Should all competitors cooperate together? A coopetition contest model.

Duc-De Ngo∗†

This draft: January 2007

Abstract

In this paper, I develop a coopetition model where firms’ efforts contribute (1) to making markets and (2) to dividing them up. Using a rent-seeking contest approach, I investigate how the market structure influences these efforts and profits of firms. In order to focus on the effect of entrants in the game, the setting is limited to the symmetric, pure strategy Nash equilibrium. Furthermore, the collaboration via an association or a governmental agency is examined. Besides, I also consider the impact of asymmetric valuations in simultaneous-move and two-stage games. Equilibrium efforts are determined and subjected to comparative static analysis. The results show that equilibrium efforts are greater in simultaneous game than in two-stage one.

Keywords: coopetition, contest, industrial organization.

JEL Classification: C70, D21, L22

∗LEO, Université d’Orléans, Rue de Blois, B.P. 6739, 45067 Orléans Cedex 2, France. Tel: +33(0)238494944, Fax: +33(0)238417380, Email: duc-de.ngo@laposte.net
†I am grateful to Professor Cyrille Piatecki, my supervisor, for his guidance and support. I would like to thank Alain Jeunemaître, Barry Nalebuff, Okura Mahito and all participants of the EIASM “2nd Workshop on Coopetition Strategy”, September 14-15, 2006, Milan for helpful suggestions. Naturally, all errors remain mine alone.
1 Introduction

From a certain level, the economy is a war among various stakeholders. At least, in any case, it is the impression given by traditional business language (winners and losers, victors and vanquished) and by the natural way people think about business. According to this theory, in order to conquer market shares, a firm has to beat off its rivals so as to impose its own conditions to its suppliers and customers. This war is summarized by a famous sentence of Gore Vidal: "It is not enough to succeed. Others must fail". This traditional philosophy is described in the game theory as zero-sum game in which winners take all and losers leave the game with empty hands.

In order to win this war, besides quantity, quality... firms can implement several strategies, either on price, advertising or even lobbying to influence policy makers ... However, these latter strategies might often be costly or even inefficient. These operations can be considered as rent-seeking activities which always lead to some socially undesirable consequences such as destruction rather than enhancement of resources available to the society, absence of growth, or even reduction of the social welfare.

But is it realistic to think that the economics field is only characterised by wars between firms, organizations and institutions? One may find the antithesis to the concept of Gore Vidal in Brandenburger & Nalebuff’s (1996) coopetition. Coopetition can be roughly defined as a strategy based on the combination between co-operation and competition. The idea is that, in many markets, there are forces that drive firms to increase industry profits — to cooperate explicitly or tacitly — and forces that drive them to increase their own respective profits at the expense of their com-

petitors’ – to compete. Furthermore, the coopetition is derived from the concept that to win, it is not necessary to make rivals lose. Instead, there could be no looser and all competitors, by cooperating, may generate and obtain more benefits at the end. Game theory baptizes this phenomenon positive-sum game in which all players get out victoriously.

Since the seminal papers of Jorde & Teece (1989), Hamel & Prahalad (1989), Brandenburger & Nalebuff (1996) and Dowling et al. (1996), coopetitive behaviour has been the focus of much attention. Since then, coopetition studies have been developed in many different fields and from various perspectives: resource-based and knowledge-based theory (Lado et al. 1997, Loebbecke et al. 1997), evolutionary economics, industrial organization (Long & Soubeyran 2001), marketing (Krishnamurthy 1999, Dearden & Lilien 2001), location cluster (Soubeyran & Weber 2002), management (MacDonald & Ryall 2004), international management (Luo 2004, Luo 2005), transaction cost, network theory (Ryll & Sorenson 2006), neo-institutional theory, economic law (Esty & Geradin 2000) among others. Lado et al. (1997) have argued that firms can generate economic rents and achieve superior, long-run performance through simultaneous competition and cooperation and Bengtsson & Kock (2000) have considered coopetition as the most advantageous relationship between competitors.

