Financial modeling practice for equity index options

Chinatrust

Commercial Bank

Jan. 2011

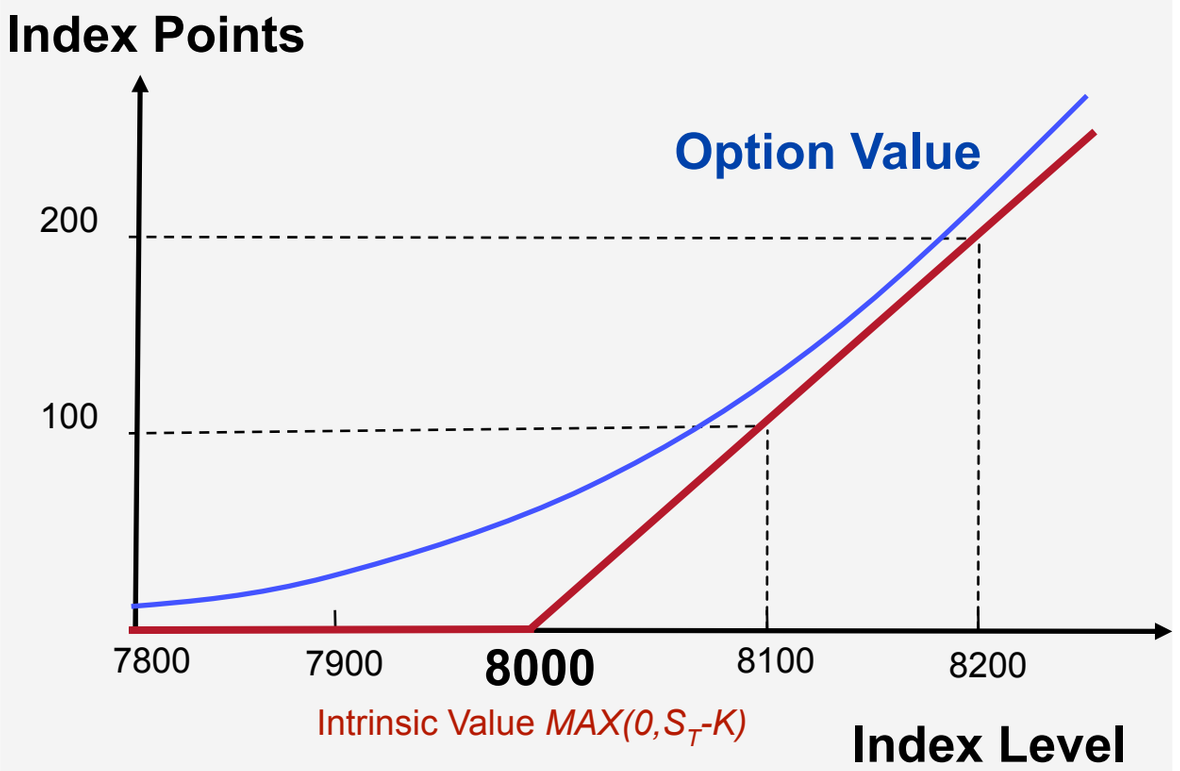
Presenter:

牛繼聖

Taiex Index Options

- European options with monthly cash-settlement
- Accompanied by Taiex index futures

Illustrations of option value



MARKET QUOTES

FT1 ↑8072s +73 8072/8073 4x29 Index OMON
At 13:45 Vol 27,923 Op 8039 Hi 8072 Lo 8026 OpInt 45,976

Template List | Edit | Contract Months | Security List | TWSE Index | Go

Option Monitor: TAIWAN TAIEX INDEX

Center 8056.56 Number of Strikes 18 -or- % from Center Exchange C (Composite)

CALLS

Strike	BSize	Bid	Ask	ASize	Last	Volume	IMid
TWSE 19 APR 07 (Contract Size: 50)							
1) 6900		1140.00	1240.00		1170.00 s	12	64.31
2) 7000		1040.00	1100.00		1070.00 s	6	46.85
3) 7100		925.00	1030.00		950.00 s		48.27
4) 7200		825.00	925.00		850.00 s	2	42.47
5) 7300		730.00	830.00		750.00 s	1	41.02
6) 7400		645.00	680.00		675.00 s	215	23.69
7) 7500		570.00	600.00		580.00 s	91	34.28
8) 7600		472.00	487.00		478.00 s	259	27.51
9) 7700		380.00	385.00		383.00 s	1086	24.00
10) 7800		290.00	294.00		291.00 s	3237	21.86
11) 7900		205.00	206.00		205.00 s	9891	19.44
12) 8000		129.00	130.00		129.00 s	37195	17.45
13) 8200		33.00	33.50		33.50 s	51061	15.32
14) 8400		3.80	3.90		3.90 s	14388	14.25
15) 8600		.30	.60		.50 s	2217	15.13
16) 8800			2.40		.10 s	1001	24.49
17) 9000		.10	.20		.10 s	1	21.79
TWSE 17 MAY 07 (Contract Size: 50)							

PUTS

Strike	BSize	Bid	Ask	ASize	Last	Volume	IMid
TWSE 19 APR 07 (Contract Size: 50)							
18) 6900		.50	.60		.60 s	467	34.15
19) 7000		.60	.70		.70 s	2316	31.86
20) 7100		.90	1.10		.90 s	513	30.44
21) 7200		1.20	1.40		1.40 s	1302	28.36
22) 7300		1.90	2.00		2.00 s	2869	26.74
23) 7400		2.60	3.10		3.10 s	3406	24.94
24) 7500		4.50	4.90		4.90 s	10518	23.54
25) 7600		7.30	8.00		7.50 s	14980	22.02
26) 7700		11.50	12.00		11.50 s	19622	20.10
27) 7800		19.00	19.50		19.00 s	27051	18.36
28) 7900		33.00	33.50		33.00 s	24367	16.84
29) 8000		57.00	59.00		57.00 s	31562	15.43
30) 8200		160.00	165.00		164.00 s	2467	13.05
31) 8400		330.00	343.00		336.00 s	107	
32) 8600		496.00	625.00		530.00 s	5	30.04
33) 8800		690.00	820.00		740.00 s		34.93
34) 9000		885.00	1360.00		940.00 s		89.93
TWSE 17 MAY 07 (Contract Size: 50)							

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2007 Bloomberg L.P.
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Option price

Black-Scholes model

Option Pricing Formula

$$dF=\mu \cdot F dt+\sigma \cdot F dz$$

$$C = e^{-rT} [F \cdot N(d_1) - X \cdot N(d_2)]$$

$$P = e^{-rT} [X \cdot N(-d_2) - F \cdot N(-d_1)]$$

where $d_1 = d_2 + \sigma T^{1/2}$

$= [\ln(F/X) + \sigma^2 T/2] / (\sigma T^{1/2})$

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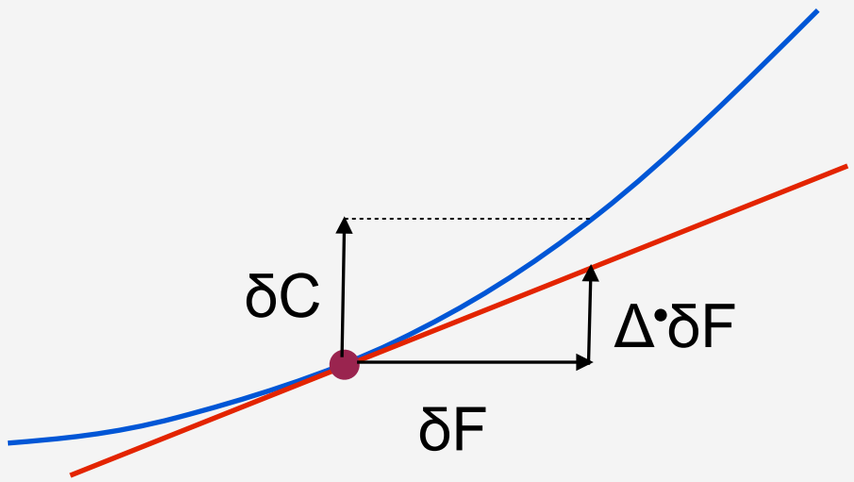
DYNAMIC HEDGING-RISK-FREE PORTFOLIO

LONG A CALL



$\delta C \simeq \Delta \cdot \delta F$; $\Delta = \partial C / \partial F$

RISK-FREE PORTFOLIO

LONG 1 CALL

+

SHORT Δ FUTURE

DYNAMIC HEDGING- RE-BALANCE

NO LONGER RISK-FREE

LONG 1 CALL

+

SHORT Δ FUTURE

when F ⟶ F'

Δ ⟶ Δ'

