

An approach to condition the transition matrix on credit cycle: an empirical investigation of bank loans in Taiwan

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Abstract

This paper presents a formal methodology for gauging the credit risk of financial institutions. Although the proposed model is based on risk-neutral probability and Markov chain model, the model is more elaborate than previous researches. First, we provide a way to condition the transition matrix on credit cycle. Second, we assume that the risk premium is time-variant. Therefore, the default probability and risk premium will be recursively endogenous. Third, we apply the methodology to bank loans in Taiwan, which is never discussed in previous studies. Furthermore, the procedure of the paper for assessing the credit risk of financial institutions is easy to follow and implement. On the whole, the credit risk modeling will be a crucial area for bank regulators in the coming years. For providing an effective credit risk review, we expect that the paper is not only helping to assess the credit risk of financial institutions, but also helping to face the Basel Capital Accord.

1. Introduction

Over a decade ago, the Basel Committee on Banking Supervision (“the Committee”) produced guidelines for determining bank regulatory capital. The objective of this accord was to level the global playing field for financial institutions and protect all against risk in the financial system. In 1988, the Committee issued “the International Convergence of Capital Standard” (or “Capital Accord of 1988”), which established regulations regarding the amount of capital that banks should hold against credit risk. Furthermore, the treatment of market and operational risk were incorporated in 1996 and 1999, respectively. The final version of the Accord was published in 2004 and will be implemented after 2006.

The Committee tried to strengthen capital requirements given the different risk exposures of financial institutions and the wide range of differences in risk management systems. The net effect is that will be able to use internally based risk assessment systems in setting capital requirements. For the credit risk, bank regulators have to develop an effective credit review process used to measure the credit risk of their loans. Therefore, credit risk modeling will be a crucial area for bank regulators in the coming years in order to provide an effective credit risk review, not only helping to detect borrowers in difficulty, but also helping to face the Basel Capital Accord.

Recently, a revolution has been brewing in the way credit risk is both measured and managed. Although the most recent recession hit at different times in different countries, most statistics showed a significant increase in credit risk. In the past, the Committee approach has been described as a “one size fit all”. Now, the Committee allows banks using “internal models” rather than the alternative regulatory (“standardized”) model to assess their credit risk. As a result, most new models and

technologies have emerged applying to analysis credit risk, compared to previous methods. Consequently, accurate credit risk analysis becomes even more important today.

Although credit risk models have developed rapidly such as Jarrow, Lando and Turnbull (1997), Kijima and Komoribayashi (1998) and Wei (2003), they do not consider the credit risk of bank loans. They only apply Markov chain model to measure credit risk of bonds. On the other hand, Lu (2006) has applied Markov chain model to bank loans, he do not consider the dependence in transition matrices, credit cycle, which is an important factor. Despite Wei (2003) incorporate credit cycle into transition matrices, he assumes the risk premium is constant, that is, time-invariant. As a result, this paper incorporate the credit risk and time-varying risk premium into Markov chain model that is more elaborate than previous studies. Furthermore, we estimate the credit risk of bank loans for thirty-one domestic banks in Taiwan. We expect that the model not only provide an effective credit risk review but also help to face the Basel Capital Accord.

The purpose of this paper is to provide an effective model to measure the credit risk. The model is more elaborate than previous studies such as Jarrow, Lando and Turnbull (1997) and Wei (2003) that ignore the credit cycle and time-varying risk premium. In this paper, we contribute to the literature in the following aspect. First, the model incorporates well-documented empirical properties of transition matrices such as their dependence on credit cycle. This paper serves as one of the first studies to adopt a conditional Markov chain model for assessing the credit risk of bank loans, which is never discussed in previous studies. In addition, we compare the difference between the observed and fitted transition matrix. The observed transition matrix is calculated from original Markov chain model that don't consider credit cycle. On the

other hand, we incorporate the credit cycle with the fitted transition matrix. The empirical results show that if we ignore the credit cycle, then estimated results will be distorted and not true. Second, the model extends the previous framework by relaxing the risk premium as time-varying parameter. Third, the estimated procedures are easy to follow and implement. The model can apply to value the credit risk of other financial institutions. On the whole, we expect that the model proposed in the paper will be helpful for Taiwan's financial institutions.