Despite this abundant literature, to the best of my knowledge, there has been no study that deals with the issue of market structure in a coopetitive relationship. The question arises is the following: if coopetition is mutually beneficial to all participants, should all competing firms cooperate simultaneously? In other words, is there an optimal structure or an optimal number of participants in game so that coopetitive strategies are fruitful to all? In order to find an answer to this question, in this paper, I used a modified model of Krishnamurthy (1999) to investigate the effect of the number of players on the efforts and the final payoff of each player.
The remainder of the paper is organized as follows. Section 2 describes the general model where competition and cooperation can exist at the same time and then investigates the effect of market structure on players’ strategy and profit. Section 3 introduces asymmetry in players’ valuation of the prize in the duopoly case. The results are computed in both simultaneous-move and two-stage games and a static comparison is provided. Section 4 contains some concluding remarks.

2 The General Model

Consider a coopetition contest in which each player spends two types of efforts: the first effort, called cooperative effort, is to enlarge the prize, and the second, called competitive effort, is to determine the winner (in the winner-take-all contest) or to capture the greater prize share. In this game, \(n\) risk-neutral players compete among them by expending irreversible effort to win. Let \(x_i\) represent this effort level spent by the player \(i\). \(x_i / \sum_{j=1}^{n} x_j\) is the firm \(i\)'s market share or in winner-take-all contests, it denotes a generalised probability function\(^2\) where \(r \geq 0\) reflects the ease of affecting the probability of winning. If \(r = 0\), then the spent effort has no effect on market share. If \(r \to \infty\), then the winner takes all, i.e. the player with the highest effort captures the whole prize. This function is identical to the one first introduced by Tullock (1980) within the rent-seeking contest literature and later axiomatized by Skaperdas (1996).

Let \(y_i\) denote the effort to contribute to increasing the pie. Effort levels are nonnegative and their costs are assumed to be a quadratic function and not a linear one as assumed by Krishnamurthy (1999) and Dearden & Lilien (2001). For example, in the field of marketing, one can imagine the following scenario. The demand for the firm \(i\)'s product is given by

\(^2\)Technically, a generalised probability function corresponds to the measure induced by a probability function.
the market size $V$ multiplied by its market share. This firm can influence
the size of the total market by contributing an effort $y_i$ to a generic ad-
vertising campaign and an effort $x_i$ to capturing its market share through
brand advertising operations.

The overall prize which is an increasing function of cooperative efforts
can be defined by the following relation:

$$V = V(y) = V_o + \left( \sum_{i=1}^{n} y_i \right)^{\gamma}$$

where $V_o$ is the initial demand without any effort spent and the power $\gamma$
is strictly positive ($\gamma > 0$). This relation is used by Krishnamurthy (1999),
Dearden & Lilien (2001) and Hanssens et al. (2002) to describe the impact
of generic advertising on demand function. To simplify and without loss
of generality, I restrict the analysis to the case of $V_o = 0$.

Then the payoff of the player $i$ is given by his revenue less his efforts’
costs.

$$\pi_i = \frac{x_i^r}{\sum_{j=1}^{n} r_j} \left( \sum_{j=1}^{n} y_j \right)^{\gamma} - \frac{x_i^2}{2} - \frac{y_i^2}{2}$$  \hspace{1cm} (1)

### 2.1 Noncooperative setting

In first place, consider the case where firms set those expenditures non-
cooperatively. All firms are supposed to simultaneously set effort expend-
itures. Specifically, given effort expenditures by other firms, the firm $i$
sets $(x_i, y_i)$ so as to maximize its profit $\pi_i$:

$$\max_{[x_i, y_i | X_{-i}, Y_{-i}]} \{ \pi_i | x_i \geq 0, y_i \geq 0 \}$$

resulting in Nash equilibria to the coopetitive game.

