Optimal Revenue Sharing Contracts with Externalities and Dual Agency

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Abstract

A model of bilateral trade between an upstream supplier (landlord) and a downstream producer (retailer) is constructed, in which the upstream supplier confers long-term property usage rights to the downstream supplier in return for a base rental fee plus a percentage of verifiable sales production. Our model allows for the possibility that downstream sales production complements other activities of the upstream supplier to increase its total revenues. An optimal contract is designed that balances investment incentives of the downstream producer with initial investment and subsequent reinvestment incentives of the upstream supplier. A number of important stylized empirical facts associated with retail lease contracting are addressed with the model, including why: i) retail leases contain base rents and often (but not always) contain an overage rental feature, ii) stores that generate greater externalities pay lower base rents and have lower overage rent percentages than stores that generate fewer externalities, iii) the overage rent option is typically well out-of-the-money at contract execution, and iv) stand-alone retail operations often sign leases that contain an overage rental feature.

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I. Introduction and Motivation

Incentives to execute formal contracts derive from the division of labor and exchange, where division of labor implies delegation of responsibility. Comparative advantage underlies division of labor, but delegation introduces costs when there are conflicting objectives between agents. Contracts are often used in an attempt to minimize the costs of conflicting objectives. *Incentive contracts* have been developed that try to align agent interests in order to approach efficient (first-best) outcomes.

Contracts first appeared in agriculture as bilateral agreements between landlords and their tenants.[1] Sharecropping arrangements were perhaps the most common such contract, in which the tenant agreed to share revenues from crop production with the landlord. The structure of these contracts has puzzled economists for a long time. The reason is that a sharing arrangement appears to reduce incentives for the tenant to exert effort to maximize production, to the detriment of the landlord. A fixed payment rental contract with incentive payments made back to the tenant, where incentive payments correlate positively with production, would seem to Pareto-dominate the standard sharecropping contract.

It wasn't until the development of agency theory that convincing arguments were offered to rationalize observed contracting practices. For example, Stiglitz (1974) emphasized risk-sharing with idiosyncratic (weather-related) production shocks. The sharecropping arrangement shifts risks of stochastically variable production from the risk-averse tenant to the well-endowed, risk-neutral landlord. Tenant effort level, chosen prior to the realization of random production shocks, is shown to approach the first-best level with an appropriately structured sharecropping contract. Others have since extended Stiglitz's basic argument to explicitly account for landlord bargaining power, tenant financial

[1] See Chapter 1 of Laffont and Martimort (2002).

constraints, and certain other supply effects (Newbery and Stiglitz (1979), Braverman and Stiglitz (1982), Eswaran and Kotwal (1985)).

Retail lease contracting offers similar puzzling features, in which retail tenants in a shopping center configuration typically pay base rents plus a percentage of sales when sales exceed an overage threshold value. Furthermore, it is known that base rents and overage rent percentages vary systematically depending on the size of the retail tenant, where larger tenants pay lower base rents and overage rent percentages (see, e.g., Benjamin et al. (1992), Gould et al. (2002)).

Using detailed shopping center data obtained from Wheaton (2000, see pp.187-191 for additional detail), in which square footage of retail area is used to proxy for external effects (i.e., more retail square footage indicates a larger allocation of space to a retailer and hence greater positive externalities), the basic relations between base rent, percentage rent, and externality can be seen. Panel A of Table 1 shows that, for all retail firms in the sample, base and percentage rents move together, and larger retailers pay lower base and percentage rents. Panel B demonstrates that the same relations hold within retail store category.

Table 1 Here

The combination of multiple tenants in a shopping center setting together with systematic variation in contract terms as a function of tenant size has sparked considerable interest among researchers. Brueckner (1993) was the first to consider externality as an explanation for tenant agglomeration and observed contracting practices, in which larger tenants (anchor stores) generate positive externalities to the benefit of smaller tenants. The landlord in this setting operates as a discriminating monopolist. In the model, prior to consideration of tenant effort, an optimal allocation of space can be achieved with base rents only, in which externality-generating tenants paying lower base rents than externality-consuming tenants. When tenant effort is considered, Brueckner shows that incentive payments made by the landlord back to the tenant is an optimal contract—an outcome that is exactly the opposite of what is generally observed in practice. Incentive payment percentages do increase with externality, however, so comparative static relations generally match up against empirically observed outcomes.

Others have offered risk-sharing arguments, but those arguments fail to explain observed retail contracting practices. For example, although it may be true that anchor tenants, which are firms with a national presence and sizable scale, are less risk-averse than smaller "mom-and-pop" retail tenants, it is doubtful that nationally recognized brand-name tenants that lease smaller spaces than anchor tenants (e.g., shoe or clothing specialty stores) are more risk-averse than the larger anchor tenants. These smaller specialty stores typically pay base and overage rental percentages that exceed rents paid by anchor stores, which accords more closely to explanations that emphasize externalities as opposed to risk-sharing (see Gould et al. (2002) for further discussion of this issue).

Wheaton (2000) argues that overage rental features are an incentive-compatible contracting mechanism. In a shopping center setting, tenants sign long-term leases. An optimal mix of tenants is easily obtainable at the outset, but tenants disappear over time for idiosyncratic reasons. The landlord controls the releasing decision, which is non-contractable. At the time of releasing, the landlord may have an incentive to sign a tenant that pays the most rent, regardless of its effect on other tenants. This high-rent-paying tenant will typically consume rather than generate externalities. Incumbent tenants may instead collectively prefer an externality-generating tenant in order to maximize total shopping center sales. The overage rental contract can achieve this objective, and Wheaton shows it is an optimal contract when landlord hold-up problems exist.

Although rich and insightful, previous literature nonetheless fails to explain several important empirical facts that relate to retail lease contracting. First, as noted by Edelman and Petzold (1996), Gould et al. (2002), and others, the overage rent percentage option is typically well out-of-the-money at contract execution. Anchor stores often do not have overage rental clauses at all in their lease contract, and pay only minimal base rents. This suggests that overage rents are a lower-powered rather than higher-power incentive mechanism—contrary to the incentive mechanisms required in the models of Brueckner (1993), Wheaton (2000), and others.

Second, models in both the sharecropping and retail lease contracting literature generally assume complete bargaining power on behalf of the landlord, where complete bargaining power for the landlord biases contract terms toward the overage rental component. While complete bargaining power for the landlord may be appropriate in a sharecropping setting, we would not expect this to be true in general in a retail setting. When bargaining power tilts towards the tenant, a fixed (or even reverse-sharing) contract emerges due to the tenant's preference for such an outcome. This provides an alternative explanation as to why smaller, more localized tenants pay higher overage percentages than larger national chain stores.

Third, and perhaps most important, it is well known that stand-alone retail establishments (e.g., "big box" retailers) often execute lease contracts with overage rent features. These retail establishments often neither generate nor receive externalities that are attributable to an optimal tenant mix (e.g., the landlord owns the single parcel of land occupied by the stand-alone, but typically does not own the surrounding properties). The tenants are also often national chain stores, which eliminates risk-sharing as a convincing explanation for contracting practices.

To demonstrate the existence of percentage rent contracts with stand-alone retailers, I have obtained data on 183 Walgreen store leases that were financed through the securitized (conduit) mortgage loan market.[2] Rental contracts are categorized by whether the Walgreen's store paid fixed rents only or signed a percentage rent contract that contained fixed plus overage rents paid by

[2] These mortgage loans secured by Walgreen stores were included in securities owned by a particular investor in the commercial mortgage-backed securities (CMBS) market. The sample includes all Walgreen store data from the investor for which detailed lease data were available.

Walgreen's to the landlord when sales exceeded a prespecified threshold value.[3] We also have data on whether the Walgreen's store was part of a shopping center or was a stand-alone separate from a shopping center. Table 2 displays the categorical results.

Table 2 Here

A total of 88 of the 183 Walgreen's stores (48.1 percent) signed percentage rent contracts, and 144 of the 183 stores (78.7 percent) were stand-alones as opposed to existing within a shopping center configuration. Observe that 78 of 88 stores (88.6 percent) that signed percentage rent contracts were stand-alone stores, where stand-alones presumably generate fewer positive (off-site) externalities accruing to the benefit of the landlord than Walgreen stores contained in a shopping center. This compares to 66 of 95 stores (69.5 percent) that signed fixed rent only contracts that were stand-alones. Another way to state the result is that only 45.8 percent of stores (66 of 144) that were stand-alones signed fixed rent only contracts, whereas 74.4 percent of shopping center configured stores (29 of 39) signed fixed rent only contracts.

Thus, this table demonstrates that: i) percentage rent contracts are commonly used with stand-alone (big box) retail establishments, and ii) percentage rent contracts are significantly more common with stand-alone stores than with shopping center configured stores, suggesting a first-order negative relation between externality and the use of percentage rent contracts. That is, the data indicate that inter-store externalities are not necessary to explain the existence of percentage rent retail contracts.

A more refined analysis of the contract type–store type relationship is presented in Table 3. Column A of the table displays logit regression results when contract type is regressed against store type (percentage rent contract and stand-alone stores are the indicated variable). Column B displays

[3] The percentage rent contract executed by the stand-alone therefore strongly resembles the contract offered to shopping center tenants. When percentage rent contracts were executed by Walgreens, there was very little variation in the overage rent percentage or the overage rent threshold, so we do not incorporate these terms into the analysis.

results from a similar specification, with the addition of control variables that include store square footage, lease term, whether or not a termination option exists for the tenant, building age (greater or less than one year since construction), loan-to-value ratio on the mortgage loan, and mortgage loan term. Results confirm relations displayed in Table 2, which show a propensity for the use of percentage rent contracts in stand-alone store settings. This in turn suggests a negative relation between externality and percentage rents in retailer–landlord lease contracting.

Table 3 Here

In summary, these findings suggest that the existing theory of retail contracting is at best incomplete. Percentage rent contracts are employed in non-shopping center settings with low-or-no-externality generating tenants. This result, combined with other results on the relation between tenant size and contract type in a shopping center setting, implies that it is the *absence of externality* – whether or not inter-store agglomerative effects matter – that accounts for percentage rent incentive contracting with retail property.