This paper is organized as follows. Section 1 provides motivation. In Section 2 reviews literature concerning models of credit risk. Section 3 presents the formal methodology of this paper. Section 4 describes the sample data used in this paper. Section 5 shows the main empirical results. Finally, section 6 includes a discussion of our findings and a conclusion.

2. Literature Review

Over the last years, credit risk modeling and credit derivatives valuation have received tremendous attention worldwide. These credit risk models can be grouped into two main categories: the structural-form model and the reduced-form model. One important difference between these two categories of models is the implicit assumption they make about managerial decisions regarding capital structure. The structural-form model is also called the asset value model for assessing in credit risk, typically of a corporation's debt. It was based on the principle of pricing option in Black and Scholes (1973) and more detailed model developed by Merton (1974). Furthermore, the basic Merton model has subsequently been extended by removing

one or more of Merton's assumptions. For instance, Black and Cox (1976) and Kim, Ramaswamy and Sundaresan (1989) suggest that capital structure is explicitly considered and default occurs if the value of total assets is lower than the value of liabilities. Brennan and Schwartz (1978) and Longstaff and Schwartz (1995) investigate stochastic interest rate correlated with the firm process. Leland (1994) endogenizes the bankruptcy while accounting for taxes and bankruptcy costs. Consequently, structural-form models rely on the balance sheet of the borrower and the bankruptcy code in order to derive the probability of default. Although these extensions of Merton's original framework, these models still suffer some drawbacks. First, since the firm's value is not a tradable asset, nor it is easily observable, the parameters of the structural-form model are difficult to estimate consistently. Second, the inclusion of some frictions like tax shields and liquidation costs would break the last rule. Third, corporate bonds undergo credit downgrades before they actually default, but structural-form models cannot incorporate these credit-rating changes.

Reduced-form models attempt to overcome these shortcomings of structural-form models, such as Jarrow and Turnbull (1995), Duffie and Singleton (1997), Jarrow, Lando and Trunbull (1997) and Lando (1998). Unlike structural-form models, reduced-form model make no assumptions at all about the capital structure of the borrowers. The calibration of this probability of default is made with respect to ratings agencies' data. The original Jarrow and Trunbull (1995) model, which was perhaps the first reduced-form model to experience widespread commercial acceptance, was worked out through the use of matrices of historical transition probabilities from original ratings and recovery values at each terminal state. In contrast, reduced-form models can extract credit risks from actual market data and are not dependent on asset value and leverage. Therefore, parameters that are related to the firm's value need not

be estimated in order to implement them.

Jarrow, Lando and Turnbull's (1997) model matches the Committee's opinion reasonably well and represents a major step forward in credit risk modeling. The model of Jarrow, Lando and Turnbull (1997) is based on the risk-neutral probability valuation model, also called the Martingale approach to pricing of securities, which derives a risk premium for the dynamic credit rating process from the Markov chain process and then estimates the default probability by transition matrix. But, they ignore some conditions such as credit cycle, which is an empirical property of transition matrix.

From a credit risk modeling perspective, variation in transition matrices attribute to credit cycle is potentially very important. For Belkin, Suchower and Forest (1998), they employ a parameter to measure the credit cycle, meaning the values of default rates and of end-of-period risk ratings not predicted by the initial mix of credit grades. As well-known, ratings changes plays a crucial role in many credit risk models and the distribution of rating changes vary across time. The Basel Committee on Banking Supervision also emphasizes the importance of the credit cycle, which may improve the accurate assessments of credit risk. Therefore, if ignore such dependence, then it may lead to inaccurate assessments of credit risk.

There are some known researches which explicitly link the impact of the dependence of transition matrices, such as Belkin, Suchower and Forest (1998) and Kim (1999). Wilson (1997) has shown that transition probabilities change over time as the state of the economy evolves and that these changes drive correlations between changes in the credit quality of different obligors. Belkin, Suchower and Forest (1998) employ a parameter to measure the credit cycle and propose a method of calculating transition matrices conditional on the credit cycle. Kim (1999) builds a credit cycle

index and his method is similar to that of Belkin, Suchower and Forest (1998). Nickell, Perraudin and Varotoo (2000) quantify the dependence of transition matrices on the industry, domicile of the obligor, and on the stage of the business cycle by employing ordered probit models. Although they attempt to incorporate credit cycle in transition matrices, they don't consider the risk premium.