A Nash equilibrium is an expenditure profile $(x^*, y^*)$ such that for
each firm $i$:

$$
\pi^*_i ((x^*_i, y^*_i), (X^*_{-i}, Y^*_{-i})) \geq \pi_i ((x_i, y_i), (X^*_{-i}, Y^*_{-i})) \text{ for any } (x_i, y_i)
$$

The objective of the firm is to maximize its expected profit. The first-order conditions associated with this maximization following the Kuhn–Tucker theorem are:

$$
\begin{cases}
\frac{\partial E\pi_i}{\partial x_i} = \frac{r x_i^{r-1} \sum_{j \neq i} x_j^r}{(\sum_{j=1}^n x_j^r)^2} \left( \sum_{j=1}^n y_j \right)^\gamma - x_i \leq 0 \\
\frac{\partial E\pi_i}{\partial y_i} = \frac{x_i^r \gamma}{\sum_{j=1}^n x_j^r} \left( \sum_{j=1}^n y_j \right)^{\gamma-1} - y_i \leq 0 \\
x_i \frac{\partial E\pi_i}{\partial x_i} = y_i \frac{\partial E\pi_i}{\partial y_i} = 0
\end{cases}
$$

with strict equality if respectively $x_i > 0$ and $y_i > 0$.

In addition, if $x_i > 0$ and $y_i > 0$ then the following second order conditions must respectively be negative:

$$
\frac{\partial^2 E\pi_i}{\partial x_i^2} = \frac{r x_i^{r-2} \left( \sum_{j \neq i} x_j^r \right) \left( \sum_{j=1}^n x_j^r \right)}{(\sum_{j=1}^n x_j^r)^4} \left( \sum_{j=1}^n y_j \right)^\gamma - 1
$$

$$
\frac{\partial^2 E\pi_i}{\partial y_i^2} = \frac{x_i^r \gamma (\gamma - 1)}{\sum_{j=1}^n x_j^r} \left( \sum_{j=1}^n y_j \right)^{\gamma-2} - 1
$$

Let players be identical, i.e. $x_i = x, y_i = y, i = 1, \ldots, n$. We will consider only situations where all firms cooperate in equilibrium. In this case, it is natural to look for a symmetric equilibrium where all effort levels are the same:

$$
\begin{align*}
\frac{\partial E\pi}{\partial x} &= \frac{r x^{r-1} (n-1) x^r (ny)^\gamma}{(nx^r)^2} - x \leq 0 \quad (2) \\
\frac{\partial E\pi}{\partial y} &= \frac{x^r (ny)^{\gamma-1}}{nx^{r\gamma}} - y \leq 0 \quad (3)
\end{align*}
$$
Solving (2) and (3) for the equilibrium efforts yields the following:

\[ x^* = [r(n - 1)\gamma^{2-\gamma}]^{1/2}n^{-1} \] and \[ y^* = \gamma^{1-\gamma}n^{-1} \]

Consider next the effect of changes in \( n \) on the optimal efforts \((x^*, y^*)\).

Totally-differentiating equations (2) and (3) then rearranging, we have:

\[
\frac{dx^*}{dn} = \frac{-\frac{\partial^2 E^\pi}{\partial x \partial n} \frac{\partial^2 E^\pi}{\partial y^2} + \frac{\partial^2 E^\pi}{\partial y \partial n} \frac{\partial^2 E^\pi}{\partial x \partial y}}{|H|}
\]

\[
\frac{dy^*}{dn} = \frac{-\frac{\partial^2 E^\pi}{\partial x \partial x} \frac{\partial^2 E^\pi}{\partial y^2} + \frac{\partial^2 E^\pi}{\partial y \partial x} \frac{\partial^2 E^\pi}{\partial x \partial y}}{|H|}
\]

where the Hessian determinant \(|H| = \frac{\partial^2 E^\pi}{\partial x^2} \frac{\partial^2 E^\pi}{\partial y^2} - \frac{\partial^2 E^\pi}{\partial x \partial y} \cdot \frac{\partial^2 E^\pi}{\partial y \partial x}
\]

**Theorem 1** Assume that \(|H| > 0\) (condition for local stability), \( r \leq 1 \) and \( \gamma \leq \frac{n-2}{n-1} \) with \( n \geq 3 \). Then as the number of firms in the game increases, the equilibrium efforts \((x^*, y^*)\) decrease.