The purpose of this paper is to construct a model that addresses the important stylized empirical facts associated with retail lease contracting. To summarize, these facts are: i) retail leases contain base rents and often (but not always) contain an overage rental feature; ii) stores that generate more externalities pay lower base rents and have lower overage rent percentages than stores that generate less externalities; iii) the overage percentage rent option is typically well out-of-the-money at contract execution, iv) the tenant often has significant bargaining power in setting contract terms, and v) stand-alone retail operations often execute leases that contain an overage rental feature.

II. Summary of Model and the Main Results

To explain the data, we offer a model of bilateral incentive contracting that incorporates spillover and dual agency. Positive externalities are generated by the tenant accrue to the benefit of the landlord. Costly effort (e.g., advertising) by the tenant affects sales production. Initial investment as well as subsequent reinvestment by the landlord also affects sales productivity. Endogenously determined variables are the optimal contracting terms (base rent, overage rent threshold value, overage rent percentage), initial investment by the tenant and landlord, and the landlord's reinvestment threshold and quantity.

Long-term leases are executed between the landlord and tenant. Unit retail sales move stochastically over time, where total sales depend on non-contractable investments made by the tenant and the landlord. Landlord investment (e.g., building shell, infrastructure) depreciates over time. Sales growth with spillover benefits, overage rents, and depreciation all provide incentives for the landlord to reinvest to increase sales and hence rental revenues. The optimal contract is constructed to maximize a weighted average of joint retailer-landlord expected profits, where weights signify relative bargaining power between the two parties. Investment incentives and hence relative profit follow from optimal contract determination.

The basic tension thus follows from balancing non-contractable specific investment incentives through the optimal contract.[4] Higher-powered overage rental features are disliked by the tenant, and have the effect of decreasing the tenant's investment to decrease total sales. Lower total sales subsequently decrease incentives for reinvestment by the landlord. Alternatively, all else equal, overage rents increase incentives for value-added reinvestment, which is preferred by the landlord. The optimal contract thus balances this tension, in which fixed base rents trade off with overage rents in the optimal contract.

[4] Joskow (1987) notes that, "Buyers and sellers make larger *ex ante* commitments to the terms of future trade, and rely less on repeated negotiations over time, when relationship-specific investments are more important."

When positive externalities accrue to the landlord as a proportion of total sales, low initial investment by the tenant depresses landlord equity value. This causes the landlord to substitute base rents for percentage rents in order to increase initial investment. Greater external flows also allow the landlord to decrease base rents, which further increases initial investment. Thus, base rents and overage rental percentages move inversely with spillover magnitudes, which is consistent with the empirical evidence.

This result does not depend explicitly on a common-agency, inter-store externality framework, where externality-consuming tenants "subsidize" externality-generating tenants by paying higher rents. Rather, in our model, external flows that accrue to the landlord *substitute* for base and overage rents. This causes an increase in initial investment, to the benefit of both landlord and tenant, while also not depressing incentives for landlord reinvestment.

This substitution effect explains overage rent features with stand-alone retail operations. Overage rents are required to compensate for the *absence* of external flows in order to provide incentives for landlord reinvestment. Since stand-alone retail operations typically contain only one tenant, reinvestment can be especially important and easy to coordinate.

As landlord bargaining power decreases, both base and overage rents decrease (see Edelman and Petzold (1996) for interview evidence). Variation in bargaining power can thus explain differences in retail lease contracts, independent of external effects. For example, variation in tenant bargaining power can explain why stand-alone retail operations sometimes execute percentage rent contracts and sometimes do not.[5]

We find that the overage rental percentage option is typically well out-of-the-money at contract execution. For example, base case results suggest 12 to 17 year average hitting times to the overage rent threshold. However, significant probabilities exist (exceeding 20 percent) of hitting the threshold

[5] Recall the stand-alone store data reported in Table 1, in which 78 stores signed percentage rent contracts and 66 signed fixed rent contracts.

within a five year time frame. Thus, a long average time to paying overage rents provides incentives for retailer to make significant up-front investments. When exceedingly high sales growth does occur, however, triggering unexpectedly early overage rent payments, the tenant doesn't mind making the payments since profits are high as well. Consequently, the optimal contract is such that base rent and overage rent percentages are relatively high when external flows are low, but the overage rent option remains well out-of-the-money.

A higher profit margin for a retailer causes base rents to increase and overage rental percentages to decrease. This result demonstrates that retail tenants prefer base rents to overage rents, with the two contract features being substitutes. It differs from our externality results, in which base rents and overage rent percentages move together as external flows substitute for both types rents. This result also suggests that one must be careful to distinguish between differences in retail categories (which is probably best measured by externality) and differences in retailers within a retail category (which can be measured by profit margin).[6]

Relative to first-best, the tenant underinvests and the landlord overinvests, both initially and dynamically. Tenant underinvestment is most severe when externalities are smaller, since initial investment is discouraged by higher overage rents in the incentive contract. To compensate for tenant underinvestment, the landlord overinvests. Overinvestment incentives increase with higher rates of external flows, causing overage rent percentages to decline to partially counteract this tendency to overinvest. In fact, when spillover magnitudes are particularly high, a reverse overage rent is paid by the landlord to the tenant.

Our model is capable of explaining contracting practices for operations other than shopping centers; indeed, although sharing contracts may be necessary in a shopping center setting, they are not sufficient. Consequently, one can ask the question of why we don't observe sharing contracts with certain other types of commercial property operations, such as office or apartment property.

[6] See Edelman and Petzold (1996), who find exactly this result.

We contend that two factors are necessary for these sharing contracts to work in practice. First, a market structure must exist where an upstream supplier (landlord) provides an input (land, infrastructure) that crucially impacts downstream supplier productivity (retail sales). This is clearly the case with retail, but is much less clearly the case with office or apartment property, where numerous other factors are probably more important than locational and physical amenities in the success or failure of the "operation." Second, output directly attributable to the provision of upstream supply must be easily measured and verified. This is not the case with many types of operations, in which the relevant information is difficult to measure or obtain at a feasible cost.

While we obviously do not believe that inter-store externalities are not necessary for overage rent contracting practices observed with shopping centers, we believe there is more to the story. In other words, our model strongly suggests that shopping centers and other agglomerative common-agency settings do not have a monopoly on sharing contracts. It is the absence of externality that makes these contracts especially attractive to landlords, as seen by use of similar contracts with stand-alone stores. Bargaining power also plays are role, where anchor stores and national chains generally have more of it than mom-and-pop retailers. In addition to retail, our model explains the existence of sharecropping contracts, in which landlord bargaining power and the absence of externality are primary reasons for the sharing feature. We would expect to observe the use of similar sharing contracts in other settings that satisfy the productivity and measurability criteria discussed above. Franchise agreements and the licensing of certain types of intellectual property are relevant examples.

III. Model

To fix ideas, consider a representative retail operation. Total current operating revenue, or sales, is Qs, where Q is the quantity or quality of retail services and s is the sales per unit of retail services. Retail services, Q, depend on contributions from both the landlord and the tenant. Landlords (real estate developers) have a comparative advantage in the provision and maintenance of locational amenities and exterior space from which retail goods are sold. In contrast, retail operators have a comparative advantage in the advertising and distribution of consumer goods.

Based on this division of labor, total retail services, Q, are separable into landlord and retailer components. The landlord provides retail services of q at any point in time. These services are then scaled by the retailer's contribution, k, to result in total retail services, $Q=kq$. Unit sales, s, are determined by the number and characteristics of consumers in households surrounding the operation. Unit sales evolve stochastically according to a geometric Wiener process, with drift parameter, μ, and volatility parameter, σ, $\mu,\sigma>0$.

The retailer's contribution to total service provision, k, is made at the start of operations at a cost of αk^{β}, $\alpha>0$, $\beta>1$. This investment is non-contractable and can be characterized as site specific physical investment (e.g., tenant improvements) as well as other activities that affect total sales, such as advertising. At the same time, conditional on retailer investment, the landlord makes its own initial investment of $q=q_0$ at a cost of $k q_0^{\gamma}$, $\gamma>1$. Note that, just as service provision from the tenant scales up total sales, it also scales up the landlord's investment cost.

Over time, landlord service provision, q, depreciates at a constant rate of δ, $\delta>0$. In general, depreciation can be related to site-specific physical capital as well as other factors that affect the productivity of the real property. Because of depreciation, the landlord will have incentives to reinvest to restore productivity. Thus, after the commencement of operations, the landlord can repeatedly reinvest in the property. Reinvestment is non-contractable and in the landlord's own interests. With each reinvestment, the landlord increases the level of services available for the retailer to $q=\overline{q}$. The cost of the instantaneous reinvestment as of time t is $k\overline{q}^{\gamma}e^{\rho t}$, $\rho\geq0$. The $e^{\rho t}$ term adjusts for time-related factors that affect the cost of reinvestment after the start of operations.

A long-term lease contract is executed prior to initial investment to compensate the landlord for its service provision. Contract terms are set to incentivize both the tenant and landlord to make relationship-specific investments that affect total sales, where each party can observe the other's contributions to the overall success of the project, but courts cannot. Both the landlord and tenant are risk neutral, with a discount rate of ι. To ensure well-behaved value dynamics, we will restrict parameter values to satisfy $\iota \geq \mu+\rho-\delta$.

The lease contract has three components: base rent, overage rent percentage (as a percentage of total sales), and the overage threshold value (above which overage rents are paid). At any point in time, t, both the base rent and the overage threshold value are proportional to current replacement cost of retail services, $kq^{\gamma}e^{\rho t}$. In other words, the base rent and the overage threshold value have the respective forms: $akq^{\gamma}e^{\rho t}$ and $bkq^{\gamma}e^{\rho t}$, $a,b\geq0$. Overage rent is paid whenever total sales, Qs, exceed the overage threshold value. The percentage of total sales, p, that is paid as overage rent can be positive or negative. When $p>0$, overage rent is paid by the tenant to the landlord. When $p<0$, the landlord subsidizes the fixed rent component by returning rent that is proportional to total sales. The constants a, b, and p completely identify the lease contract.