Furthermore, Wei (2003) propose multi-factor, Markov chain model that consider the credit cycle and risk premium on transition matrix. Wei (2003) allows the transition matrix to evolve according to credit cycle. Although his model is more general than previous studies, he assumes that the risk premium is kept constant over time. According to Kijima and Komoribayashi (1998) and Lu (2006), they propose a procedure to estimate the risk premium, which is not time-invariant but is actually always time-variant. As a result, there are two important factors, the credit cycle and risk premium, for assessing credit risk. The paper relaxes assumptions of previous researches by incorporating the credit cycle and time-variant risk premium into Markov chain model for assessing the credit risk of bank loans, which is more elaborate than previous studies. Consequently, we expect that the paper can improve the accuracy of assessment of credit risk and helpful for Taiwan's banking industry.

3. Model Specification

3.1 Observed transition matrix

Credit ratings of firms are published in a timely manner by rating agencies, such as Standard & Poor's or Moody. They provide investors with invaluable information to assess firms' abilities to meet their debt obligations. If a company's credit quality

has improved or deteriorated significantly over time, such a review will prompt the agency to raise or lower its rating. Since the market considers companies with high ratings to be less risk than those with low ratings, credit ratings change from time to time to indicate firms' credit risk ratings. In recent years, it has become common to use a Markov chain model to describe the dynamics of firm credit ratings, as in Jarrow and Turnbull (1995) and Jarrow, Lando and Turnbull (1997).

To be more specific, let x_t represent the credit rating at time t of a bank's borrower. We assume that $x = \{x_t, t=0,1,2,\dots\}$ is a time-homogeneous Markov chain on the state space $S=\{1, 2, \dots, C, C+1\}$, where state 1 represents the highest credit class; and state 2 the second highest, ..., state C the lowest credit class; and state $C+1$ designates default. It is usually assumed for the sake of simplicity that the default state $C+1$ is absorbing. Furthermore, let $f_{ij} = P(x_{t+1} = j | x_t = i)$, $i, j \in S$, $t=0,1,2,\dots$ denote the probability of state i transiting to state j through the actual probability measure. That is, f_{ij} and P represent the one-step transition probability and actual probability measure, respectively. Then, the discrete time and the regime-switching of credit class i transiting to credit class j can be represented by a time-homogeneous transition matrix as following, is called observed transition matrix

$$F = \begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1C} & f_{1,C+1} \\ f_{21} & f_{22} & \cdots & f_{2C} & f_{2,C+1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ f_{C1} & f_{C2} & \cdots & f_{CC} & f_{C,C+1} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{D} \\ \mathbf{O} & \mathbf{1} \end{bmatrix} \quad (1)$$

$\begin{matrix} \text{---} & \text{---} \\ \text{---} & \text{---} \\ \text{---} & \text{---} \\ \text{---} & \text{---} \\ \text{---} & \text{---} \end{matrix}$

where $f_{ij} > 0 \quad \forall i, j$ and $\sum_{j=1}^C f_{ij} = 1 \quad \forall i$. The submatrix $\mathbf{A}_{(C \times C)}$ is defined on non-absorbing states $\hat{S} = \{1, 2, \dots, C\}$. The components of submatrix \mathbf{A} denote the regime-switching of credit classes for the bank's borrower, however it excludes

default state $C+1$. $\underset{(C \times 1)}{D}$ is the column vector with components $f_{i,C+1}$, which represent the transition probability of banks' borrower for any credit class, i.e., $i=1, 2, \dots, C$, switching to default class, i.e., $j=C+1$. We assume for the sake of simplicity that bankruptcy (state $C+1$) is an absorbing state, so that $\underset{(1 \times C)}{O}$ is the zero column vector giving a transition probability from the default state at initial time until final time. Once the process enters the default state, it would never return to credit class state, so that $f_{C+1,C+1} = 1$. In such a case, we would say that default state $C+1$ is an absorbing state.