Sell

Δ'-Δ

Future

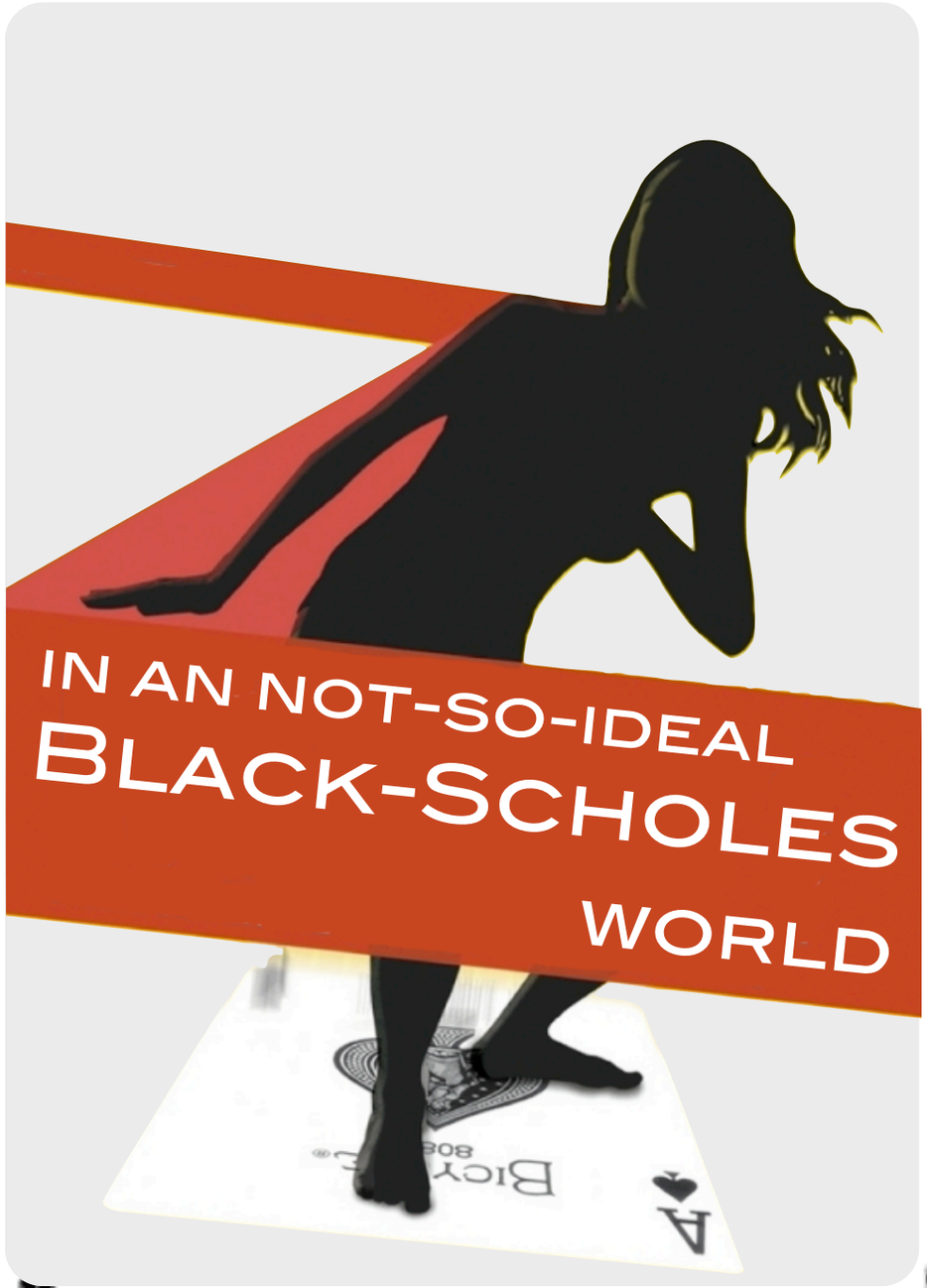
NEW RISK-FREE PORTFOLIO

LONG 1 CALL

+

SHORT Δ' FUTURE

Now, Practitioners Take Over



if we can accommodate or ignore

- Price of index futures follows Brownian Motion
- Hedge can be done continuously
- No transaction costs
- and more……

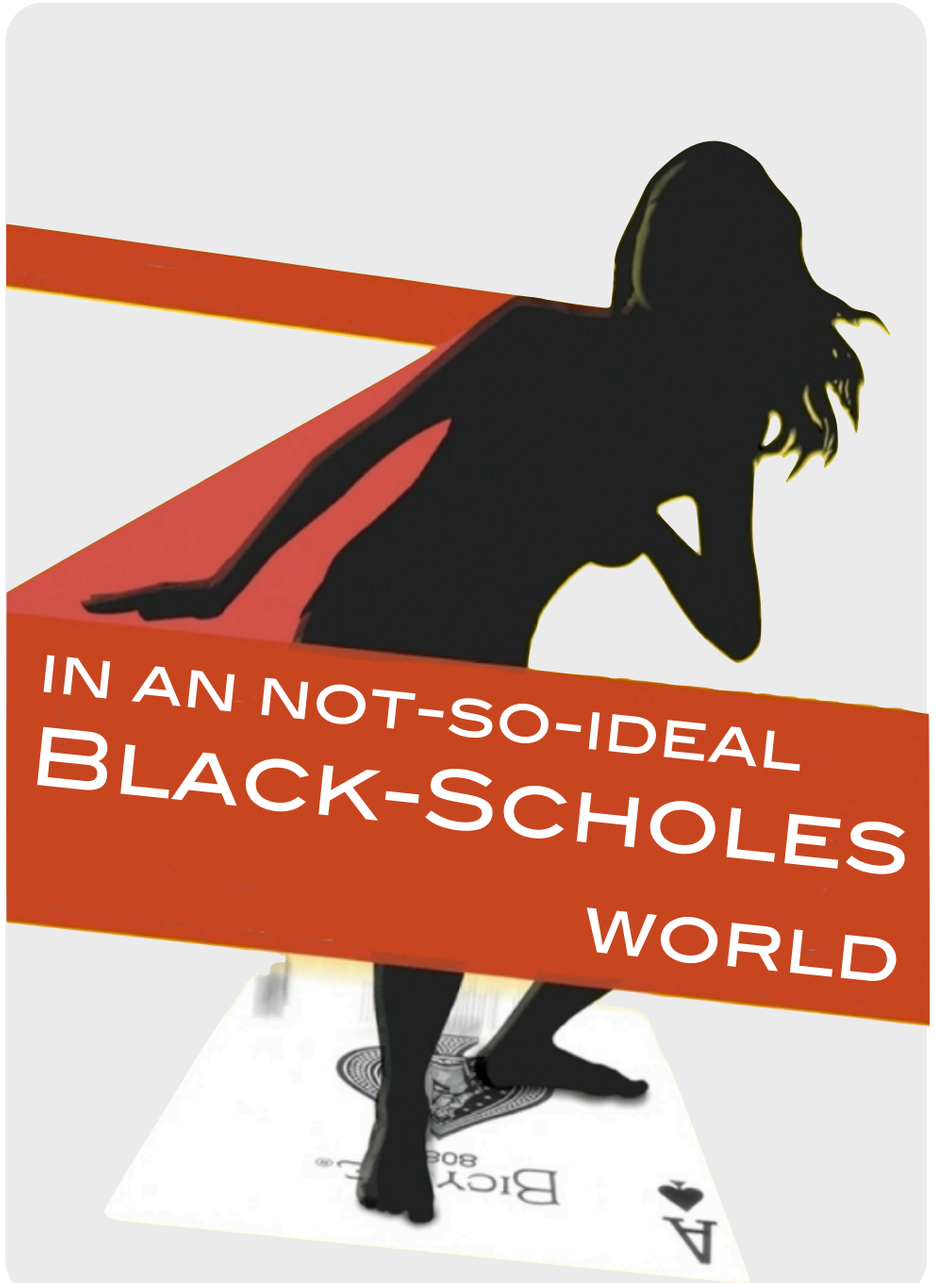
Now, Practitioners Take Over

We can calculate the prices and also know how to hedge the options perfectly.....

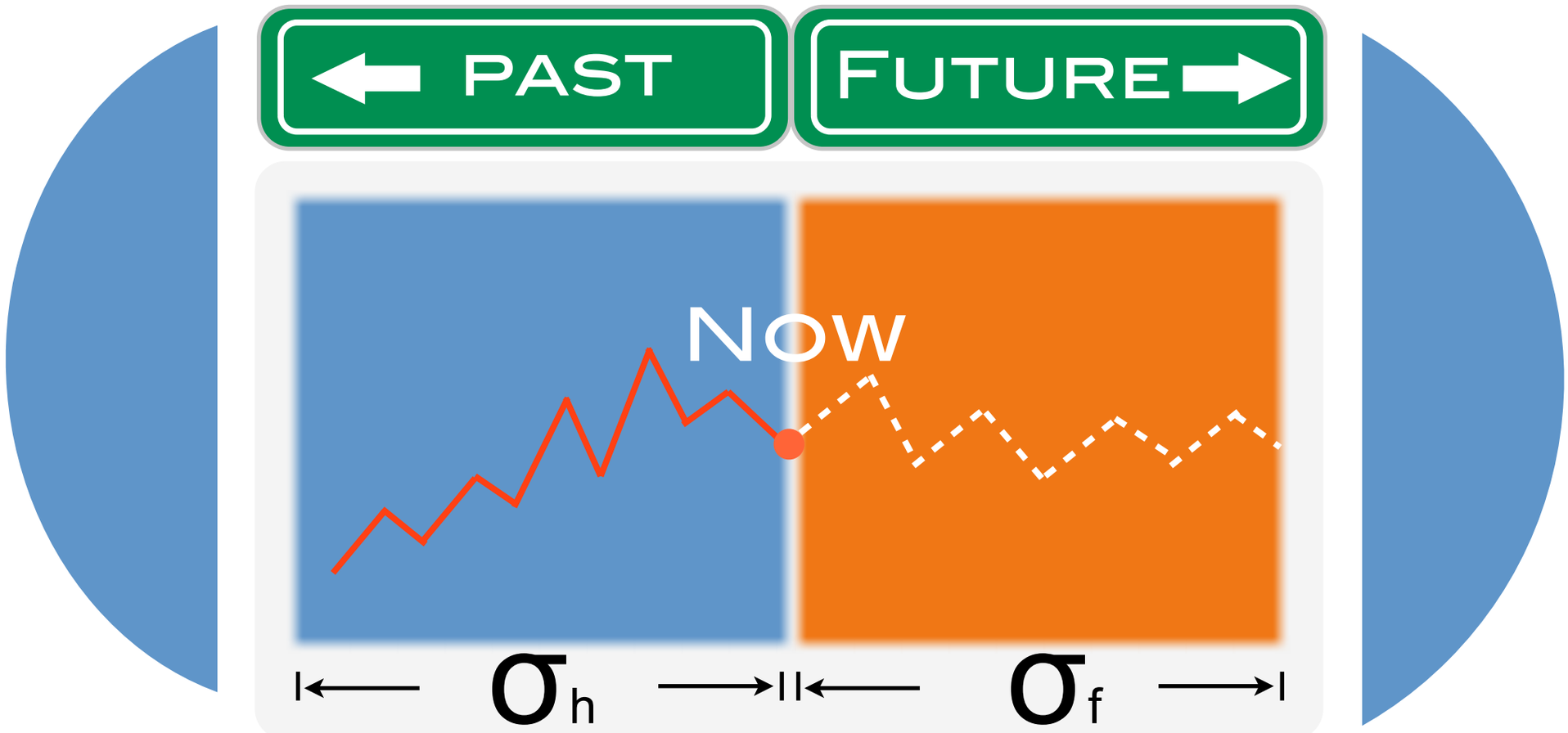
Let's input the parameters into Black-Scholes Formula

