	79894	79958
2009	73	11088	11382	10595	11088	8625	11655	11689	11559	11689
2003 ~ 09	1517	46892	46895	46649	46891	46996	47077	47211	47185	47226

[5] The calculation of VIX before its announcement from TAIFEX in December 2006 is based on a formula given by SinoPac Futures.

Panel B: Sharpe Ratio of Hedging Strategies

S.R. (T=20)		Volatility-Model-Free					Model-Free			
Year	N	Δ-H	Δ-F	Δ-Imp	adΔ	Δ-Γ	SL	adSL-H	adSL-F	adSL-V
2003	93	**0.2022**	0.0894	(0.0129)	**0.2022**	(0.3496)	0.0779	0.0281	0.1065	(0.0243)
2004	218	0.0343	**0.1322**	0.0451	0.0343	(0.2842)	0.0900	0.0213	(0.0425)	(0.0270)
2005	191	(0.3556)	(0.3538)	(0.4451)	(0.3556)	(0.6966)	**(0.1100)**	(0.1624)	(0.1268)	(0.1940)
2006	223	(0.1316)	(0.1394)	(0.2658)	(0.1317)	(0.4981)	(0.0367)	(0.0217)	(0.0759)	**(0.0141)**
2007	326	(0.0331)	(0.0437)	(0.0926)	(0.0331)	**0.0366**	(0.0256)	(0.0908)	(0.0454)	(0.0637)
2008	393	0.6676	0.6698	0.5987	0.6676	**0.6811**	0.6727	0.6680	0.6683	0.6677
2009	73	(0.1229)	(0.1217)	(0.1611)	(0.1229)	(0.1094)	**(0.1040)**	(0.1227)	(0.1078)	(0.1227)
2003 ~ 09	1517	0.2908	0.2912	0.2648	0.2908	0.2927	**0.2936**	0.2883	0.2888	0.2878

Figure 1: Evolution of Sharpe ratios of hedging strategies on SPX given time to maturity T from 2 to 30.