3.2 Fitted transition matrix

For considering the credit cycle, there are some procedures to calculate the fitted transition matrix. The first step is to devise a mapping through which the transition probability can be translated into credit scores. The paper employs the normal distribution which is easy to calculate. Since the summation of each row in a transition matrix is always equal to 1. We can invert the cumulative normal distribution function starting from the default function as Belkin, Suchower and Forest (1998), Kim (1999) and Wei (2003). Therefore, the transition matrix as equation (1) can be converting as follow

$$Y = \begin{bmatrix} y_{12} & y_{13} & y_{14} & \cdots & y_{1C} \\ y_{22} & y_{23} & y_{24} & \cdots & y_{2C} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_{C,2} & y_{C,3} & y_{C,4} & \cdots & y_{C,C} \end{bmatrix} \quad (2)$$

Equation (2) is $(C \times C)$ because of no need to covert the row for the absorbing state, default state. If we convert equation (2) into a probability transition, then we also get a transition matrix as equation (1). As in Belkin, Suchower and Forest (1998), we

decompose Y of time- t into two factors:

$$Y_t = \alpha L_t + \sqrt{1 - \alpha^2} \varepsilon_t \quad (3)$$

where L_t is systematic component that shared by all borrowers, meaning “credit cycle”. The credit cycle will be positive in good year, which imply that for each initial credit rating, a lower than average default rate and higher than average rate of upgrades to downgrades. On the other hand, the credit cycle will be negative in bad year. In any year, the observed transition matrix as above will deviate from the norm, that is $L_t = 0$. The ε_t is non-systematic, idiosyncratic factor that unique to a borrower. We also assume that L_t and ε_t are unit normal variable and mutually independent. The coefficient α is an unknown coefficient, which represents the correlation between Y_t and credit cycle, L_t .

We find the coefficient of each row of equation (2) to minimum the weight, mean-squared discrepancies between the observed transition probabilities and observed transition probabilities. We define

$$P_t(i, j) = \Phi(y_{i,j+1}) - \Phi(y_{i,j}) \quad (4)$$

$$P(y_{i,j+1}, y_{i,j} | L_t) = \Phi\left(\frac{y_{i,j+1} - \hat{\alpha} L_t}{\sqrt{1 - \hat{\alpha}^2}}\right) - \Phi\left(\frac{y_{i,j} - \hat{\alpha} L_t}{\sqrt{1 - \hat{\alpha}^2}}\right) \quad (5)$$

where $\Phi(\cdot)$ represents the standard normal cumulative distribution function and equation (4) and (5) represent the observed and fitted transition probability of state i transfer to j observed in time- t , respectively. The least square problem takes the form as Belkin, Suchower and Forest (1998)

$$\min_{L_t} \sum_i \sum_j \frac{n_{t,i} [P_t(i, j) - P_t(y_{i,j+1}, y_{i,j} | L_t)]^2}{P(y_{i,j+1}, y_{i,j} | L_t) [1 - P_t(y_{i,j+1}, y_{i,j} | L_t)]} \quad (6)$$

where $n_{t,i}$ is the number of borrowers from initial state i transfer to state j . In

addition, the weighting factor is $\frac{n_{t,i}}{P_t(y_{i,j+1}, y_{i,j}, L_t)[1 - P_t(y_{i,j+1}, y_{i,j}, L_t)]}$. Therefore, we

can get fitted transition probability via equation (6). Then, we construct the fitted transition matrix as follow

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1C} & m_{1,C+1} \\ m_{21} & m_{22} & \cdots & m_{2C} & m_{2,C+1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ m_{C1} & m_{C2} & \cdots & m_{CC} & m_{C,C+1} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \quad (7)$$

3.3 Risk premium and default probability

In addition, the paper uses the risk-neutral probability approach to assess the credit risk of bank loans. Although a traditional risk-neutral probability approach is used to assess the default probability of corporate debt, the methods and models for assessing the credit risk on bank loans and bonds are similar.¹ Accordingly, we conclude that a risk-neutral probability approach also can apply to bank loans.