F K T r σ

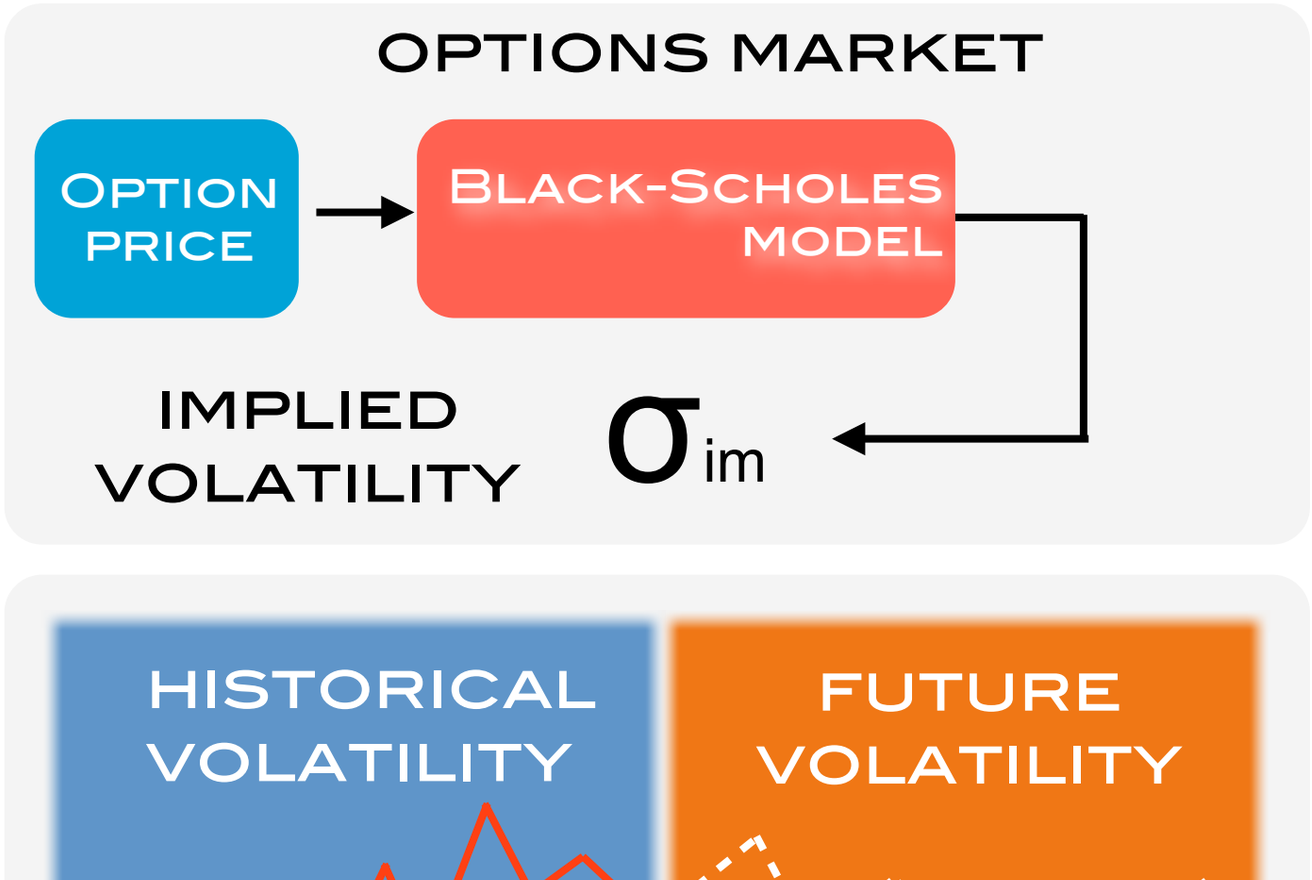
Wait a minute!