**Proof**

\[
\frac{\partial^2 E^\pi}{\partial x^2} = r(n - 1)[(r - 1)n - 2r]n^{\gamma-3}x^{-2}y^{\gamma-1} - 1
\]

Since the third term \([(r - 1)n - 2r] < 0\) then \(\frac{\partial^2 E^\pi}{\partial x^2} < 0\)

\[
\frac{\partial^2 E^\pi}{\partial y^2} = n^{\gamma-3}\gamma(\gamma - 1)y^{\gamma-2} - 1 < 0 \text{ as } (\gamma - 1) < 0
\]

\[
\frac{\partial^2 E^\pi}{\partial x \partial y} = r(n - 1)\gamma n^{\gamma-3}x^{-1}y^{\gamma-1} > 0
\]

\[
\frac{\partial^2 E^\pi}{\partial y \partial x} = r[-n + 2 + (n - 1)\gamma]n^{\gamma-3}y^{\gamma-1}x^{-1} \leq 0
\]

\[
\frac{\partial^2 E^\pi}{\partial y \partial n} = \gamma(\gamma - 2)n^{\gamma-3}y^{\gamma-1} < 0
\]

Then \(\frac{dx^*}{dn} < 0\), \(\frac{dy^*}{dn} < 0\) Q.E.D.

**Corollary 1.1** An increase in the number of players in the game decreases their expected profits.

**Proof**

\[
\frac{d\pi}{dn} = \frac{\partial E^\pi}{\partial x} \frac{dx}{dn} + \frac{\partial E^\pi}{\partial y} \frac{dy}{dn} + \frac{\partial E^\pi}{\partial n}
\]
\[ \frac{\partial E_\pi}{\partial x} = 0, \frac{\partial E_\pi}{\partial y} = 0 \text{ by definition of equilibrium.} \]

Moreover, \[ \frac{\partial E_\pi}{\partial n} = (\gamma - 1) n^{\gamma - 2} y^{\gamma} < 0 \] Q.E.D.

The corollary 1.1 shows that merger of players will increase each non-merging player’s expected profit and vice versa the entrants are not welcome by incumbents in this coopetition game. This result is the opposite of the following claim sustained by Brandenburger & Nalebuff (1996):

\[ \ldots \text{imitation of win-win strategy is healthy, not harmful. So if you come up with a win-win strategy, you don’t have to keep it secret. It’s not a problem if your strategy becomes widely known and widely imitated. In fact, that’s all to the good. The more competitors that adopt your strategy, the better for you. – (Brandenburger & Nalebuff 1996, pp 143)} \]

Yet, according to our result, from \( n = 3 \), the more are competitors that adopt coopetition strategy, the worse is for incumbents.

At this stage, it would be interesting to compare the results obtained so far with a situation in which an regulator supervises the game. His task consists of controlling the number of participants so as to maximize the welfare of the industry.

**Theorem 2** The regulator will set the number of participant \( n_r \) less than \( n_o \) (the zero-profit number of firms in the noncooperative model).

**Proof** The regulator maximises the welfare with first-order condition:

\[ \frac{\partial n E_\pi}{\partial n} = \gamma n^{\gamma - 1} y^\gamma - \frac{x^2}{2} - \frac{y^2}{2} = 0 \]

therefore, \( n_r^{\gamma - 1} = \left( \frac{x^2}{2} + \frac{y^2}{2} \right)^\frac{1}{\gamma} y^{-\gamma} \)

The zero-profit number of firms satisfies \( \frac{1}{n} (ny)^\gamma - \frac{x^2}{2} - \frac{y^2}{2} = 0 \). Hence \( n_o^{\gamma - 1} = \left( \frac{x^2}{2} + \frac{y^2}{2} \right) y^{-\gamma} \)

But \( \gamma \leq 1 \). Together these equations yield \( n_r < n_o \) Q.E.D.
2.2 Cooperative setting

In a number of markets, either a governmental agency or an industry association often sets a budget for a collective action on behalf of the industry and then allocates that budget amongst the firms that are expected to benefit. It is the case in generic advertising (Forker & Ward 1993) or in collective lobbying (Greenwood & Aspinwall 1998) among others.