For the pricing of the defaultable borrower, we need to consider the corresponding stochastic process $\tilde{x} = \{\tilde{x}_t, t = 0, 1, 2, \dots\}$ of credit rating under the risk-neutral probability measure. For valuation purposes, the fitted transition matrix needs to be transformed into a risk-neutral fitted transition matrix under the equivalent martingale measure where we let $\tilde{\mathbf{M}}$ denote such a matrix. Although the transition matrix under the new measure need not be Markovian, it is an absorbing Markov

¹ First, in essence, both loans and bonds are contracts that promise fixed payments of principle and interest in the future. Second, loans and bonds stand ahead the claims of a firm's equity holders if the firm goes into default. Third, there are covenants on activities the borrower may undertake while the loans or bonds are outstanding, including limits on the type and amount of new debt, investments, and asset sales that can enhance the probability of repayment. For any given cash flow, the higher dividend payout to stockholders, the less are available for repayment to bondholders and lenders. Finally, the rate of loans are similar to bond yields, usually reflecting risk premiums that vary with the perceived quality of the borrowers and the collateral or security backing of the debt.

chain, which may not be time-homogeneous. Thus the fitted transition matrix under the risk-neutral probability measure is given by

$$\tilde{\mathbf{M}}(t, t+1) = \begin{bmatrix} \tilde{m}_{11}(t, t+1) & \cdots & \tilde{m}_{1C}(t, t+1) & \tilde{m}_{1,C+1}(t, t+1) \\ \tilde{m}_{21}(t, t+1) & \cdots & \tilde{m}_{2C}(t, t+1) & \tilde{m}_{2,C+1}(t, t+1) \\ \vdots & \ddots & \vdots & \vdots \\ \tilde{m}_{C1}(t, t+1) & \cdots & \tilde{m}_{CC}(t, t+1) & \tilde{m}_{C,C+1}(t, t+1) \\ \hline 0 & \cdots & 0 & 1 \end{bmatrix} \quad (8)$$

$$= \begin{bmatrix} \tilde{\mathbf{A}}(t, t+1) & \tilde{\mathbf{D}}(t, t+1) \\ \hline \tilde{\mathbf{O}} & \mathbf{1} \end{bmatrix}$$

(C×C) (C×1)
(1×C) (1×1)

where $\tilde{m}_{ij}(t, t+1) = \tilde{\mathbf{P}}\{\tilde{x}_{t+1} = j | \tilde{x}_t = i\}$, $i, j \in S$. \tilde{m}_{ij} and $\tilde{\mathbf{P}}$ represent the risk-neutral fitted transition probability and risk-neutral probability measure. The conditions for equation (1) must be satisfied here, together with the equivalence condition that $\tilde{m}_{ij}(t, t+1) > 0$ if and only if $m_{ij} > 0$.

Note that the risk premium plays a crucial role for assessing the credit risk. The zero risk rate (risk-free rate) and risky rate (loan's rate) can capture the credit risk of the bank loans for every rating class with the risk-neutral probability measure. First, let $V_0(t, T)$ be the time- t price of a risk-free bond maturing at time T , and $V_i(t, T)$ be its higher risk, that is, risky counterpart for the rating class, i . However, a loan does not lose all interest and principal when the borrower defaults. Realistically, we assume that a bank will receive some partial repayment even if the borrower goes into bankruptcy. Let δ be the proportions of the loan's principle and interest, which is collectable on default, where in general δ will be referred to as the recovery rate. If there is no collateral or asset backing, then $\delta = 0$. On the contrary, the recovery rate is $0 < \delta \leq 1$.

As shown by Jarrow, Lando and Turnbull (1997), it can be assumed that

$\tilde{m}_{i,j}(t, t+1) = \lambda_{ij}(t) \cdot m_{ij}$, $i, j \in S$, and $\lambda_{ij}(t) = \lambda_i(t)$, for $j \neq i$ and their procedure for risk premium as

$$\lambda_i(0) = \frac{V_0(0,1) - V_i(0,1)}{(1-\delta)V_0(0,1)m_{i,C+1}} \quad (9)$$

In equation (9), it is apparent that a zero or near-zero default probability, i.e., $m_{i,C+1} \approx 0$, would cause the risk premium estimate to explode and it is also implied that the credit rating process (including default state) of every borrower is independent, which is inappropriate and irrational for bank loans. However, if the borrower defaults, then we should never estimate the default probability in the future. As a result, we modify the assumption that every borrower's credit rating class is independent only before entering default state. We redefine the risk premium as

$$\ell_i(t) = \frac{1}{1 - m_{i,C+1}} \sum_{j=1}^C \tilde{m}_{ij}^{-1}(0, t) \frac{V_i(0, t) - \delta V_0(0, t)}{(1-\delta)V_0(0, t)}, \quad i=1,2,\dots,C \text{ and } t=1,\dots,T \quad (10)$$