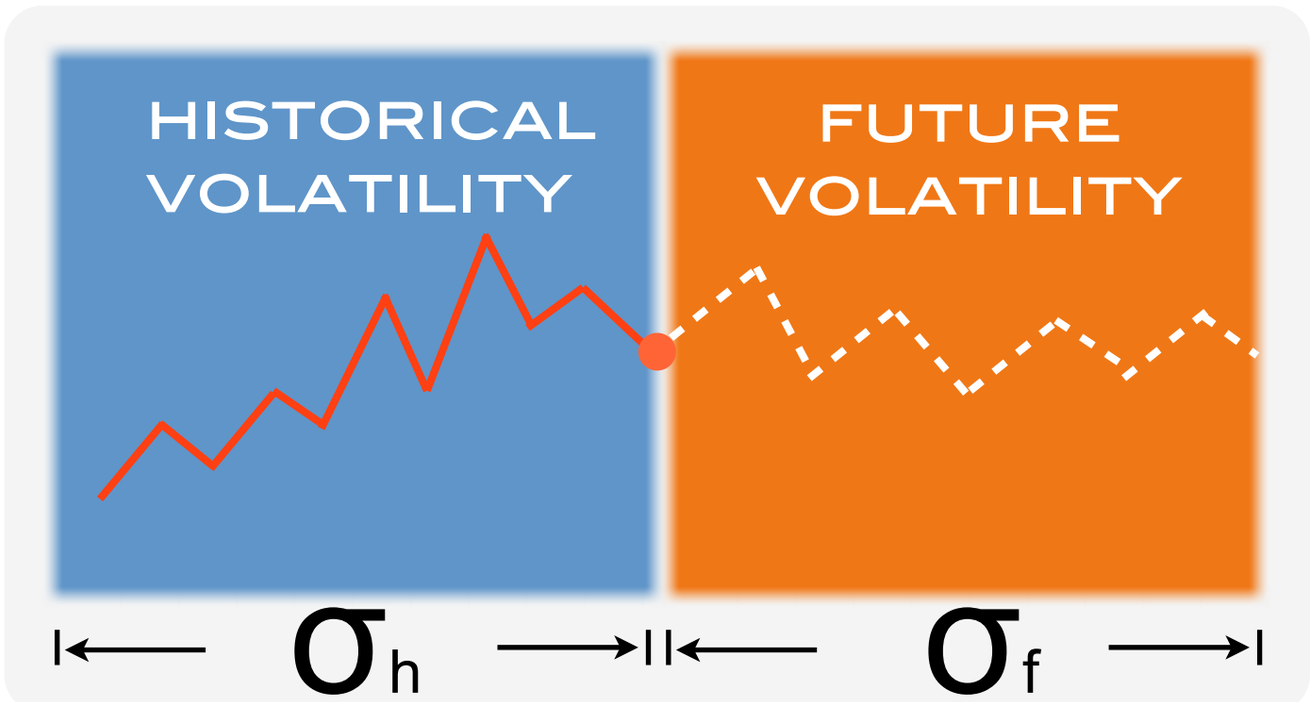


Volatilities



VOLATILITIES





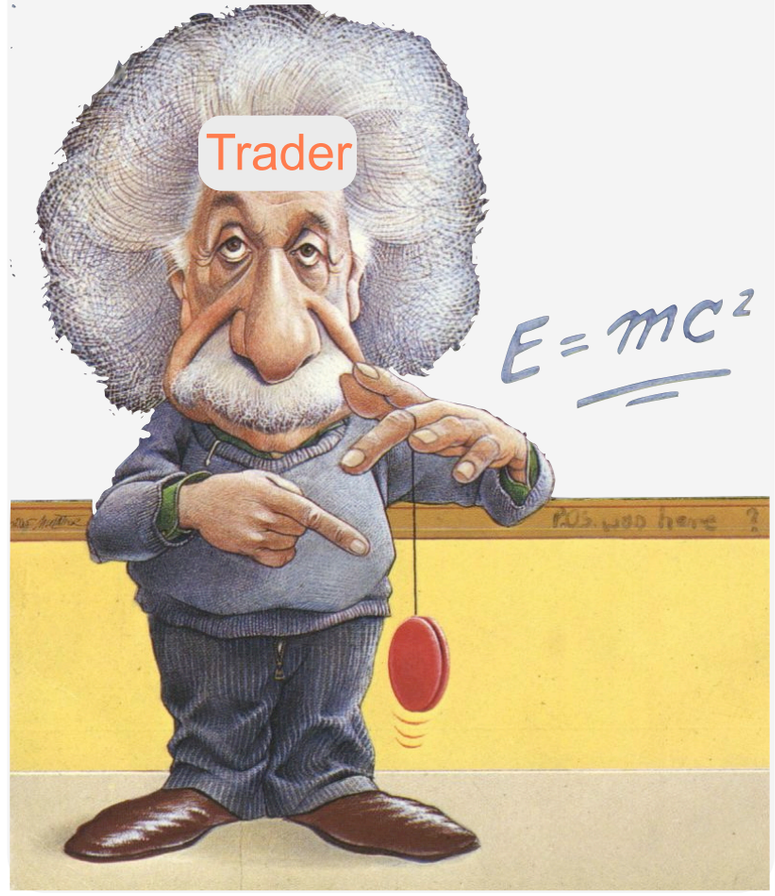
VOLATILITIES

OPTIONS MARKET

OPTION PRICE → BLACK-SCHOLES MODEL → IMPLIED VOLATILITY σ_{im}

MODEL VOLATILITY

σ_M = Non-trivial



HISTORICAL VOLATILITY

FUTURE VOLATILITY

σ_h σ_f

CASE STUDY

CASE 1

AN

ARBITRAGER

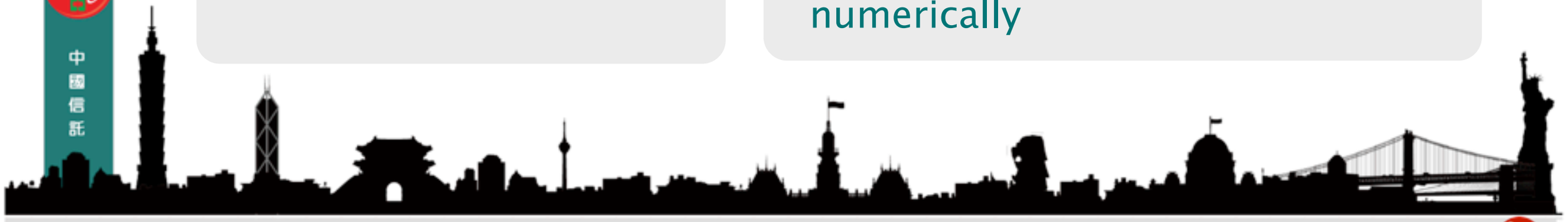
There is an under-priced call option offered in the market; how can an arbitrageur take advantage of it without taking extra risks?

Buy 1 Call

Sell Δ Futures

Dynamically hedge it with Futures until expiry

Knowing the world isn't perfect, he needs to test the idea numerically

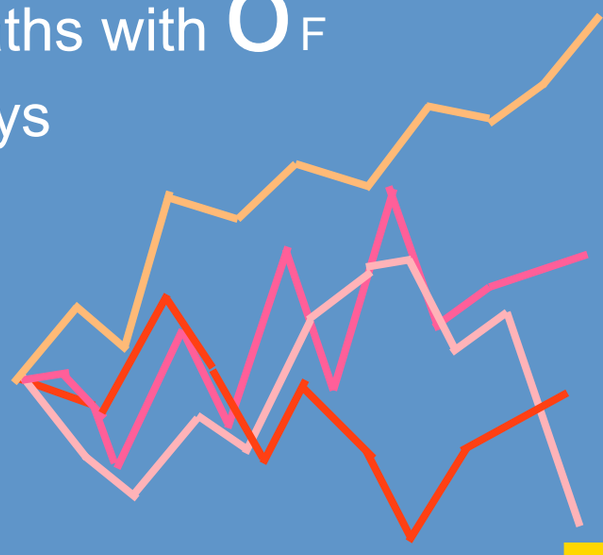


Monte Carlo Simulation

50,000 paths with σ_F

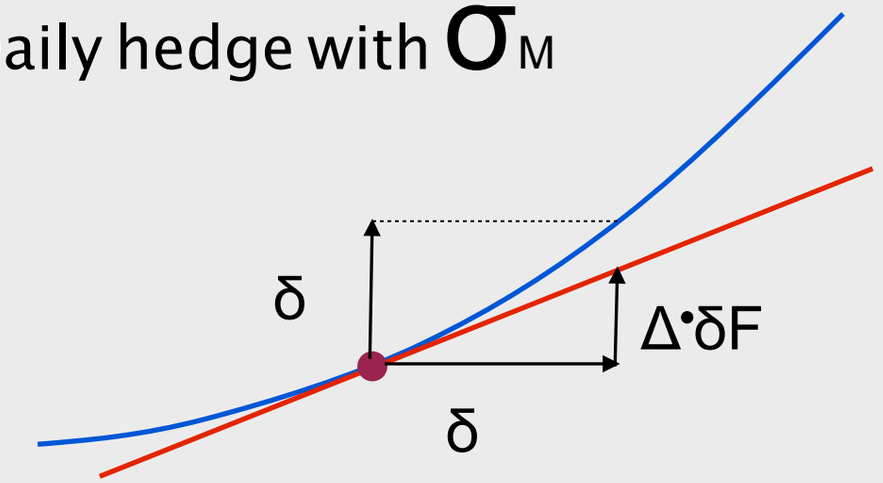
T = 30 days

F_0=8000

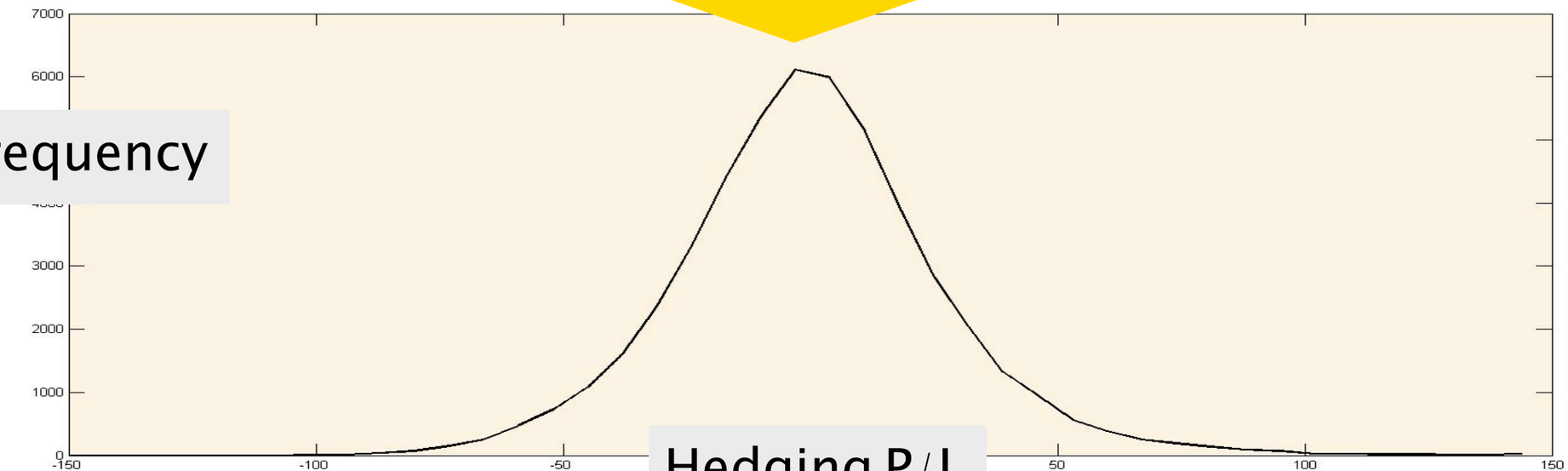


Call option K=8000

Daily hedge with σ_M



Frequency



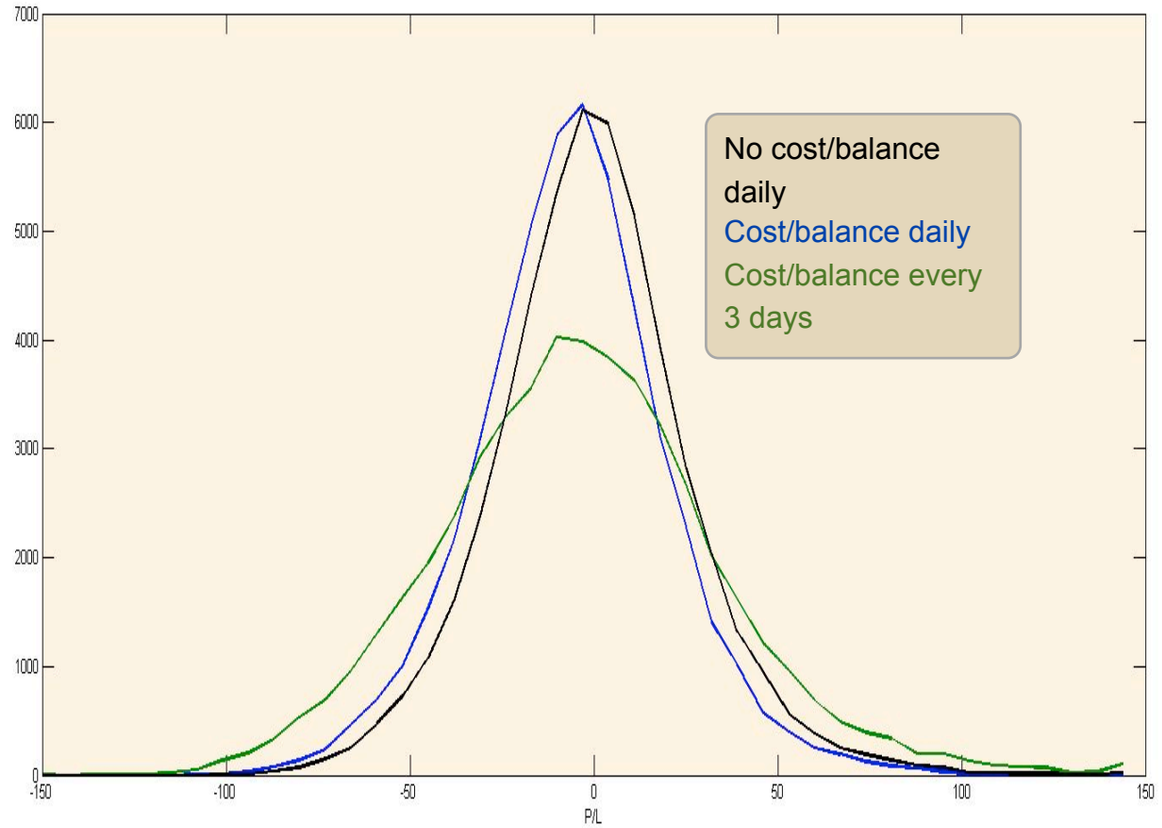
Hedging P/L

Preliminary - Real World Imperfection

Assumption:

$\sigma_F = 16\%, \sigma_M = 16\%$

Effects of transactional cost and re-balance frequency



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Effect of Inaccurate Estimation of σ_F

Assumption:

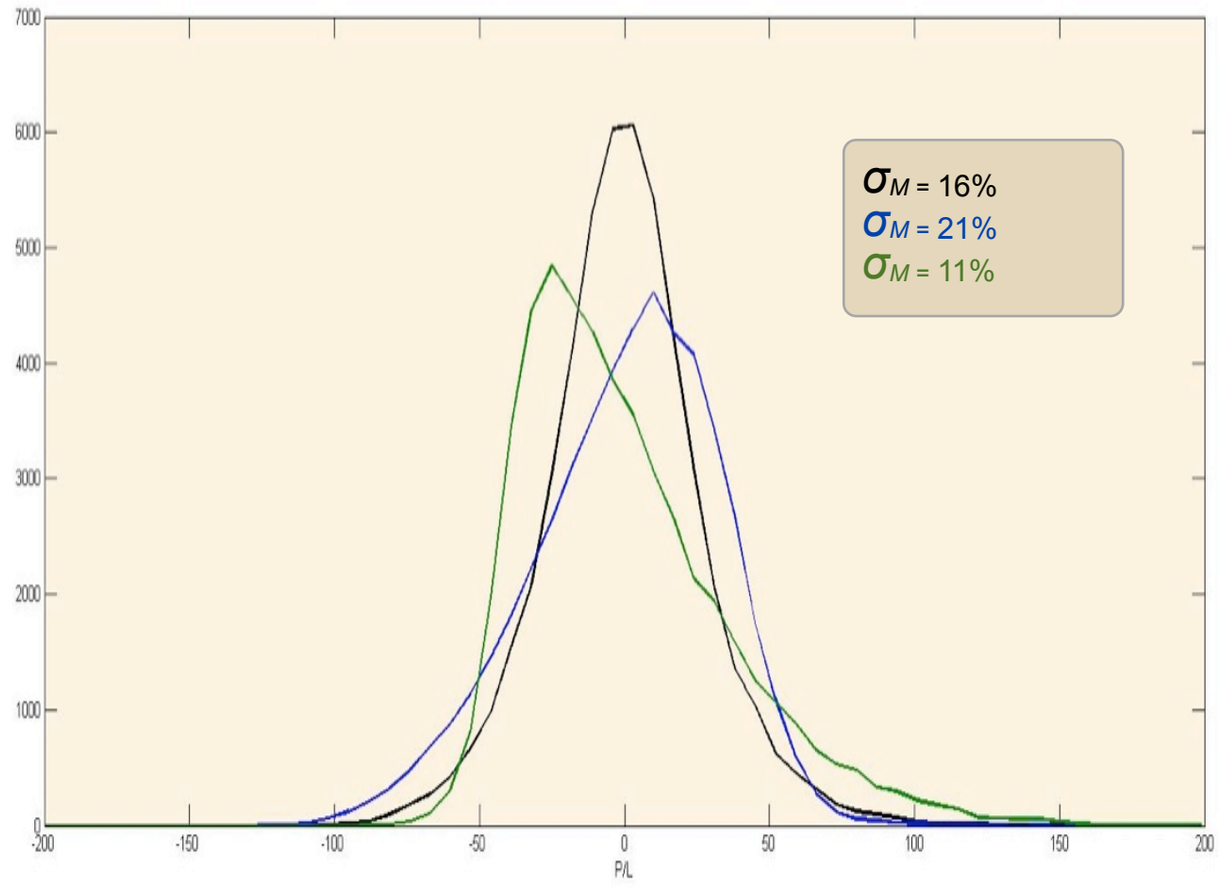
$\sigma_F = 16\%$,

no

transactional

cost,

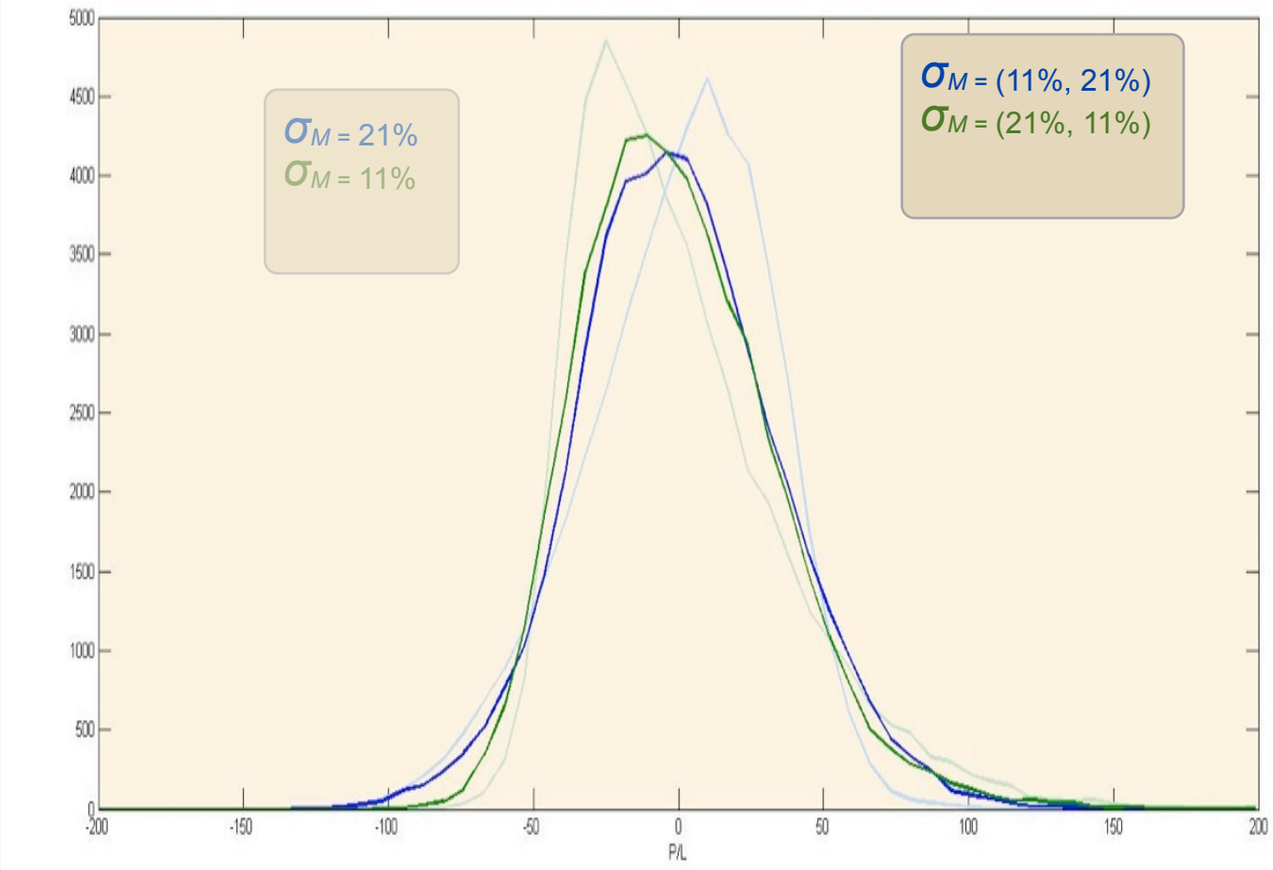
daily hedge



Effect of Variable σ_M

Assumption:

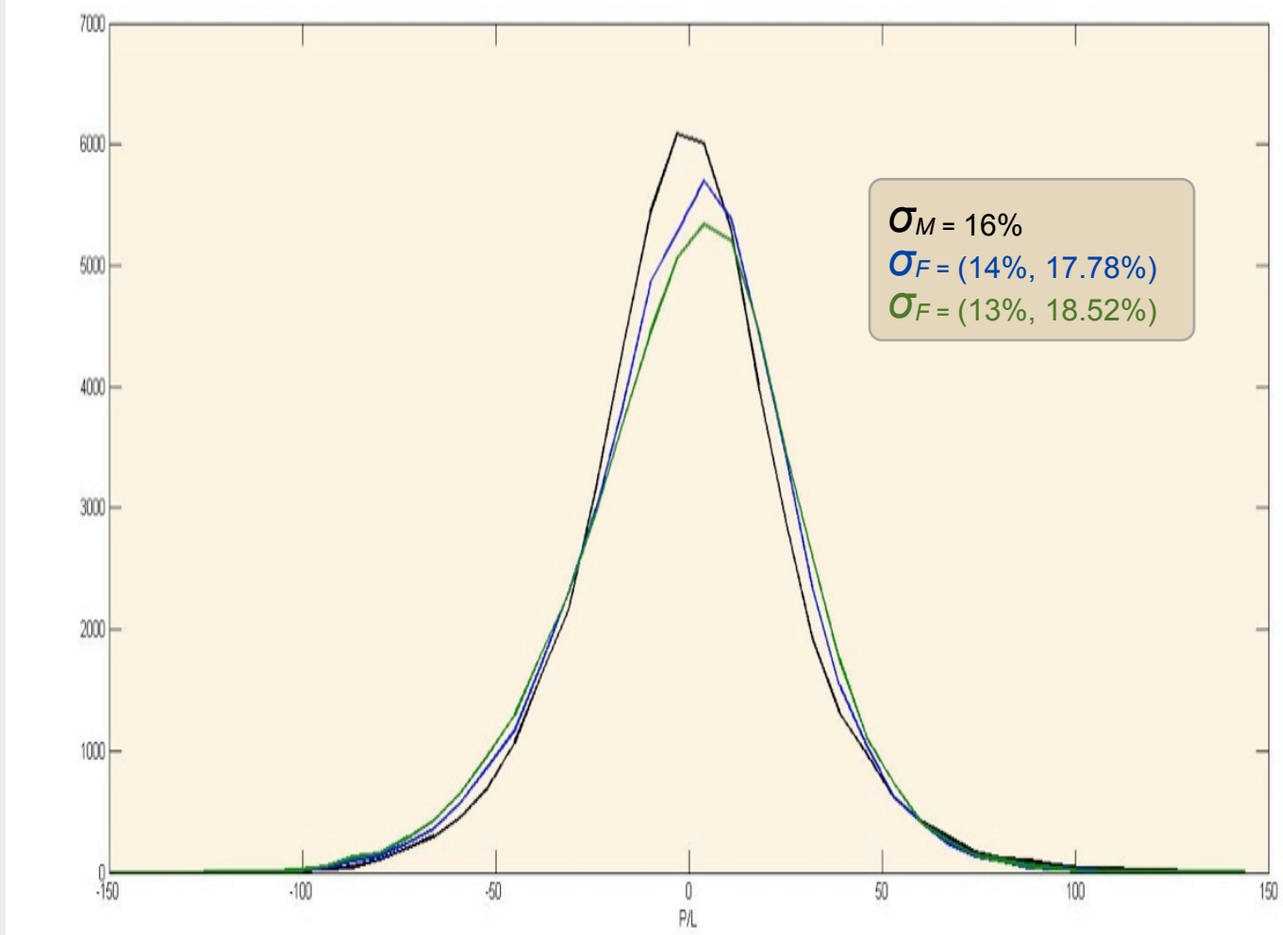
Different σ_M's are used for first 15 days and second 15 days



Effect of Variable σ_F

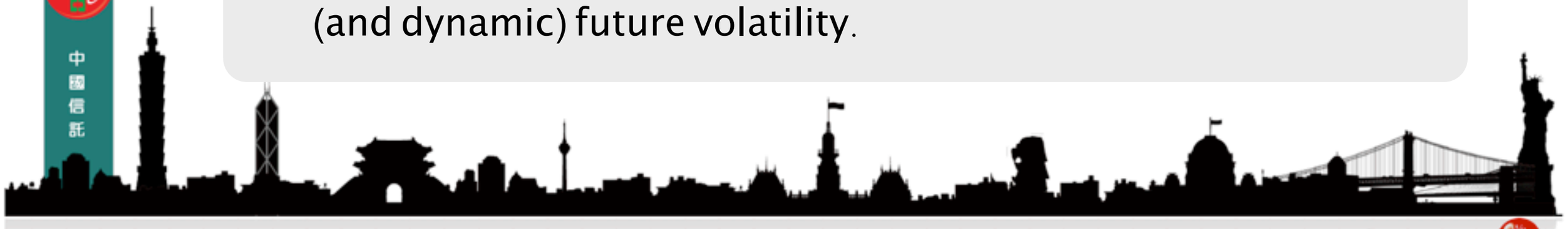
Assumption:

Different σ_F's are used for first 15 days and second 15 days,with average variance = $(\sigma_M)^2 = (16\%)^2$



WHAT WE'VE LEARNED

- Higher rebalance frequency increases P/L accuracy of arbitrage, but decreases the expectation value due to higher transactional cost
- Our goal is make the model volatility as close to the future volatility as possible.
- Inaccurate estimation of the future volatility would not destroy the arbitrage entirely, but increase the uncertainty of the P/L outcome
- The above argument is stable even in the cases of variable future volatility or model volatility. That means we can dynamically adjust the model volatility to track the estimated (and dynamic) future volatility.

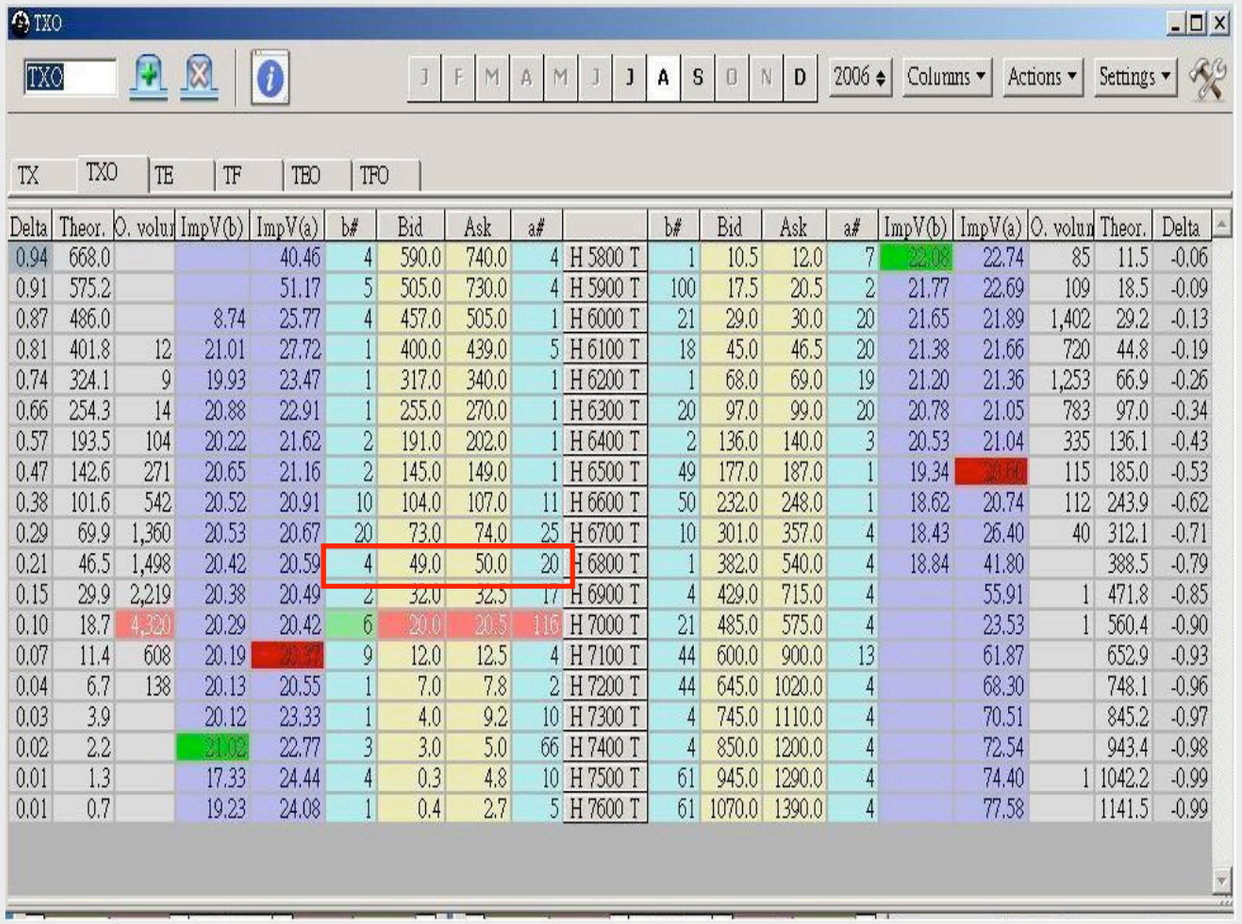


CASE STUDY

CASE 2

AN MARKET MAKER

A market maker actively provides 2-way quotes for many contracts, and constantly holds a complicated but low-risk portfolio of options and futures