S&P Index Option

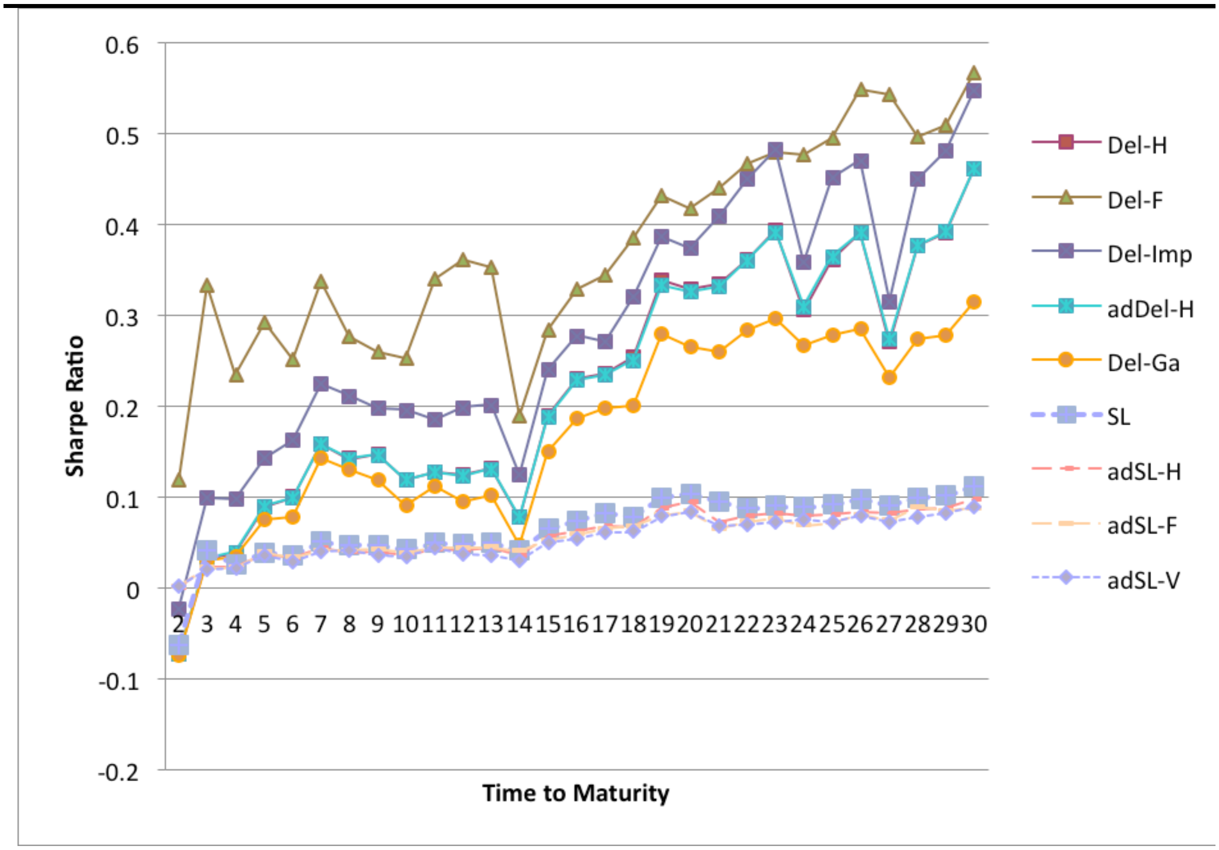


Figure 2: Evolution of Sharpe ratios of hedging strategies on TXO given time to maturity T from 2 to 30.