I model that situation as follows. Let $Y_c$ denote the efforts that the firms collectively choose. By choosing $Y_c$, the objective of the firms in the industry is to maximize the sum of industry profits. Besides, they also choose efforts $x_i$ to compete individually and noncooperatively. Under these assumptions, an equilibrium $(x^*, Y^*_c)$ exists and can be characterised by Kuhn-Tucker first-order conditions:

$$\begin{cases}
\frac{\partial E_\pi}{\partial x} = \frac{rx^{-1}(n-1)x^r}{(nx^r)^2} (ny)^{\gamma} - x \leq 0 \\
\frac{\partial \sum E_\pi}{\partial y_c} = \gamma (ny)^{\gamma - 1} - y \leq 0 \\
x_i \frac{\partial E_\pi_i}{\partial x_i} = Y_c \frac{\partial E_\pi_i}{\partial y_i} = 0
\end{cases} (4)
$$

Solving the system (4) yields the following equilibrium efforts:

$y^* = \gamma^{1/(2-\gamma)} n^{(2-\gamma)/2-1}$ and $x^* = [r(n - 1)n^{(2\gamma - 4)/(2-\gamma)} \gamma^{\gamma/(2-\gamma)}]^{1/2}$

If we compare the noncooperative and cooperative equilibrium efforts, we can see that association promotes both cooperative and competitive efforts. Actually, the efforts that maximize the sum of the profits of the firms in the cooperative situation are greater than the efforts in the noncooperative one. By collectively determining the cooperative efforts to expend, the firms can reach the optimum.
3 Asymmetric case

The previous section has analysed the symmetric setting. Now to extend to asymmetric case, I consider a specific case in which \( n = 2 \) with asymmetric prize valuation \( V_i = m_i (y_1 + y_2)^\gamma \). The payoff function of player \( i \) becomes as follows:

\[
\pi_i = \frac{x_i^r}{x_1^r + x_2^r} m_i (y_1 + y_2)^\gamma - \frac{x_i^2}{2} - \frac{y_i^2}{2} \quad \text{with} \quad i = 1, 2. \tag{5}
\]

3.1 Simultaneous-moves

First, I examine the simultaneous-moves case where the players simultaneously decide their effort levels.

The first order conditions for an interior Nash equilibrium of the model are:

\[
\frac{\partial \pi_i}{\partial x_i} = \frac{rx_i^{r-1}x_i}{(x_1^r + x_2^r)^2} m_i (y_1 + y_2)^\gamma - x_i = 0 \tag{6}
\]

\[
\frac{\partial \pi_i}{\partial y_i} = \frac{x_i^r}{x_1^r + x_2^r} m_i \gamma (y_1 + y_2)^{\gamma - 1} - y_i = 0 \tag{7}
\]

Rewrite the first-order conditions (6),(7) and take the ratio of them to get the following relationship between equilibrium effort levels and unit profit margins:

\[
\frac{x_1}{x_2} = \left( \frac{m_1}{m_2} \right)^{1/2} \tag{8}
\]

\[
\frac{y_1}{y_2} = \left( \frac{m_1}{m_2} \right)^{r/2+1} \tag{9}
\]

These relationships are summarized in the next proposition.

**Proposition 3** The firm who values the prize more (i.e. \( m_i \) is bigger) spends more effort in equilibrium but both players allocate proportional fraction of their valuations.
Proposition 3 raises interesting questions about the structure of relative advantages and payoff of the players. Nti (1999) obtained this result for the competitive effort and I have shown that it is also right for cooperative effort.