$$\tilde{A}(0, t+1) = \tilde{A}(0, t)\tilde{A}(t, t+1) \quad (11)$$

where $\tilde{m}_{ij}^{-1}(0, t)$ are the components of the inverse matrix $\tilde{A}^{-1}(0, t)$ and $\tilde{A}(0, t)$ will be invertible. Note that the $m_{i,C+1}$ is the transition probability of fitted transition matrix by procedure as above. The denominator of equation (10) is not that $m_{i,C+1}$, but that $(1 - m_{i,C+1})$, which can avoid the problem in equation (9). For equation (11), $\tilde{A}(t, t+1) = \Lambda(t) \cdot A$ and $\Lambda(t)$ is the $(C \times C)$ diagonal matrix

with diagonal components being the risk premium adjusted to $\ell_j(t)$, $j \in \hat{S}$. In particular, the risk premium of $t=0$ is

$$\ell_i(0) = \frac{1}{1 - m_{i,C+1}} \frac{V_0(0,1) - \delta V_i(0,1)}{(1-\delta)V_0(0,1)}, \quad \text{for } i=1,2,\dots,C \quad (12)$$

Therefore, we can estimate risk premium by a recursive method for all loan periods,

$t=0, 1, \dots, T$. On the whole, we also find that risk-neutral transition matrix varies over time to accompany the changes in the risk premium by equation (10) and (12). Then, we assume the indicator function to be

$$1_{\{i\}} = \begin{cases} 1, & \text{if } I \in \{\tau > T\} \quad (\text{not default before time } T) \\ \delta, & \text{if } I \in \{\tau \leq T\} \quad (\text{default before time } T) \end{cases} \quad (13)$$

Since the Markov processes and the interest rate are independent under the equivalent martingale measure, the value of the loan is equal to

$$\begin{aligned} V_i(t, T) &= V_0(t, T) \left\{ \tilde{E}_t [1_{\{\tau > T\}}] + \tilde{E}_t [\delta 1_{\{\tau \leq T\}}] \right\} \\ &= V_0(t, T) \left\{ \tilde{Q}_t^i(\tau > T) + \delta [1 - \tilde{Q}_t^i(\tau > T)] \right\} \\ &= V_0(t, T) \left\{ \delta + (1 - \delta) \tilde{Q}_t^i(\tau > T) \right\} \end{aligned} \quad (14)$$

where $\tilde{Q}_t^i(\tau > T)$ is the probability under the risk-neutral probability measure that the loan with rating i will not be in default before time T . It is clear that

$$\begin{aligned} \tilde{Q}_t^i(\tau > T) &= \frac{V_i(t, T) - \delta V_0(t, T)}{(1 - \delta) V_0(t, T)} \\ &= \sum_{j=1}^C \tilde{m}_{ij}(t, T) = 1 - \tilde{m}_{i, C+1}(t, T) \end{aligned} \quad (15)$$

which holds for time $t \leq T$, including the current time, $t=0$. Similarly, the default probability occurs before time T as

$$\tilde{Q}_t^i(\tau \leq T) = \frac{V_0(t, T) - V_i(t, T)}{(1 - \delta) V_0(t, T)}, \text{ for } i=1, \dots, C \text{ and } T=1, 2, \dots \quad (16)$$

Consequently, we can estimate the default probability of bank loans under risk-neutral probability measure that incorporate with credit cycle and time-varying risk premium.

4. Data

In Taiwan, there are two rating agencies, the Taiwan Rating and the Taiwan Economic Journal. The sample data come from two databases of the Taiwan Economic Journal (TEJ), including the Taiwan Corporate Risk Index (TCRI) and long and short-term bank loans. The sample period is between 1997 and 2003.

The TCRI is a complete history of short and long-term rating assignments for Taiwan's corporations. The definitions of the ratings categories of TCRI for long-term credit are similar to Standard & Poor's and Moody. TEJ applies a numerical class from 1 to 9 and D for each rating classification. The categories are defined in terms of default risk and the likelihood of payment for each individual borrower. Obligation rated number 1 are generally considered as being the lowest in terms of default risk, which is similar to the investment grade for Standard & Poor's and Moody. Obligation rated number 9 are the most risky and the rating class D denotes the default borrower. Therefore, the rating categories used by TEJ, Standard & Poor's and Moody are quite similar, though differences of opinion can lead in some cases to different ratings for specific debt obligations. On the other hand, since the borrowers' obligation rated numbers are not consistent in every year, we combine the number 1~4 as a new rating class, denoted as 1*. Similarly, we combine number 5~6 and 7~9 as two new rating classes, denoted as 2* and 3*, respectively. Thus, there are four rating classes, 1*, 2*, 3* and D.