Thus, to summarize, at each instant in time the retailer pays the landlord the total rent:

$$R = R(k,q,s,t) = akq^{\gamma}e^{\rho t} + pMax\{0, Qs - bkq^{\gamma}e^{\rho t}\} \qquad (1)$$

Total sales generate both profits for the retailer and possibly externalities that benefit the landlord. Retailer profits are proportional to total sales, where the profit margin, π, $0<\pi<1$, can depend on the type of product sold (the category of sales). Externalities that accrue to the landlord are also proportional to total sales, where the externality parameter is denoted as λ, $\lambda \geq 0$. At each instant in time the landlord thus realizes total benefits of $R+\lambda Qs$. Concurrently, the retail tenant realizes net revenue of $\pi Qs-R$.

Determination of the optimal investment polices and lease contract require that we value the landlord and tenant's revenue streams conditional on their actions. Equilibrium is determined in four stages, which we state in reverse order as a dynamic programming problem. In the first stage, conditional on the lease terms $\{a,b,p\}$ and initial investment by the tenant of k, the landlord determines the optimal level and timing of reinvestment in the property as a function of states q and s. The landlord's resulting equity value has the expected present value, $V^L(q,s,t|a,b,p,k)$. Between reinvestment dates, this present value satisfies the pde:

$$0=\frac{1}{2}\sigma^2 s^2 V^L_{ss} + \mu s V^L_s - \delta q V^L_q + V^L_t - \iota V^L + R + \lambda Qs \tag{2}$$

The level of retail services at which reinvestment optimally occurs is $\underline{q}^*$. At this point, the landlord reinvests to increase the level of retail services to $\overline{q}^*$. Both the new level of retail services and the point at which reinvestment occurs maximize the landlord's net investment gain:

$$0=\max_{\underline{q},\overline{q}}\left\{V^L(\overline{q},s,t) - k\overline{q}^{\gamma} e^{\rho t} - V^L(\underline{q},s,t)\right\} \tag{3}$$

The landlord's minimum equity value occurs if per unit sales become zero. In this case, the tenant pays the base rent of $akq^{\gamma}e^{\rho t}$ in perpetuity with no incentives for landlord reinvestment.[7] The resulting lower bound on landlord equity value is:

$$V^L(q,0,t)=\frac{akq^{\gamma}e^{\rho t}}{\gamma\delta+\iota-\rho} \tag{4}$$

where parameter values are chosen to satisfy $\gamma\delta+\iota-\rho > 0$.

The tenant's equity value is similarly valued. The tenant has expected present value, $V^T(q,s,t|a,b,p,k)$. Between reinvestment points, this present value satisfies the pde:

$$0=\frac{1}{2}\sigma^2 s^2 V_{ss}^T + \mu s V_s^T - \delta q V_q^T + V_t^T - \iota V^T + \pi Q s - R \tag{5}$$

At the landlord's optimal reinvestment point, the tenant's present value must be continuous:

$$V^T(\overline{q}^*,s,t)=V^T(\underline{q}^*,s,t) \tag{6}$$

Finally, if sales reach zero, the tenant is obligated to make the fixed lease payment. This obligation mirrors the landlord's equity value stated in equation (4):

$$V^T(q,0,t)=\frac{-akq^{\gamma}e^{\rho t}}{\gamma\delta+\iota-\rho} \tag{7}$$

[7] For simplicity we assume a "credit" tenant, in the sense that the tenant has other resources to fund the contracted rental payments should sales become zero. Alternatively, payoffs between zero and the perpetual base rent could be considered as a result of zero sales, a change that would not alter the basic problem structure.

The second stage problem is to determine the landlord's optimal initial investment, q_0^*. Although might expect initial investment to equal $\bar{q}^*$, this outcome is not guaranteed since, unlike the first-stage reinvestment problem, there is no opportunity cost of reinvestment as related to $V^L(\underline{q},s,t)$. Consequently, initial landlord investment is specified by:

$$W^L(a,b,p) = \max_{q_0}\left\{V^L(q_0,1,0) - kq_0^\gamma\right\} \tag{8}$$

where initial sales are normalized to 1 and $W^L(a,b,p) \geq 0$ is imposed as a participation requirement.

The third stage of the problem involves determination of the tenant's optimal initial investment, k^*. Conditional on the lease terms $\{a,b,p\}$, at time $t=0$ the tenant chooses the initial quantity, k, to maximize its present value:

$$W^T(a,b,p) = \max_{k}\left\{V^T(k,q_0^*,1,0) - \alpha k^\beta\right\} \tag{9}$$

and where a participation constraint of $W^T(a,b,p) \geq 0$ is imposed. The landlord's equity has the resulting value, $W^L(a,b,p) = V^L(k^*,q_0^*,1,0) - k^* q_0^{*\gamma}$.

The fourth stage of the model is a determination of the optimal lease contract. The optimal contract, $\{a^*,b^*,p^*\}$, maximizes a weighted average of the landlord's and tenant's initial equity value:

$$\max_{a,b,p}\left\{\omega W^L(a,b,p) + (1-\omega)W^T(a,b,p)\right\} \tag{10}$$

with $0 \le \omega \le 1$. The weight ω reflects the bargaining power of the landlord relative to the tenant. We would expect dominant landlords and those who provide space in highly desirable locations to have more bargaining power relative to tenants. Larger tenants and those with nationally recognized brands will typically have more bargaining power relative to landlords.

The first-best solution is one that maximizes the joint equity value of the landlord and tenant, $\hat{V} = V^L + V^T$. In between investment, joint equity value satisfies:

$$0 = \frac{1}{2}\sigma^2 s^2 \hat{V}_{ss} + \mu s \hat{V}_s - \delta q \hat{V}_q + \hat{V}_t - \iota \hat{V} + Qs(\lambda + \pi) \tag{11}$$

If sales become zero, so does the joint equity value. First-best reinvestment is triggered by the retail service level, $\underline{\underline{q}}$. At that point, reinvestment occurs to increase the level of retail services to $\overline{\overline{q}}$. The first-best reinvestment policy is thus the pair $\left\{\underline{\underline{q}}^*, \overline{\overline{q}}^*\right\}$ that solves the problem:

$$0 = \max_{\underline{\underline{q}}, \overline{\overline{q}}} \left\{ \hat{V}(\overline{\overline{q}}, s, t) - k\overline{\overline{q}}^{\gamma} e^{\rho t} - \hat{V}(\underline{\underline{q}}, s, t) \right\} \tag{12}$$

Optimal initial investment in the first-best case can be analyzed in two stages. First, the optimal initial q is determined, denoted as q_{00}^*. This initial value is governed by:

$$\max_{q_{00}} \left\{ \hat{V}(\hat{k}, q_{00}, 1, 0) - \hat{k} q_{00}^{\gamma} \right\} \tag{13}$$

Then, the optimal $\hat{k}$, denoted as $\hat{k}^*$, is determined to maximize joint equity value at t=0. This initial investment solves the problem:

$$\max_{\hat{k}} \left\{ \hat{V}(\hat{k}, q^*_{00} 1,0) - \alpha \hat{k}^{\beta} \right\} \tag{14}$$

There is no one unique lease contract in the first-best case, since lease payments are a zero-sum wealth transfer between agents. Rather, the tenant's profit margin parameter, π, together with the landlord's externality parameter, λ, determine joint equity value as a function of total sales. Total sales in turn depend on the optimal initial investment, $Q^*_{00} = \hat{k}^* \; q^*_{00}$, and the subsequent optimal reinvestment levels, $\left\{ \underline{\underline{q}}^*, \overline{\overline{q}}^* \right\}$, that fall out of the earlier optimization problems. The resulting time t=0 total equity value in the first-best case is:

$$\hat{V}(\hat{k}^*, q^*_{00}, 1,0) - \hat{k}\left(q^*_{00}\right)^{\gamma} - \alpha\left(\hat{k}^*\right)^{\beta} \tag{15}$$

IV. Transformation and Solutions

The equity value equations (2) and (5) are partial differential equations that do not offer obvious general solutions. The objective of this section is to transform the partial differential equations into ordinary differential equations and then to solve these equations in order to characterize the optimal investment and contracting policies.

To start, recognized that the equity value functions, V, and the landlord (re)investment cost functions are homogeneous of degree one in k. To exploit this relation, define the transform variable as $y=q^{1-\gamma}se^{-\rho t}$ and let $kF^i(y)=q^{-\gamma}e^{-\rho t}V^i(q,s,t)$, where i=0 indicates the landlord and i=1 indicates the tenant. Value dynamics depend on whether an overage rent is in effect. For $0 \leq y \leq b$, no overage rent is paid. We indicate this region with j=0. Otherwise, for $y>b$, an overage rent is in effect in addition to the base rent. This region is indicated by j=1. Lastly, construct the three constants: $\theta = \frac{\gamma}{\gamma - 1}$; $\psi = \delta + \iota - \mu$; $\phi = \delta\gamma + \iota - \rho$, all of which are positive.

With this transformation, and using the indicated notation, equity value equations (2) and (5) simplify to the following ode:

$$0 = \frac{1}{2}\sigma^2 y^2 F^i_{yy} + (\phi - \psi) y F^i_y - \phi F^i + \psi\zeta_{i,j} y + \phi\xi_{i,j} \qquad (16)$$

where $\zeta_{i,j} = [(1-i)\lambda + i\pi + p(1-2i)j]/\psi$ and $\xi_{i,j} = (1-2i)(a-pbj)/\phi$ for i,j=0,1.

The zero sales boundary conditions expressed in equations (4) and (7) become:

$$F^i(0) = \xi_{i,0} \qquad (17)$$

for i=0,1.

Transformed optimal reinvestment by the landlord, as originally expressed in equation (3), is restated as:

$$0 = \max_{\underline{y},\bar{y}} \left\{ \underline{y}^{-\theta}\left(F^0(\underline{y}) - 1\right) - \bar{y}^{-\theta} F^0(\bar{y}) \right\} \qquad (18)$$

Note that, because y moves inversely with q, $\underline{y}^*$ is the new level for y immediately after reinvestment occurs and $\bar{y}^*$ is the threshold at which reinvestment is triggered.