The efforts spent in equilibrium are:

\[
\begin{align*}
  x_i &= (A_i A_{-i} r (y_1 + y_2)^\gamma m_i)^{1/2} \\
  y_i &= A_i m_i^\gamma r^{1/2} (A_1 m_1 + A_2 m_2)^{(\gamma-1)/2}
\end{align*}
\]

where \( A_i = \frac{m_i^{r/2}}{m_1^{r/2} + m_2^{r/2}} \)

3.2 Two-stage game

It is interesting to investigate which game structure — simultaneous-move or two-stage game — is more suitable. In this two-stage game, at the first stage, players decide how much of effort they simultaneously spend to enlarge the pie. And then at the second stage, they choose efforts to determine their share.

The equilibrium is computed by backward induction.

The first-order conditions in the second stage:

\[
\frac{\partial \pi_i}{\partial x_i} = \frac{rx_i^{r-1}x_{-i}^r}{(x_1^r + x_2^r)^2} m_i (y_1 + y_2)^\gamma - x_i = 0
\]

Then

\[
\begin{align*}
  x_1 &= \frac{rx_1^{r-1}x_2^r}{(x_1^r + x_2^r)^2} m_1 (y_1 + y_2)^\gamma \\
  x_2 &= \frac{rx_2^{r-1}x_1^r}{(x_1^r + x_2^r)^2} m_2 (y_1 + y_2)^\gamma
\end{align*}
\]

Dividing \( x_1 \) by \( x_2 \) and rearranging yields:

\[
\begin{align*}
  x_i^2 &= A_i A_{-i} r (y_1 + y_2)^\gamma m_i
\end{align*}
\]
Substituting $x_i$ in equation (5), then differentiate with respect to $y_i$, we obtain the first-order conditions of the first stage. Thus, we find player’s efforts in enlarging the pie in equilibria:

$$y_i = B_i \gamma^{1/2-\gamma} (B_i + B_{-i})^{\gamma-1}$$

(14)

where $B_i = m_i A_i (1 - 1/2r A_{-i})$.

Compare these equilibrium efforts with the simultaneous-move, we have:

**Proposition 4** The total of efforts spent by players to enlarge the pie in the simultaneous game is greater than the efforts in the two-stage game. In a similar manner, players spend more in competitive processus.

**Proof**

$$y_1^t + y_2^t \equiv \gamma^{1/2-\gamma} (B_1 + B_2)^{\gamma-1} < \gamma^{1/2-\gamma} (A_1 m_1 + A_2 m_2)^{\gamma-1} \equiv y_1^s + y_2^s$$

Thus $x_1^t < x_1^s$ and $x_2^t < x_2^s$

$Q.E.D.$

An important implication of this proposition is for contest design in effort maximization. There are, in fact, many situations where a contest designer may want to extract maximum efforts from the players. Typical examples include: employment tournament where a firm wants employees to exert their best efforts to win rewards, sport competition where spectators enjoy intensely matches.

### 4 Conclusion

Following Krishnamurthy (1999), this paper extends the Tullock rent-seeking game, which considers a situation where the players spend effort not only in rent-seeking but also in enlarging the prize. By this way, it
can be used to describe a new phenomena in the world of business, the coopetition.

It was shown that in coopetitive situation, the more the number of firms is, the less are efforts spent and their profits. The analysis also indicated that regulation, association of coopetition could be useful tools to promote firms’ efforts. Unlike Brandenburger & Nalebuff’s (1996) result, I found that imitation of a coopetition strategy might have negative effects on incumbents. Furthermore, examining a situation where the players may have asymmetric prize valuations reveals that both competitive and cooperative efforts are proportional with this asymmetry. The results and insights obtained here might be applied to illuminate issues in work team, lobbying, alliance, network competition . . .

In future research, it would be interesting to derive results in two-stage game in which players choose first effort levels to determine the share and then effort levels to enlarge the prize.

References