Delta	Theor.	O. volun	ImpV(b)	ImpV(a)	b#	Bid	Ask	a#		b#	Bid	Ask	a#	ImpV(b)	ImpV(a)	O. volun	Theor.	Delta
0.94	668.0			40.46	4	590.0	740.0	4	H 5800 T	1	10.5	12.0	7	22.08	22.74	85	11.5	-0.06
0.91	575.2			51.17	5	505.0	730.0	4	H 5900 T	100	17.5	20.5	2	21.77	22.69	109	18.5	-0.09
0.87	486.0		8.74	25.77	4	457.0	505.0	1	H 6000 T	21	29.0	30.0	20	21.65	21.89	1,402	29.2	-0.13
0.81	401.8	12	21.01	27.72	1	400.0	439.0	5	H 6100 T	18	45.0	46.5	20	21.38	21.66	720	44.8	-0.19
0.74	324.1	9	19.93	23.47	1	317.0	340.0	1	H 6200 T	1	68.0	69.0	19	21.20	21.36	1,253	66.9	-0.26
0.66	254.3	14	20.88	22.91	1	255.0	270.0	1	H 6300 T	20	97.0	99.0	20	20.78	21.05	783	97.0	-0.34
0.57	193.5	104	20.22	21.62	2	191.0	202.0	1	H 6400 T	2	136.0	140.0	3	20.53	21.04	335	136.1	-0.43
0.47	142.6	271	20.65	21.16	2	145.0	149.0	1	H 6500 T	49	177.0	187.0	1	19.34	20.80	115	185.0	-0.53
0.38	101.6	542	20.52	20.91	10	104.0	107.0	11	H 6600 T	50	232.0	248.0	1	18.62	20.74	112	243.9	-0.62
0.29	69.9	1,360	20.53	20.67	20	73.0	74.0	25	H 6700 T	10	301.0	357.0	4	18.43	26.40	40	312.1	-0.71
0.21	46.5	1,498	20.42	20.59	4	49.0	50.0	20	H 6800 T	1	382.0	540.0	4	18.84	41.80		388.5	-0.79
0.15	29.9	2,219	20.38	20.49	2	32.0	32.5	17	H 6900 T	4	429.0	715.0	4		55.91	1	471.8	-0.85
0.10	18.7	4,320	20.29	20.42	6	20.0	20.5	116	H 7000 T	21	485.0	575.0	4		23.53	1	560.4	-0.90
0.07	11.4	608	20.19	20.37	9	12.0	12.5	4	H 7100 T	44	600.0	900.0	13		61.87		652.9	-0.93
0.04	6.7	138	20.13	20.55	1	7.0	7.8	2	H 7200 T	44	645.0	1020.0	4		68.30		748.1	-0.96
0.03	3.9		20.12	23.33	1	4.0	9.2	10	H 7300 T	4	745.0	1110.0	4		70.51		845.2	-0.97
0.02	2.2		21.02	22.77	3	3.0	5.0	66	H 7400 T	4	850.0	1200.0	4		72.54		943.4	-0.98
0.01	1.3		17.33	24.44	4	0.3	4.8	10	H 7500 T	61	945.0	1290.0	4		74.40	1	1042.2	-0.99
0.01	0.7		19.23	24.08	1	0.4	2.7	5	H 7600 T	61	1070.0	1390.0	4		77.58		1141.5	-0.99

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Market makers' quotes

All Market Quotes

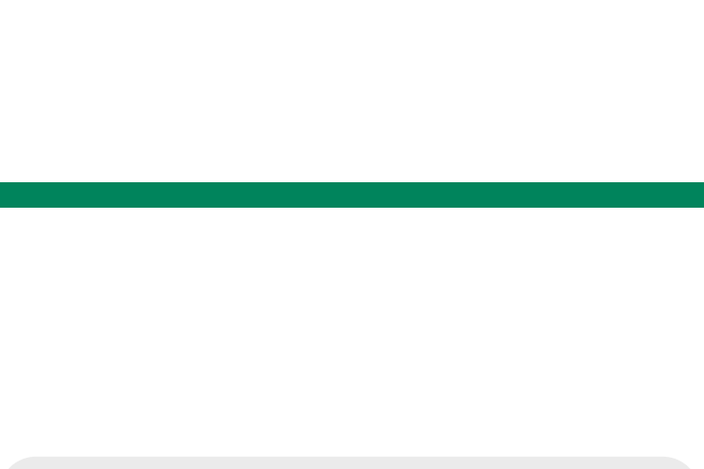
Bid	6800 C	Offer
	54	48
	53	6
	52	125
	51	36
	50	20
4	49	
42	48	
80	47	
15	46	
220	45	

Market Makers' Quotes

Bid	6800 C	Offer
	54	
	53	
	52	50
	51	20
	50	20
	49	
20 20	48	
50	47	
	46	
	45	

Makes sense to use Implied Volatilities!

IMPLIED VOLATILITY CURVE/SURFACE



TXO

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TX | TXO | TE | TF | TEO | TFO

Delta	Theor.	O. volu	ImpV(b)	ImpV(a)	b#	Bid	Ask	a#		b#	Bid	Ask	a#	ImpV(b)	ImpV(a)	O. volu	Theor.	Delta
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0.91	575.2			51.17	5	505.0	730.0	4	H 5900 T	100	17.5	20.5	2	21.77	22.69	109	18.5	-0.09
0.87	486.0		8.74	25.77	4	457.0	505.0	1	H 6000 T	21	29.0	30.0	20	21.65	21.89	1,402	29.2	-0.13
0.81	401.8	12	21.01	27.72	1	400.0	439.0	5	H 6100 T	18	45.0	46.5	20	21.38	21.66	720	44.8	-0.19
0.74	324.1	9	19.93	23.47	1	317.0	340.0	1	H 6200 T	1	68.0	69.0	19	21.20	21.36	1,253	66.9	-0.26
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0.47	142.6	271	20.65	21.16	2	145.0	149.0	1	H 6500 T	49	177.0	187.0	1	19.34	20.80	115	185.0	-0.53
0.38	101.6	542	20.52	20.91	10	104.0	107.0	11	H 6600 T	50	232.0	248.0	1	18.62	20.74	112	243.9	-0.62
0.29	69.9	1,360	20.53	20.67	20	73.0	74.0	25	H 6700 T	10	301.0	357.0	4	18.43	26.40	40	312.1	-0.71
0.21	46.5	1,498	20.42	20.59	4	49.0	50.0	20	H 6800 T	1	382.0	540.0	4	18.84	41.80		388.5	-0.79
0.15	29.9	2,219	20.38	20.49	2	32.0	32.5	17	H 6900 T	4	429.0	715.0	4		55.91	1	471.8	-0.85
0.10	18.7	4,320	20.29	20.42	6	20.0	20.5	116	H 7000 T	21	485.0	575.0	4		23.53	1	560.4	-0.90
0.07	11.4	608	20.19	20.37	9	12.0	12.5	4	H 7100 T	44	600.0	900.0	13		61.87		652.9	-0.93
0.04	6.7	138	20.13	20.55	1	7.0	7.8	2	H 7200 T	44	645.0	1020.0	4		68.30		748.1	-0.96
0.03	3.9		20.12	23.33	1	4.0	9.2	10	H 7300 T	4	745.0	1110.0	4		70.51		845.2	-0.97
0.02	2.2		21.02	22.77	3	3.0	5.0	66	H 7400 T	4	850.0	1200.0	4		72.54		943.4	-0.98
0.01	1.3		17.33	24.44	4	0.3	4.8	10	H 7500 T	61	945.0	1290.0	4		74.40	1	1042.2	-0.99
0.01	0.7		19.23	24.08	1	0.4	2.7	5	H 7600 T	61	1070.0	1390.0	4		77.58		1141.5	-0.99