The tenant's continuity condition at the point of reinvestment, as originally expressed in equation (6), becomes:

$$\left(\underline{y}^*\right)^{-\theta} F^1(\underline{y}^*) = \left(\overline{y}^*\right)^{-\theta} F^1(\overline{y}^*) \tag{19}$$

Optimal initial investment by the landlord, as seen in equation (8), is reexpressed as:

$$G^0(a,b,p) = \max_{y_0} \left\{ k y_0^{-\theta} \left(F^0(y_0) - 1 \right) \right\} \tag{20}$$

Equation (9), the tenant's optimal initial investment, is transformed to:

$$G^1(a,b,p) = \max_{k} \left\{ k \left(y_0^* \right)^{-\theta} F^1(y_0^*) - \alpha k^{\beta} \right\} \tag{21}$$

in which k^* is the optimal value.

Finally, from equation (10) the optimal contract, $\{a^*, b^*, p^*\}$ is determined by:

$$\max_{a,b,p} \left\{ \omega G^0(a,b,p) + (1-\omega) G^1(a,b,p) \right\} \tag{22}$$

The first-best problem also simplifies as a result of the transformation. The transformed first-best value dynamics becomes:

$$0 = \frac{1}{2} \sigma^2 y^2 \hat{F}_{yy} + (\phi - \psi) y \hat{F}_y - \phi \hat{F} + (\lambda + \pi) y \tag{23}$$

The absorbing boundary is such that $\hat{F}(0)=0$. Optimal reinvestment is determined analogously to the second-best form expressed in equation (18). Optimal initial investment in the first-best case similarly follows the second-best forms expressed in equations (20) and (21). Total value at time *t*=0 conditional on the optimal investment and reinvestment policies is:

$$\hat{G}=\hat{k}^{*}\left(y_{00}^{*}\right)^{-\theta}\left[\hat{F}(y_{00}^{*})-1\right]-\alpha\left(\hat{k}^{*}\right)^{\beta} \tag{24}$$

The general solutions to equation (16) are:

$$F^{i}(y)=\begin{cases}F_{i,0}(y), & 0\le y\le b\\ F_{i,1}(y), & y>b\end{cases} \tag{25}$$

with

$$F_{i,j}(y)=A_{i,j}^{1}y^{\eta_1}+A_{i,j}^{2}y^{\eta_2}+\zeta_{i,j}y+\xi_{i,j} \tag{26}$$

for *i,j*=0,1. $A_{i,j}^{1}$ and $A_{i,j}^{2}$ are constants to be determined and $\eta_1,\eta_2=\left[\frac{1}{2}-\frac{\phi-\psi}{\sigma^2}\right]\pm\sqrt{\left[\frac{1}{2}-\frac{\phi-\psi}{\sigma^2}\right]^2+\frac{2\phi}{\sigma^2}}$. It follows from the definitions of ϕ and ψ that $\eta_1>1$ and $\eta_2<0$.

Particular solutions require solving for the constants $A_{i,j}^{1}$ and $A_{i,j}^{2}$, *i,j*=0,1, as well as optimal reinvestment values, $\underline{y}^{*}$ and $\overline{y}^{*}$. We will consider the landlord (*i*=0) problem first. Because there is an absorbing lower bound, $A^{2}_{0,0} = 0$. This leaves five unknowns, $A^{1}_{0,0}$, $A^{1}_{0,1}$, $A^{2}_{0,1}$, $\underline{y}^{*}$, $\overline{y}^{*}$, implying that five equations are required for identification.

At the overage threshold value, b, value matching and smooth pasting are required. This implies that:

$$F_{00}(b) = F_{01}(b) \text{ and } F'_{00}(b) = F'_{01}(b) \tag{27}$$

Additional restrictions are provided by equation (18) and the associated first-order conditions. To facilitate the analysis, we will assume that a base rental contract is in effect initially and immediately after reinvestment. That is, $y_0 \leq b$ and $\underline{y} \leq b$ is required. This is what is seen in practice (base rents only at execution of retail lease contract), and is a constraint that we don't expect to bind for realistic parameter values. Then, because the overage rent region also includes base rent, we will require that p=0 when the reinvestment trigger point, $\overline{y}$, is such that $\overline{y} \leq b$; otherwise, for $\overline{y} > b$, p can assume positive or negative values. This stipulation allows for the general possibility that reinvestment can be triggered in either the base rental or the overage rental regions, while also streamlining the analysis.

With this structure, equation (18) can be rewritten as:

$$0 = \max_{\underline{y},\overline{y}} \left\{ \underline{y}^{-\theta}\left(F_{00}(\underline{y}) - 1\right) - \overline{y}^{-\theta} F_{01}(\overline{y}) \right\} \tag{28}$$

The associated first-order conditions are:

$$\underline{y}^{*} F'_{00}(\underline{y}^{*}) - \theta\left(F_{00}(\underline{y}^{*}) - 1\right) = 0 \tag{29a}$$

$$\overline{y}^* F'_{01}(\overline{y}^*) - \theta F_{01}(\overline{y}^*) = 0 \tag{29b}$$

Finally, the solution must also satisfy the second-order conditions:

$$\left(\underline{y}^*\right)^2 F''_{00}(\underline{y}^*) - 2\theta \underline{y}^* F'_{00}(\underline{y}^*) + \theta(\theta+1)\left(F_{00}(\underline{y}^*) - 1\right) < 0 \tag{30a}$$

$$\left(\overline{y}^*\right)^2 F''_{01}(\overline{y}^*) - 2\theta \overline{y}^* F'_{01}(\overline{y}^*) + \theta(\theta+1) F_{01}(\overline{y}^*) > 0 \tag{30b}$$

Equations (27) through (29) identify the system. By taking the general solution in (26), solving for equation (29a), and subsequently applying the value-matching and smooth-pasting conditions stated in (27), we see that:

$$A^1_{00}(\underline{y}^*) = \frac{\left(\underline{y}^*\right)^{-\eta_1}}{\eta_1 - \theta}\left[(\theta-1)\zeta_{0,0}\underline{y}^* + \theta\left(\xi_{0,0} - 1\right)\right] \tag{31a}$$

$$A^1_{01}(\underline{y}^*) = A^1_{00}(\underline{y}^*) - A^2_{01} b^{\eta_2 - \eta_1} - p b^{1-\eta_1}\left(\frac{1}{\psi} - \frac{1}{\phi}\right) \tag{31b}$$

$$A^2_{01} = p\frac{b^{1-\eta_2}}{\eta_1 - \eta_2}\left[(1-\eta_1)\frac{1}{\psi} + \eta_1\frac{1}{\phi}\right] \tag{31c}$$

The parameter space must be such that these values are positive. Note that A^1_{00} and A^1_{01} depend on the optimal reinvestment point, $\underline{y}^*$, which is yet to be determined. Also observe that $A^1_{01} = A^2_{01}$=0 when p=0. When this condition holds, optimal reinvestment occurs (or occurs as if) in the base rental region, as you would expect when percentage rents are not being paid.

By referencing the first-order conditions in (29), we obtain the following relations for $\underline{y}^*$ and $\overline{y}^*$:

$$\underline{y}^* = \overline{y}^* \left[\frac{F_{01}(\overline{y}^*)}{F_{00}(\underline{y}^*) - 1} \right]^{\frac{-1}{\theta}} \tag{32a}$$

$$\overline{y}^* = \theta \left[\frac{F_{01}(\overline{y}^*)}{F'_{01}(\overline{y}^*)} \right] \tag{32b}$$

which must satisfy the second-order conditions stated in (30). Equations (31) and (32) close the system, with resulting solutions providing the landlord's equity value and optimal reinvestment policy.

With the necessary variables identified (the three valuation constants and the two reinvestment bounds), conditions for optimal *initial* investment by the landlord follow directly from the reinvestment policy relations. Specifically, beginning with equation (20), we see that optimal initial investment, y_0^*, must satisfy the first-order condition expressed in equation (29a) as well as the second-order condition (30a) (with y_0^* replacing $\underline{y}^*$ in both equations). Substituting the specific solution to $F_{00}(y)$ into these equations provides the relations needed for determining optimality. The first-order condition for y_0, for example, must specifically satisfy:

$$A_{00}^{1} y_0^{\eta_1}(\eta_1 - \theta) + \frac{\lambda y_0}{\psi}(1-\theta) + \theta\left(1 - \frac{a}{\phi}\right) = 0 \tag{33}$$

Tenant equity value is stated generally in equation (26). As with the landlord problem, because there is an absorbing lower bound, $A_{1,0}^{2}$ =0. Furthermore, because $\underline{y}^*$ and $\overline{y}^*$ are exogenously determined by the landlord, there are only the three constants, $A^1_{1,0}$, $A^1_{1,1}$, $A^2_{1,1}$, to be determined to obtain specific solutions for tenant equity value.

The identifying relations are first, the value-matching and smooth pasting conditions required when moving from a fixed-rent-only regime to a fixed-rent and percentage rent regime:

$$F_{10}(b) = F_{11}(b) \text{ and } F'_{10}(b) = F'_{11}(b) \tag{34}$$

And second, there is the value-matching condition required at the point of reinvestment:

$$\left(\underline{y}^*\right)^{-\theta} F_{10}(\underline{y}^*) = \left(\overline{y}^*\right)^{-\theta} F_{11}(\overline{y}^*) \tag{35}$$

After substituting the general solutions for $F_{1j}(y)$ into equations (34) and (35), it is immediately apparent that relations are all linear in the unknown constants, $A^1_{1,0}$, $A^1_{1,1}$, $A^2_{1,1}$. This simplifies the problem considerably, where we find that:

$$\left[A^1_{1,0} \quad A^1_{1,1} \quad A^2_{1,1}\right] = \left[\frac{pb(\psi - \phi)}{\psi\phi} \quad \frac{-p}{\psi} \quad \chi\right] \Xi^{-1} \tag{36}$$

in which

$$\chi = \frac{\left(\overline{y}^*\right)^{1-\theta}(\pi - p) - \left(\underline{y}^*\right)^{1-\theta}\pi}{\psi} - \frac{(a - pb)\left(\overline{y}^*\right)^{-\theta} - a\left(\underline{y}^*\right)^{-\theta}}{\phi} \tag{37a}$$

and

$$\Xi = \begin{bmatrix} b^{\eta_1} & \eta_1 b^{\eta_1 - 1} & \left(\underline{y}^*\right)^{\eta_1 - \theta} \\ -b^{\eta_1} & -\eta_1 b^{\eta_1 - 1} & -\left(\overline{y}^*\right)^{\eta_1 - \theta} \\ -b^{\eta_2} & -\eta_2 b^{\eta_2 - 1} & -\left(\overline{y}^*\right)^{\eta_2 - \theta} \end{bmatrix} \tag{37b}$$

With tenant equity value identified, we can find the tenant's optimal initial investment. Because the landlord value and cost functions are homogeneous in *k*, initial investment by the tenant does not affect the subsequent optimal investment and reinvestment policies of the landlord. Consequently, the tenant undertakes a simple (myopic) optimization with respect to its initial investment, *k*, as stated in equation (21). This results in

$$k^* = \left[\frac{\left(y_0^*\right)^{-\theta} F_{10}(y_0^*)}{\alpha\beta} \right]^{\frac{1}{\beta - 1}} \tag{38}$$

which also satisfies the necessary second-order condition.