Taiwan Index Option

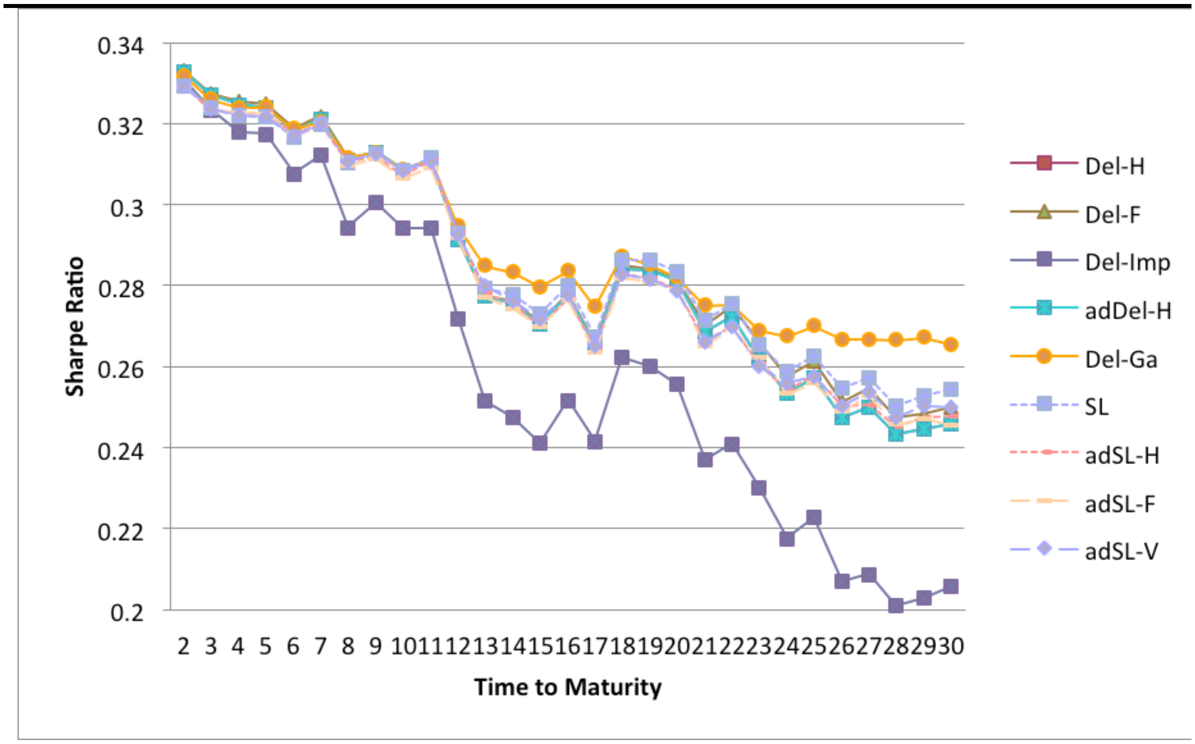
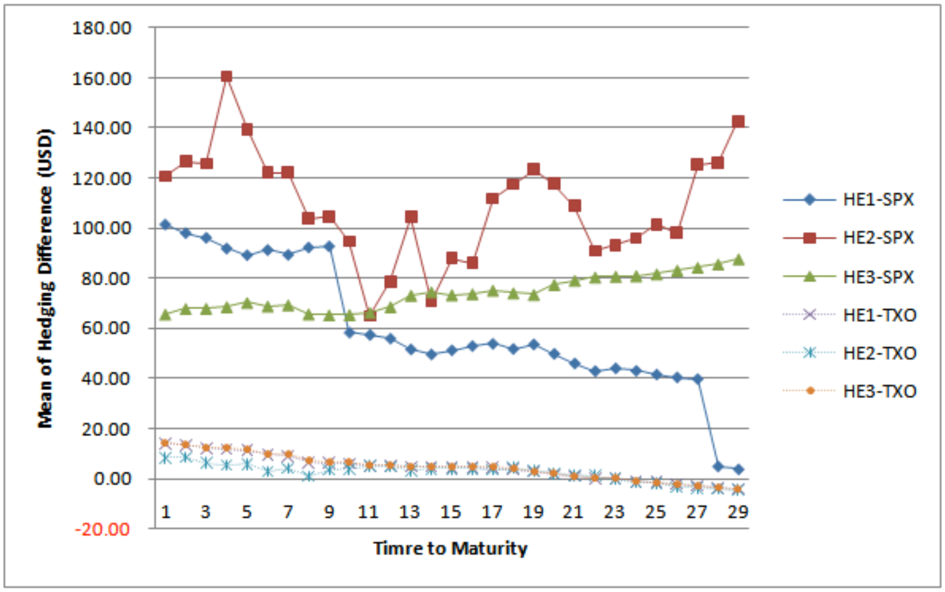
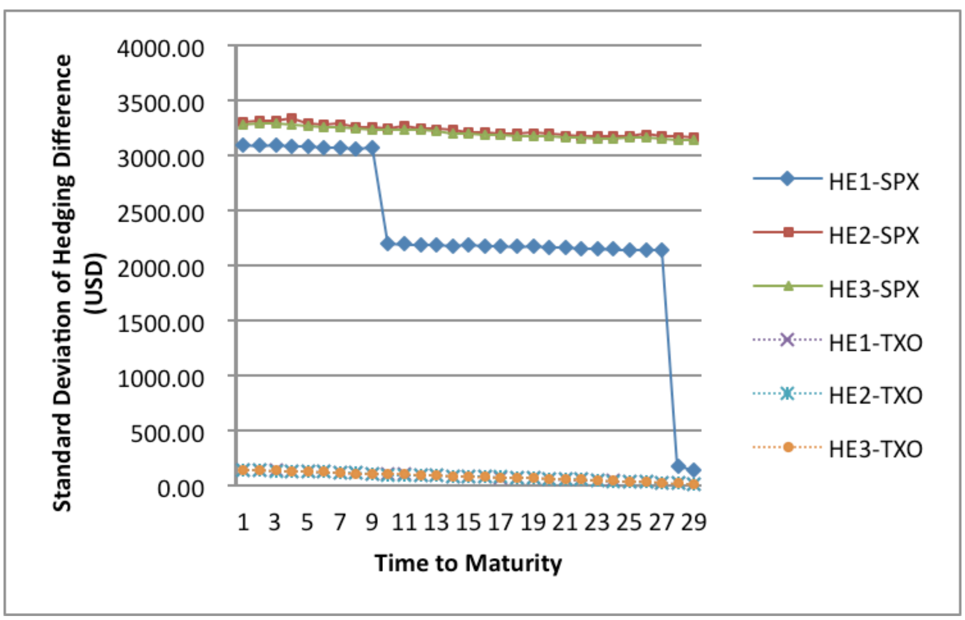


Figure 3: Comparisons of differences of hedging performances between Delta and Stop-Loss strategies.

(a) Mean of Hedging Differences



(b) Standard Deviation of Hedging Differences



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