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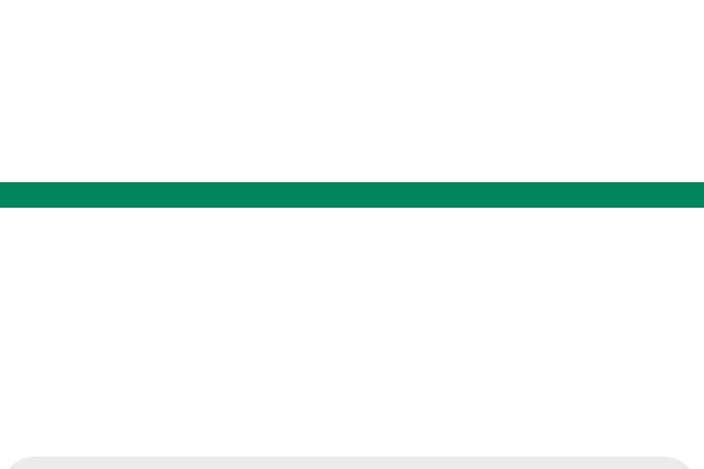
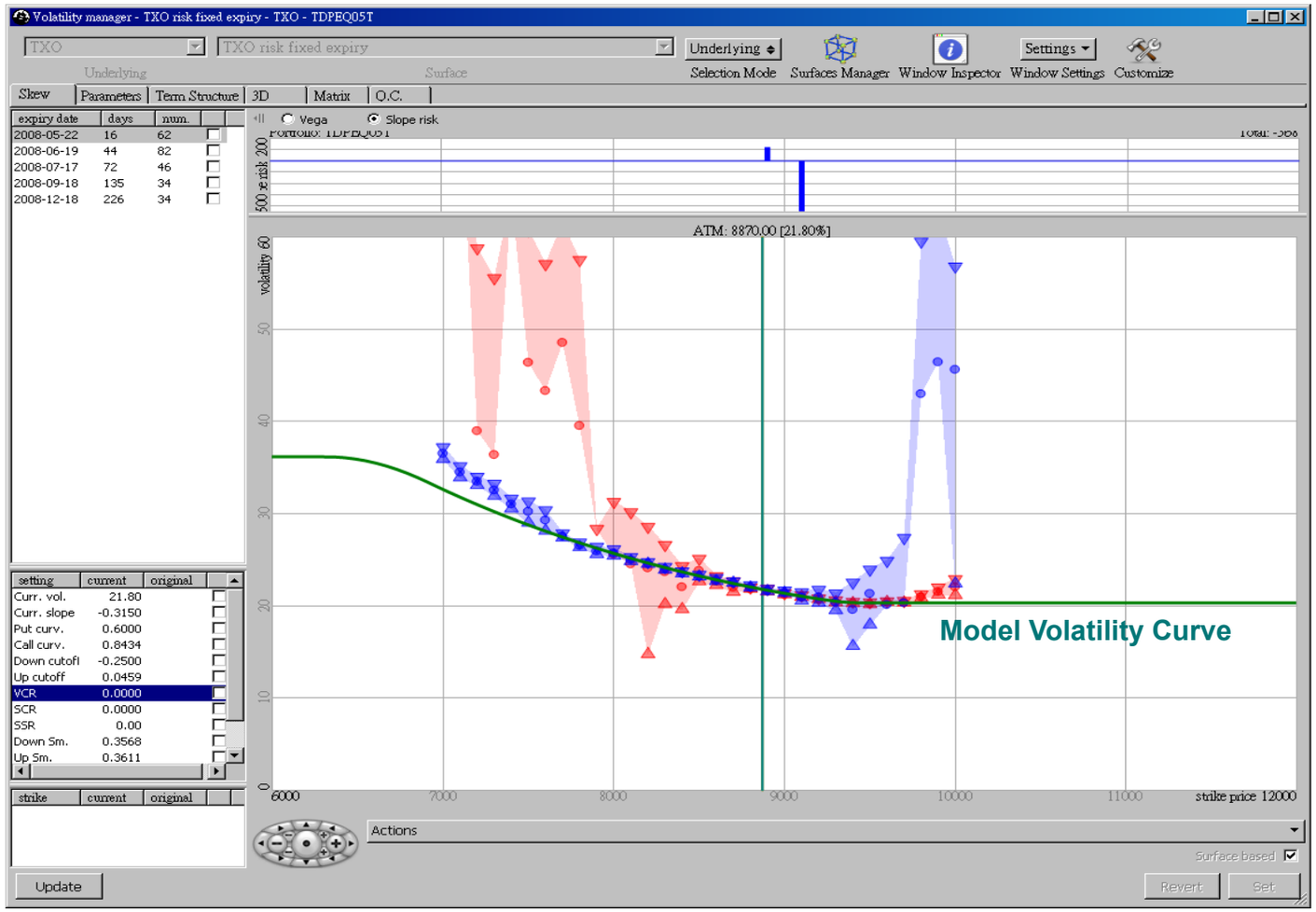
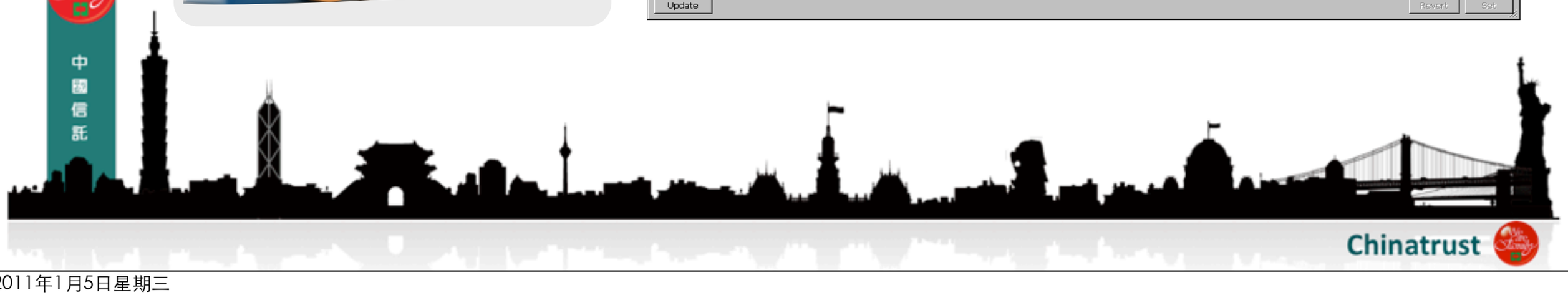
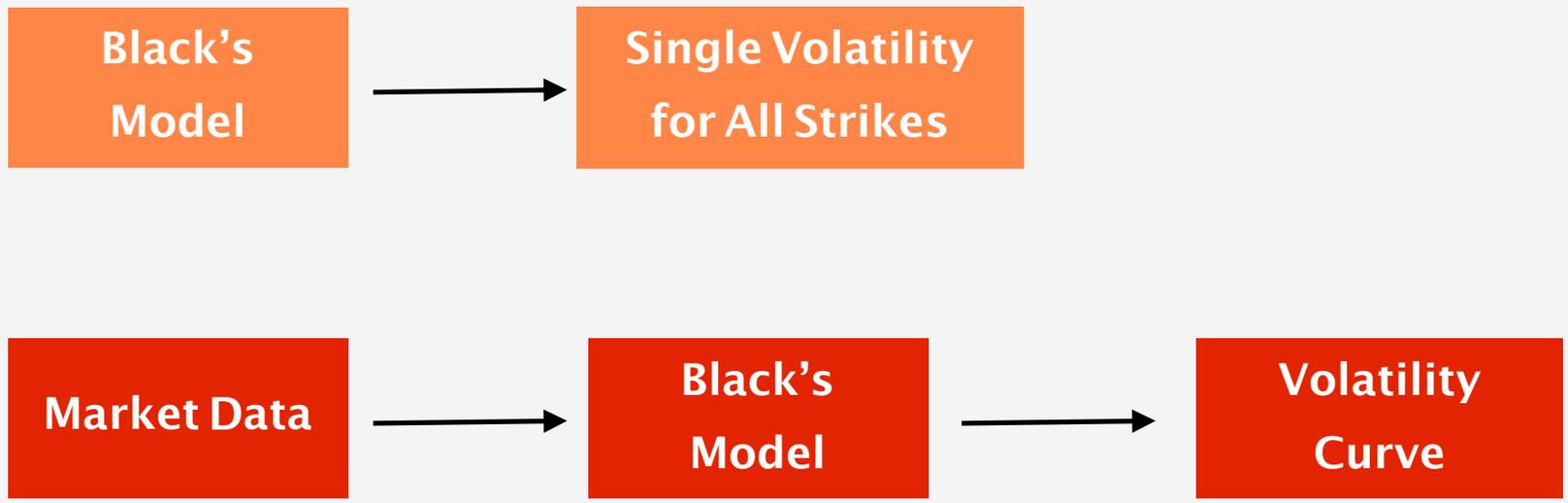
IMPLIED VOLATILITY CURVE/SURFACE







Serious inconsistency



Conceptually, the implied volatilities on the curve must have a different meaning from the volatility in Black's Model.

Do we need to develop a new model?

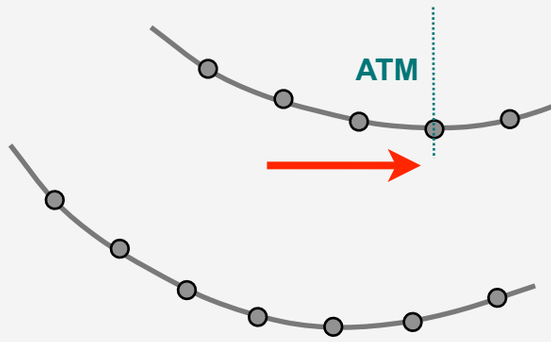
Not from a Practical Perspective

Well-capturing short-term P/L using factor-sensitivity methods

A very good descriptive tool for price dynamics

Parallel shift of curve with futures

Skewness of the curve

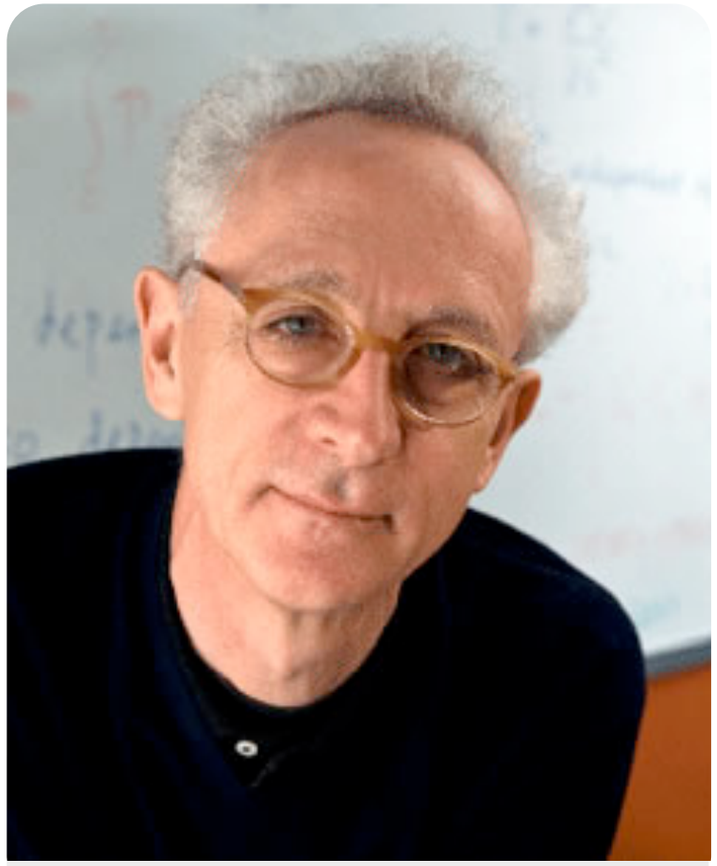


It has become industrial common language because of its simplicity

CHINATRUST 中國信託

Chinatrust

Phenomenological point of view



E.DERMAN

- Physicists who do so-called phenomenology work out the detailed and observable consequences of a theory, providing the practical link between principles and experiment...
- ...(phenomenologists) create heuristic approximations to engineer the theory into a pragmatic tool...
- When I moved to Wall Street, I found quantitative finance to resemble phenomenology much more than it resembled pure theory
- Quantitative finance is concerned with techniques that people use to value financial contracts and, given the fluctuations of the human psyche, it is a pragmatic study of surfaces rather than a principled study of depths

CONCLUSION

Even for good models, parameter estimation is sometimes hard. Normally, the greater the inaccuracy, the greater the uncertainty of hedging results, as demonstrated in the numerical study.

In many cases, this inaccuracy is not as disastrous as we might think, thanks to the law of large numbers. In some cases, substantial cushion is taken in derivatives pricing.

The industry tends to act like a phenomenologist rather than a theorist. To a phenomenologist, a simple model with intuitive “fixes” is more attractive than a complex one which introduces extra parameters that are either less intuitive or harder to estimate.

Q & A