The last step in the second-best problem is to use equation (22) to solve for the optimal contract, $\{a^*, b^*, p^*\}$, conditional on $\underline{y}^*$, $\overline{y}^*$, y_0^* and k^*. This can be done numerically with straightforward application of iterative techniques.

In the case of first-best, the general solution for equity value expressed in equation (23) is given by:

$$\hat{F}(y) = Ay^{\eta_1} + (\zeta_{0,0} + \zeta_{1,0})y \tag{39}$$

where *A* is a constant to be determined.

The value matching condition analogous to that stated in equation (28) can be used to identify the system in which reinvestment boundary values, $\underline{\underline{y}}^{*}$ and $\overline{\overline{y}}^{*}$, are to be determined in addition to A. Value-matching requires that:

$$\left(\underline{\underline{y}}^{*}\right)^{-\theta}\left(\hat{F}(\underline{\underline{y}}^{*})-1\right)=\left(\overline{\overline{y}}^{*}\right)^{-\theta}\hat{F}(\overline{\overline{y}}^{*}) \tag{40}$$

which produces:

$$A(\underline{\underline{y}}^{*},\overline{\overline{y}}^{*})=\frac{\left(\zeta_{0,0}+\zeta_{1,0}\right)\left(\left(\overline{\overline{y}}^{*}\right)^{1-\theta}-\left(\underline{\underline{y}}^{*}\right)^{1-\theta}\right)+\left(\underline{\underline{y}}^{*}\right)^{-\theta}}{\left(\underline{\underline{y}}^{*}\right)^{\eta_1-\theta}-\left(\overline{\overline{y}}^{*}\right)^{\eta_1-\theta}} \tag{41}$$

The associated first-order (smooth-pasting) conditions that follow from (40) can used to determine $\underline{\underline{y}}^{*}$ and $\overline{\overline{y}}^{*}$. After deriving these conditions, and with some algebraic manipulation and simplification, we find that:

$$\underline{\underline{y}}^{*}=-\gamma\left[\left(\zeta_{0,0}+\zeta_{1,0}\right)\left(\left(\frac{\underline{\underline{y}}^{*}}{\overline{\overline{y}}^{*}}\right)^{\eta_1-1}-1\right)\right]^{-1} \tag{42}$$

$$\left(\frac{\underline{\underline{y}}^{*}}{\overline{\overline{y}}^{*}}\right)=\left[\theta(\eta_1-1)\left[(\theta-1)\eta_1\left(\frac{\underline{\underline{y}}^{*}}{\overline{\overline{y}}^{*}}\right)^{\eta_1-1}+\eta_1-\theta\right]^{-1}\right]^{\frac{1}{1-\theta}} \tag{43}$$

in which $\underline{\underline{y}}^*$ and $\overline{\overline{y}}^*$ also must satisfy necessary second order conditions.

The final step in the first-best case is to determine the optimal initial investment quantities of y_{00}^* and $\hat{k}^*$. By referencing equations (20) and (21), and substituting the specific solution for $\hat{F}(y)$ into the equations, we find that y_{00}^* and $\hat{k}^*$ must satisfy the following relations:

$$A(\eta_1-\theta)y_{00}^{\eta_1}+(\zeta_{0,0}+\zeta_{1,0})(1-\theta)y_{00}+\theta=0 \tag{44}$$

$$\hat{k}=\left[\frac{\left[A\left(y_{00}^*\right)^{\eta_1-\theta}+(\zeta_{0,0}+\zeta_{1,0})\left(y_{00}^*\right)^{1-\theta}\right]}{\alpha\beta}\right]^{\frac{1}{\beta-1}} \tag{45}$$

where a second-order condition is also required to hold for y_{00}^*.

V. Optimal Lease Contract and Investment Policy

For reference, the appendix summarizes the numerous parameters and variables contained in the model. The optimal investment and contracting problem is solved as a dynamic program for each stage (reinvestment, initial investment, contract). A modified Newton-Raphson method is used to calculate the landlord's optimal investment and reinvestment policies. The optimal contract is obtained with the application of the Simulated Annealing Method, which is a probabilistic algorithm used to solve global optimization problems.[8]

[8] See Kirkpatrick et al. (1983). Each step of the Simulated Annealing Method replaces the current solution with a "nearby" solution chosen with a probability that depends on the difference between the corresponding value functions and a global parameter that is gradually decreased during the solution process. Introduction of the global parameter in combination with probabilistic replacement saves the method from becoming stuck at local optimums, which is a standard problem with other solution methods.

V.A. Base Case

To assess the optimal contract and reinvestment policy, numerical solutions are obtained for a variety of eligible parameter value constellations. For purposes of specifying a *base case*, the following parameter values are employed: γ=2.0; δ=.03; ι=.05; ρ=.03; μ=.03; σ=.10; α=1.0; β=2.0; π=.30; ω=.50. We believe these parameters values to be realistic and representative of those typically encountered in a retail operating environment.

For the base case we will vary the externality parameter, λ. We do this because the effects of inter-store externality in a shopping center setting has sparked considerable interest in the literature, and because the data cited previously provide useful empirical relations that can be used to assess our model.

Figure 1 displays optimal contract outcomes as a function of λ. Panel A shows variation in the initial base rent, akq_0^γ, as a percentage of initial sales, Q;[9] Panel B displays variation in overage percentage rent variable, p; and Panel C shows how initial investment, y_0, in relation to the overage rent threshold value, b, varies depending on external benefits accruing to the landlord.

Figure 1 Here

As seen in Panels A and B of Figure 1, both base rent as a percentage initial sales and the overage rent percentage decrease monotonically as externality benefits increase. Indeed, as λ increases toward a value of .20, base rent approaches zero and the overage rent percentage becomes negative to provide a subsidy back to the tenant.

To understand why this occurs, Figure 2 displays initial investment of the tenant in Panel A and initial investment of the landlord in Panel B as a function of λ. Total initial sales is displayed in Panel

[9] Recall that unit sales, s, is normalized to 1 at time t=0.

C. These graphs show that initial investment by the tenant and total initial sales are increasing in λ, whereas initial investment by the landlord is decreasing in λ.

Figure 2 Here

As λ increases, revenues to the landlord increase in proportion to total sales. Total initial sales are determined by the product of the tenant's and landlord's initial investments. Thus, as external benefits increase, the landlord decreases rent levels to encourage initial tenant investment to increase the level of total sales. Greater initial investment by the tenant allows the landlord to decrease its initial investment to some extent, while still resulting in an increase in total initial sales.

External benefits accruing to the landlord are therefore a *substitute* for base and percentage rents. The surprising aspect of this result is not that externality-generating tenants pay little or no base and percentage rent, but rather that our model predicts that we should commonly observe percentage rent lease contracts when few or no external benefits accrue to the landlord. Although we do observe this outcome with retail property, including stand-alone retail stores, we generally don't observe this contract with other types of commercial property.

Why not? I conjecture it has to do with: i) the verifiability of total site productivity (sales) in a retail store setting, ii) the relative importance of initial coinvestment by the retail tenant (advertising, customized build out) and the landlord (provision of location and infrastructure upon which to conduct retail sales activity), and iii) the need to provide incentives to the landlord to repeatedly reinvest in the asset to restore and enhance productivity. Low-cost verifiability of production is difficult or impossible in non-retail settings, and landlord (re)investment does not typically exert a first-order effect on tenant productivity.

Panel C of Figure 1 measures how far the overage rental component is from being in-the-money at contract execution. Complementary measures of relative distance are displayed in Figure 3. Panel A shows expected time to hitting the overage rent threshold conditional on initial investment by the landlord and Panel B displays the probability of hitting the threshold within 3 or 5 years. As seen in these figures, the distance measures are non-monotonic in λ, where distance to the overage rent threshold initially increases and then decreases as external benefits increase.

Figure 3 Here

To better understand this result, observe that Panel A of Figure 3 indicates relatively long expected times to hitting the overage rental threshold of 12 to 17 years, suggesting that percentage rents are a lower-powered incentive mechanism. However, Panel B indicates that there are non-trivial probabilities of hitting the threshold within a relatively short period of time, implying that overage rents are higher-powered than the expected hitting time metric might suggest. In the case of higher-than-expected sales growth, the tenant does not mind paying these rents since higher profits can subsidize the overage rent payments.

Overage rents are therefore useful in aligning the incentives of both agents for a whole range of external benefits. When λ is smaller, significant overage rent percentages are required to encourage reinvestment by the landlord as a result of strong sales growth. When λ is larger, the landlord has less need for percentage rents to incentivize reinvestment as a result of high sales. The landlord as a result reduces the overage rent percentage (and eventually reverses it) to encourage initial investment by the tenant, the benefits of which accrue to the immediate benefit of the landlord. Incentives are thus aligned by varying the contract variables a and p, while keeping the distance to the overage threshold, y_0/b, relatively constant.

Wheaton (2000) rationalizes percentage rent contracting as an incentive compatibility mechanism that aligns the *ex post* incentives of the landlord to maintain a proper mix of shopping center tenants. Our model is similar to Wheaton's in that we both offer models of optimal contracting with externalities when *ex post* actions of the landlord matter. Where we different is that Wheaton's results rely on multi-tenant inter-store external effects. Retail tenants agree to insert high-powered incentives *vis-à-vis* the overage rent component to reduce *ex post* opportunism by the landlord when reconfiguring the tenant mix. Overage rents are lower-powered in our model, since initial investment by the tenant is a crucial input to the landlord's profit function. And, as noted previously, it is the absence of spillover effects (not common agency *per se*) that result in the necessity of a percentage rent contract in our model.

In Figure 4 we display optimal reinvestment policy outcomes as they depend on λ. First, we note that initial investment by the landlord, q_0^*, equals the optimal reinvestment quantity, $\overline{q}^*$, for all λ. This implies no difference in investment-reinvestment policy in terms of quantities. In panel A of Figure 4 we display the ratio of $\overline{q}^*$ to $\underline{q}^*$, which measures the relative distance between reinvestment trigger points. Panel B displays the expected time between reinvestment, which is a complementary measure of distance.

Figure 4 Here

The expected time to reinvestment are seen to decline monotonically as λ increases. External benefits accruing to the landlord provide strong incentives for reinvestment, since these benefits accrue in direct proportion to total sales (which depend on the control variable, q). When λ is smaller, however, percentage rents are the driving force for reinvestment. In this case, although fixed rents paid by the tenant are adjusted upwards as a result of reinvestment, reinvestment resets the lease contract so that overage rents are again out-of-the-money, which reduces the landlord's incentive to undertake reinvestment in the first place.

V.B. Comparison to First-Best

Initial investment and reinvestment policies can be compared to first-best outcomes, as seen in Figure 5 as a function of λ. Panel A of Figure 5 shows initial investment by the tenant as a proportion of first-best tenant investment, $k^*/\hat{k}^*$; Panel B displays a similar ratio based on initial landlord investment, q_0^* / q_{00}^*; and Panel C graphs the expected time to reinvestment in the first-best case as a proportion of the expected time to reinvestment in the dual-agency case.

Figure 5 Here

These graphs show that the tenant underinvests relative to first-best and that the landlord overinvests, both initially and dynamically. Tenant underinvestment is most severe when externalities are small or non-existent. This is the result of a relatively high base rent and overage rent percentage required by the landlord to compensate for the absence of external flow benefits.

To compensate for tenant underinvestment, the landlord overinvests at time t=0. Interestingly, the relative amount of overinvestment does not vary much as a function of λ. This follows because, even though higher marginal initial landlord investment is not required to offset increases to tenant investment as λ increases, greater external benefits accruing (exclusively) to the landlord provide powerful incentives to invest to increase revenue flows. This effect causes total initial investment, $k^* q_0^*$, to increase relative to the first-best quantity as λ increases.

The landlord over*reinvests* as a result of external benefits accruing to its benefit. Overreinvestment happens in two ways. First, because the quantity of initial investment equals the quantity of reinvestment, Panel B of Figure 5 also indicates overreinvestment in quantities. Panel C shows that overreinvestment also occurs dynamically, where expected times to reinvestment decrease relative to first-best as λ increases. Increasing external benefits distort landlord incentives to reinvest, both in quantities and dynamically, since reinvestment has a powerful effect on landlord revenue flows without any countervailing negative effects from the tenant side. Indeed, the tenant prefers this outcome, particularly as p becomes negative.

Our results suggest that investment incentive problems are only partially solved with revenue sharing contracts, and lead to the somewhat surprising result that overinvestment is most problematic when externalities are the largest. This outcome may explain why there persists a separation between landlord and tenant responsibilities with stand-alone retailers (which generate little or no externality, and hence result in more efficient investment outcomes), and in part why local governments tend to get more involved in development in which agglomerative impacts are largest.

V.C. Comparative Statics

Comparative static relations are considered for several prominent model parameters. Table 4 displays relations for the following specific parameters: ω (landlord bargaining power), δ (rate of depreciation in q), γ (landlord cost of investment), μ (drift rate in sales), σ (volatility of sales), π (tenant profit margin). The endogenous quantities considered are: a^* (base rent multiplier), p^* (overage rent percentage), k^* (tenant investment), q_0^* (landlord investment), and $\overline{q}^* / \underline{q}^*$ (distance between reinvestment trigger points). Base case parameter values are used to generate the comparative static results. We have examined the comparative static relations for different values of λ between 0.0 and 0.2 to verify consistency of the results.

Table 4 Here

Consider first the *relative degree of bargaining power*, ω, that exists between the landlord and the tenant. We note that externality and bargaining power are often correlated in the data, where tenants that generate greater externalities are thought to have relatively more bargaining power. We would expect to see frequent exceptions to this rule of thumb, however. In certain markets or at certain desirable locations (in which the landlord is a spatial monopolist), landlords will enjoy market power relative to tenants—even with those tenants that might be highly desirable due to their brand name recognition. Conversely, market conditions or location might dictate that even small, non-externality-generating tenants enjoy considerable bargaining power relative to the landlord.

An increase in the bargaining power of the landlord increases the base rent term, a, and the overage rent percentage, p. The landlord obviously prefers more rental revenue to less rental revenue, which it obtains by increasing both a and p as its bargaining power increases. On the other hand, initial investment by both the tenant and the landlord decrease as landlord bargaining power increases. The tenant decreases initial investment due to the increase in rent payment parameters. The landlord also decreases its initial investment, since rental flow increases at the margin to decrease investment incentives.

These results demonstrate the importance of distinguishing between externality and bargaining related effects. An increase in bargaining power for the landlord increases the base rent parameter, a, to cause a decrease in initial investment. In contrast, an increase in the external flow causes the landlord to decrease base rent in equilibrium (see Figure 2). This facilitates initial investment by the tenant due the importance of the increase in external flows on equity value.

Interestingly, an analysis of data on stand-alone big-box retail operations reveals that overage rents only sometimes appear in the lease contract. Our model provides an explanation for this outcome: In certain cases, landlords may enjoy greater relative bargaining power, allowing them to secure overage rents. In other cases, tenants may have the advantage and, as a result, negotiate to exclude the overage rental feature.

Our no-externality, sharing contract result can also explain sharecropping contracts, where, in our context, the landlord makes major periodic investments in the land to enhance its productivity. In a typical sharecropping setting, we would expect the landlord to possess significant bargaining power, which further enhances the overage rent component of the optimal contract. Our model thus offers an alternative perspective on sharecropping arrangements, which typically rely on risk-sharing and capital constraints to justify observed contracting outcomes.

Reinvestment by the landlord occurs more often as its bargaining power increases. However, as seen by the level of initial investment (which also equals the level of reinvestment), the intensity of reinvestment decreases. Thus, reinvestment occurs more often but with less intensity as ω increases. Increases to both a and p provide incentives for the landlord to invest more often (since revenue flows are higher as a result), but to invest with less intensity since reinvestment costs are convex in quantity.

Next consider the effects of changes in the *rate of depreciation*, δ, as it applies to the quality/quantity of retail services provided by the landlord, q. Generally speaking, an increase in δ has a negative impact on total sales, to the detriment of the retail tenant and the landlord (when it benefits from external flows). It also causes the landlord to have to invest at a higher intensity in order to restore the productivity of the asset. As a result, the landlord increases both the base rent and the overage rent percentage to compensate for the higher rate of depreciation. The increase in rent causes the tenant to decrease its initial investment.

Changes in the *landlord investment cost parameter*, γ, has effects similar to an increase in depreciation. Lower investment costs cause the landlord to decrease the fixed rent and overage rent percentage parameters. This in turn causes the tenant to invest more. In response to the increase in tenant investment, landlord investment is indeterminate. This happens because there are two opposing effects: lower landlord investment cost increases the incentive to invest more, but a strong increase in tenant investment decreases the need to invest initially. Finally, lower investment cost causes reinvestment to occur more frequently.

This comparative static result can help explain the empirical findings of Jacoby and Mansuri (2007), who find that more closely supervised sharecropping tenants are significantly more productive than less closely supervised tenants. In our model, the cost of reinvestment is analogous to the cost of supervision by the landlord. Less costly supervision (investment) allows the landlord to reduce rents and supervise (invest) more often, which significantly increases tenant productivity. The intensity of supervision (investment) is, in general, indeterminate, and will depend on other factors specific to the problem setting.

An increase in the *drift rate of sales*, μ, causes the landlord to decrease the overage rent percentage to compensate for an expected decrease in time to hitting the percentage rent boundary. Because an increase in the drift rate of sales increases the profitability of the retail tenant, the landlord can increase the base rent to its benefit without adversely affecting net tenant profitability. Initial investment for both the landlord and tenant increase to further enhance the effect of increasing sales over time. The steady-state reinvestment ratio, $\overline{q}^* / \underline{q}^*$, increases to compensate for a higher drift rate in sales. Thus, landlord reinvestment occurs less often when the drift rate of sales increases, but with greater intensity.

An increase in the *volatility of sales*, σ, is beneficial for the landlord, as it increases the value of the overage rent option. This causes a decrease in the equilibrium base and overage rent percentage parameters. Increased volatility in sales is also beneficial for the tenant. The combination of lower rent and increased profitability increases initial investment. Initial investment by the landlord is indeterminate, and follows because the increase in tenant investment is typically so strong that the landlord can reduce its investment levels below what one might otherwise expect. The steady-state reinvestment ratio, $\overline{q}^* / \underline{q}^*$, increases to compensate for the fact that an increase in volatility reduces the expected time to reinvestment.

As the *tenant's profit margin*, π, increases, base rent and overage rent percentage increase. An increase in profit margin provides the landlord the opportunity to raise rents without negatively effecting net tenant profitability. This result is consistent with empirical estimation results reported by Wheaton (2000), in which both base rents and overage rent percentages are positively related to sales per square foot after controlling for store size (which proxies for externality/category of sales).[10] Higher rents in turn causes both agents to increase the quantity of initial investment. Finally, reinvestment occurs less often, but with greater intensity, as the tenant's profit margin increases.

VI. Conclusion

We have constructed a model of bilateral trade between an upstream supplier (landlord) that confers property usage rights to downstream producer (tenant). In return for usage rights, the downstream producer pays a base user fee (rent) plus a percentage of verifiable sales production that exceeds an overage threshold value. Our model allows for the possibility that downstream production complements other activities of the upstream supplier to increase its total revenues. The model also incorporates different levels of bargaining power that may exist between agents. In designing an optimal contract, the upstream supplier wants to provide incentives to the downstream producer to make high initial investments while also maintaining its own incentives to reinvest to enhance productivity.

[10] We would caution against interpreting π in our model as indicating a separate "category of retail stores". For example, anchor stores generally have lower sales per square foot than smaller specialty stores, where anchor stores also generally have lower base rents and overage rent percentages in their leases. Sales per square foot in this case correlates closely with externality and relative bargaining power, which are the real causes of differentials in the optimal rental contract, and which generate comparative statics that are consistent with the data in terms of retail store categories. The appropriate way to interpret the comparative static with respect to π is as a change in profitability of a store *within* a particular retail category (such as men's apparel or a jewelry store).

Endogenously determined quantities in our model are optimal contracting terms (base rent, overage rent threshold, overage rent percentage), initial investment by the downstream producer and upstream supplier, and the upstream supplier's reinvestment threshold and quantity. In this paper we specifically consider a retail lease contracting environment. We find that when positive externalities accrue to the benefit of the landlord, they substitute for both base and overage rent in the optimal contract. Lower rents cause higher initial investment by the tenant, which enhances landlord as well as tenant equity value.

Our main finding is that it is the *absence* of external effects that explains the existence of overage rents in settings where production is verifiable and highly dependent on upstream usage rights (e.g., location in the case of land). This allows us to explain the existence of overage rental contract features with stand-alone retail operations and for other settings such as sharecropping and licensing agreements.

Sharecropping and related contracting literature generally presume complete bargaining power on behalf of the landlord. When bargaining power is allowed to vary, we find that strong landlord bargaining power causes an overage rental contract to emerge with higher overage rent percentages in addition to higher base rents. Bargaining power that is more balanced or that favors the tenant results lower overage rent percentages, and can even eliminate the overage rent feature altogether. Variation in bargaining power can thus explain cross-sectional differences in overage rental contract terms, independent of external effects.

Our model also explains other important empirical facts documented with retail lease contracting, including the fact that overage rents are typically well out-of-the-money at contract execution.

We conclude by observing that, while inter-store externalities are certainly relevant to contract design in a multi-agent retail setting, there are other factors such as relative bargaining power and landlord reinvestment incentives that are central to the contracting problem. Indeed, we can explain observed relations in retail contracting without explicit consideration of inter-store externality, and then go beyond existing models to explain overage rent contract features with stand-alone retail operations. Our setting suggests it is the importance of upstream supply to downstream sales production and the verifiability of these sales that are necessary for these sharing contracts to work in practice, as opposed to inter-store externalities *per se*.

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Appendix

Summary of Model Parameters and Variables

Parameters and Variables in the Basic Model

q: Quantity or quality of retail services supplied by the landlord. At time t=0, a choice variable of the landlord, with initial investment of q_0. After time t=0, also a choice variable of the landlord as related to reinvestment. Each reinvestment produces a new quantity/quality of retail services, $\overline{q}$. Reinvestment occurs when quantity/quality of retail services reaches the lower bound, $\underline{q}$.

k: Quantity or quality of retail services supplied by the tenant. A choice variable of the tenant at time t=0.

Q: Total quantity or quality of retail services at any point in time, equal to kq.

s: Sales per unit of retail services. Sales evolve stochastically according to a geometric Wiener process with drift parameter μ and volatility σ. The initial value of unit sales is normalized to 1.

δ: Constant rate of depreciation of the quantity/quality of retail services, q.

ι: The constant riskless rate of interest. This quantity satisfies the following inequality: $\iota>\mu-\delta$.

αk^{β} : Production function for initial investment by the tenant, α>0 and β>1.

kq_0^{γ}: Production function for initial investment by the landlord, γ>1.

$k\overline{q}^{\gamma} e^{\rho t}$: Production function for follow-on reinvestment by the landlord, γ>1 and $0\leq\rho<\delta+\iota-\mu$.

$akq^{\gamma} e^{\rho t}$: Base rent paid by retail tenant, a>0, where a is a choice variable as part of optimal contract determination.

$bkq^{\gamma} e^{\rho t}$: Overage rent threshold value, b>0, where b is a choice variable as part of optimal contract determination.

pQs: Overage rent paid when total sales, Qs, are greater than the overage threshold value, $bkq^{\gamma} e^{\rho t}$. Overage rental percentage, p, is a choice variable as part of optimal contract determination.

R: Total rent paid by retail tenant to landlord at a particular point in time, where $R = R(k,q,s,t) = akq^{\gamma} e^{\rho t} + pMax\{0, Qs - bkq^{\gamma} e^{\rho t}\}$.

πQs: Profits to the retail tenant prior to payment of rent, with profit margin π, $0<\pi<1$, which can depend on the category of sales.

λQs: Externalities captured by the developer/landlord, $\lambda>0$, which accrue in addition to the base rent, R.

$V^L(q,s,t)$: Landlord equity value for $t>0$.

$V^T(q,s,t)$: Tenant equity value for $t>0$.

$W^L(a,b,p)$: Landlord equity value at $t=0$ conditional on optimal initial investment and reinvestment policies of the landlord and tenant. Value is net of the cost of initial landlord investment.

$W^T(a,b,p)$: Tenant equity value at $t=0$ conditional on optimal initial investment and reinvestment policies of the landlord and tenant. Value is net of the cost of initial tenant investment.

ω: Bargaining power of the landlord relative to the tenant at the time of lease contract execution, $0<\omega<1$.

$\hat{V}(q,s,t)$: Aggregate equity value in the first-best problem.

$\underline{\underline{q}}$, $\overline{\overline{q}}$, q_{00}, $\hat{k}$: Optimal reinvestment threshold value, new quantity/quality of retail services from reinvestment, and optimal initial investment values, respectively, that result from first-best solution.

<u>Transform Variables and Further Variable Definitions</u>

Transform variable: $y=q^{1-\gamma}se^{-\rho t}$

Transformed equity value function: $kF^i(y)=q^{-\gamma}e^{-\rho t}V^i(q,s,t)$, $i=0$ indicates the landlord, $i=1$ indicates the tenant.

.

$$\theta = \frac{\gamma}{\gamma - 1}.$$

$\psi=\delta+\iota-\mu.$

$\phi=\delta\gamma+\iota-\rho.$

$\zeta_{i,j}=[(1-i)\lambda+i\pi+p(1-2i)j]/\psi, \quad i,j=0,1.$

$\xi_{i,j}=(1-2i)(a-pbj)/\phi, \quad i,j=0,1.$

$\underline{y}$: Transformed new level for y immediately after reinvestment occurs.

$\overline{y}$: Transformed threshold at which reinvestment is triggered.

y_0: Transformed optimal level of initial landlord investment.

$G^0(a,b,p)$: Transformed landlord equity value function at t=0, net of cost of initial landlord investment.

$G^1(a,b,p)$: Transformed tenant equity value function at t=0, net of the cost of initial tenant investment.

$$F^i(y)=\begin{cases} F_{i,0}(y), & 0 \le y \le b \\ F_{i,1}(y), & y > b \end{cases}.$$

$$\eta_1,\eta_2=\frac{1}{2}-\frac{\phi-\psi}{\sigma^2}\pm\sqrt{\left(\frac{1}{2}-\frac{\phi-\psi}{\sigma^2}-\right)^2+\frac{2\phi}{\sigma^2}}.$$

$A^1_{i,j}, A^2_{i,j}, i,j=0,1$: Constants from particular equity value solutions.

$\hat{F}(y)$: Transformed aggregate equity value for the first-best problem.

A: Constant from particular solution for the first-best problem.

$\underline{\underline{y}}$: Transformed new level for y immediately after reinvestment occurs in the first-best problem.

$\overline{\overline{y}}$: Transformed threshold at which reinvestment is triggered in the first-best problem.

y_{00}: Transformed optimal level of initial investment in the first-best problem.

$\hat{G}(y)$: Transformed aggregate equity value for the first-best problem at t=0, net of cost of initial investment.

Table 1

Base Rent and Overage Rent Percentage Relations

Panel A: Without Store Category Controls

Independent Variable	Dependent Variable: Base Rent	Base Rent	Overage Rent %
Intercept	50.86*** (36.03)	-7.72** (-2.36)	0.068*** (91.95)
Square Footage	-0.00038*** (-7.29)		-2.72E-08*** (-16.45)
Overage Rent %		866.6*** (18.30)	
N	1035	1035	1035
R^2	0.049	0.245	0.208

Panel B: With Store Category Controls

Independent Variable	Dependent Variable: Base Rent	Base Rent	Overage Rent %
Intercept	105.32*** (30.89)	69.53*** (13.12)	0.09*** (40.33)
Square Footage	-0.00011 (-1.58)		-1.43E-07*** (-3.25)
Overage Rent %		395.89*** (8.60)	
N	1035	1035	1035
R^2	0.670	0.692	0.504

*** indicates statistical significance at the 1 percent level and ** indicates significance at the 5 percent level. Store categories: 1=anchor; 2=apparel/accessory; 3=apparel/unisex; 4=apparel/child; 5=apparel/woman/specialty; 6=women’s apparel; 7=men’s apparel; 8=shoes; 9=jewelry; 10=miscellaneous; 11=discount department store; 12=drug and variety; 13=books, gifts; 14=services; 15=home furnishing; 17=hobby, special interest; 18=audio-visual; 20=amusement and theatre; 21=restaurant; 22=specialty food; 23=fast food.

Table 2

Walgreen Stores Categorized by Contract Type and Store Type

	Store Type		
Contract Type	Stand-Alone	Within a Mall	Total
Overage Rent Clause Included	78	10	88
Fixed Rent Only	66	29	95
Total	144	39	193

Table 3

Logit Regression of Contract Type (1=Overage Rent Clause Included)

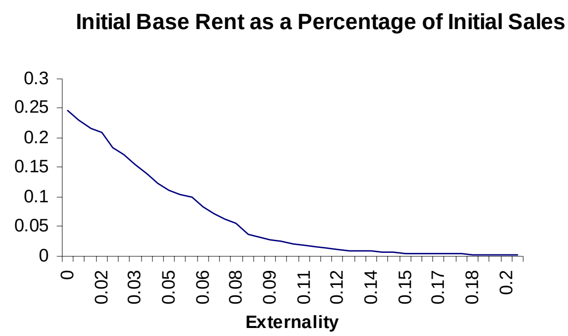
Independent Variables	Model 1	Model 2
Intercept	-0.93^{***} (-2.63)	-3.59^{*} (-1.93)
Stand-Alone	1.07^{***} (2.73)	1.01^{**} (2.07)
Square Footage		3.64E-05 (0.47)
Lease Term		0.0018 (1.33)
Termination Option		0.88 (0.99)
New Structure		-0.43 (-1.13)
Loan-to-Value Ratio		0.63 (0.50)
Loan Term		0.00018 (0.05)
N	183	159
p-Value (LR Test)	0.0044	0.0017

*** indicates statistical significance at the 1 percent level, ** indicates significance at the 5 percent level, and * indicates significance at the 10 percent level.

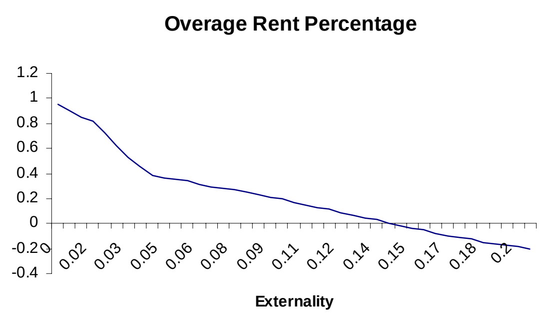
Figure 1

Optimal Contract Terms as a Function of External Effects

Panel A



Panel B



Panel C

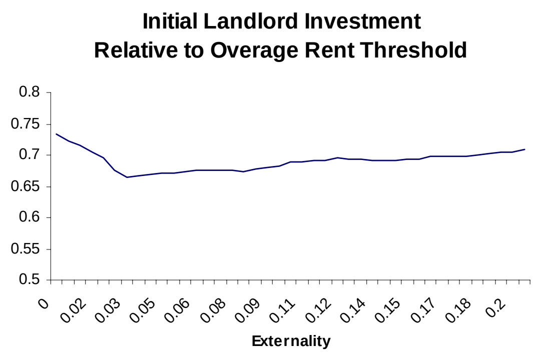
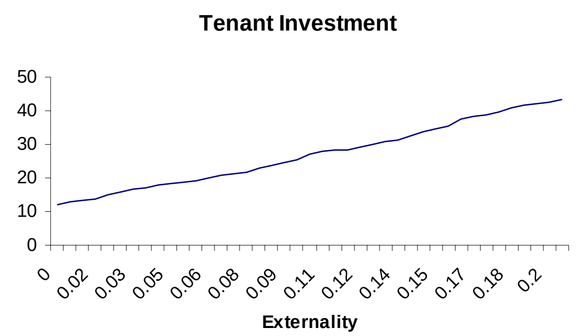


Figure 2

Tenant and Landlord Investment as a Function of External Effects

Panel A



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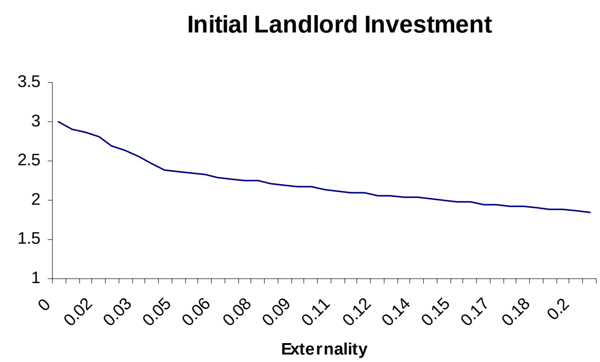

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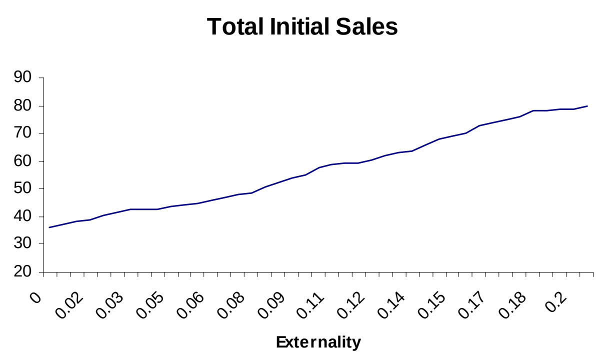
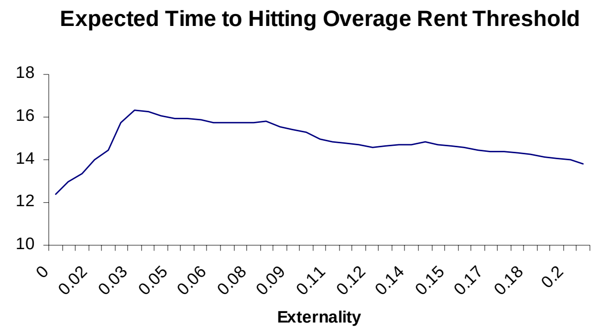


Figure 3

Distance to Overage Rent Threshold

Panel A



Panel B

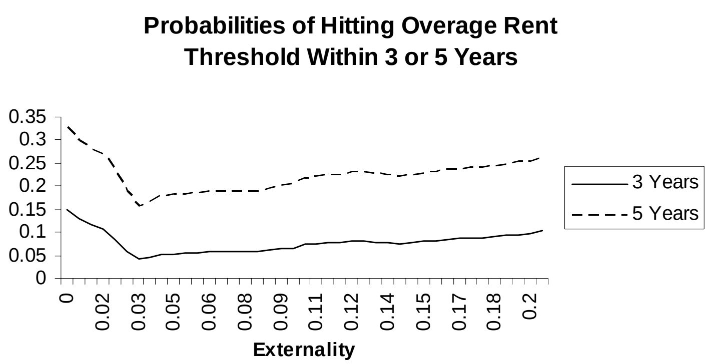
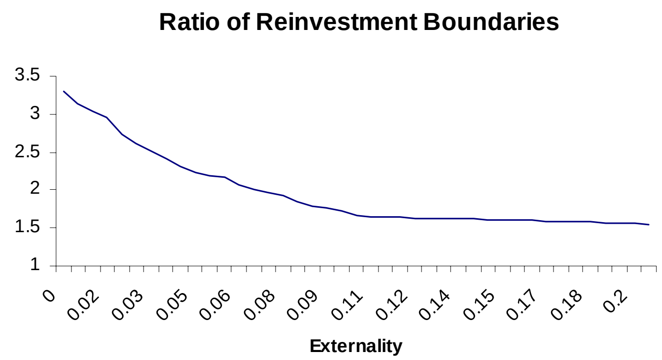


Figure 4

Landlord Reinvestment Measures as a Function of External Effects

Panel A



Panel B

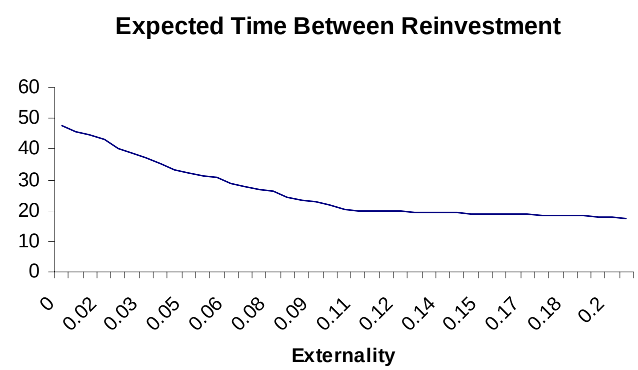
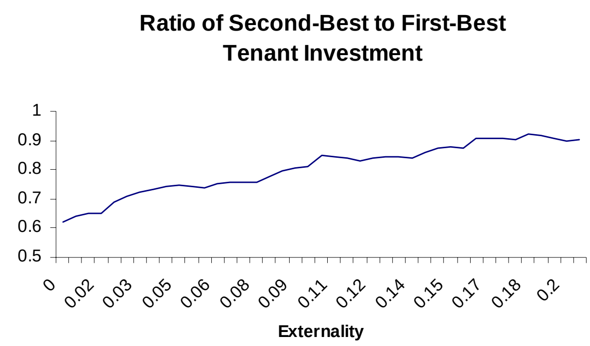


Figure 5

Investment Comparisons to First-Best as a Function of External Effects

Panel A



Panel B

Panel C

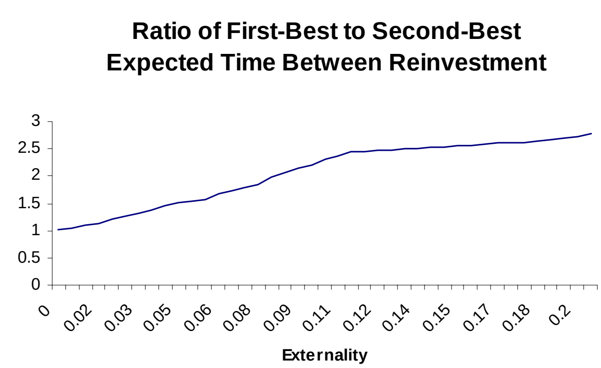


Table 4

Comparative Static Relations for Selected Parameters

Endogenous Variable	*Exogenous Parameter Value* ω	δ	γ	μ	σ	π
a^*	+	+	+	+	–	+
p^*	+	+	+	–	–	+
k^*	–	–	–	+	+	+
q_0^*	–	+	+/–	+	+/–	+
$\bar{q}^* / \underline{q}^*$	–	+	+	+	+	+