The long and short-term bank loan database is record all debts of corporations in Taiwan, including lender names, borrower names, rate of debt, and debt issuance dates, etc. For the viewpoint of banks, we can analyze the credit rating class of borrowers to investigate the credit risk of bank loans.

On the other hand, the risk-free rate is published by the Central Bank in Taiwan.

We take the government bond's yield as a proxy for the risk-free rate. The yields of government bonds for various maturities those published by the Central Bank in Taiwan. Since the maturity of bank loans and government bonds are different, we have to adjust the yields of government bonds, so we interpolate the yield of government bond whose maturity is closest and take it as the risk-free rate.

Finally, the recovery rate plays an important role for making lending decisions that serves as security for bank loans. In general, banks will set a recovery rate according to kinds, liquidity, and value of collateral before lending. Fons (1987) assumed a constant recovery rate of 0.41 according to the historic level. Longstaff and Schwartz (1995) and Briys and de Varenne (1997) also assumed a constant recovery rate. Carty and Lieberman (1996) assessed the recovery rate on a small sample of defaulted bank loans and found that it averaged over 71%. Copeland and Jones (2001) assumed that the recovery rate is equal to zero in all sample years. On the other hand, Lu and Kuo (2005) suggested taking the recovery rate as the exogenous variable from 0.1 to 0.9. According to previous studies, there is no clearly definition of the recovery rate and the data on recoveries on defaulted loans is clearly incomplete. Consequently, we assume the recovery rate as exogenous variables from 0.1 to 0.9 in this paper following the assumption of Lu and Kuo (2005).

In conclusion, we analyze default risk for at least a one-year horizon and therefore exclude observations for short-term loans and incomplete data. We also exclude loans that have an overly low rate because they are likely to have resulted from aggressive accounting politics and will bias the estimated results. Since the data without posting collateral are insufficient for gauging credit risk, we do not consider these loans.² That is, we analyze the credit risk of mid-and long-term loans with

² In general, a bank exposes itself to higher risks if it lends without collateral. As a result, banks always

posting collateral for thirty-one domestic banks in Taiwan.

5. Empirical Results

In this paper, we estimate the credit risk of thirty-one domestic banks in Taiwan.³ We compare differences of the credit risk between observed and fitted transition matrix. There are four steps. First, we calculate the observed transition matrix by equation (1). We show the average observed transition matrix of thirty-one banks from 1998 to 2003 in Table 1. There is an interesting phenomenon in Table 1. The default probability is higher in high rating class, 1*, than that in low rating class, 2*. The phenomenon may be due to credit cycle and risk-neutral probability measure. Therefore, we estimate credit cycle and risk premium in further steps.

Table 1. Observed transition matrix, 1998-2003

Initial Rating	Rating at the end of year			
	1*	2*	3*	D
1*	0.707	0.195	0.024	0.075
2*	0.035	0.738	0.186	0.042
3*	0.004	0.066	0.820	0.110

request borrowers to post collateral.

³ The thirty-one domestic banks include: (1) Bank of Taiwan; (2) Bank of Overseas Chinese; (3) Bowa Bank; (4) Central Trust of China; (5) Chang Hwa Commercial Bank; (6) Chaio Tung Bank; (7) Chinatrust Commercial Bank; (8) Chinfon Commercial Bank; (9) Cosmos Bank, Taiwan; (10) EnTie Commercial Bank; (11) E. Sun Commercial Bank; (12) Far Eastern International Bank; (13) First Commercial Bank; (14) Fubon Commercial Bank; (15) Fuhwa Commercial Bank; (16) Grand Commercial Bank; (17) Hua Nan Commercial Bank; (18) Hsinchu International Bank; (19) Jih Sun International Bank; (20) Land Bank of Taiwan; (21) Taipei International Bank; (22) Taiwan Cooperative Bank; (23) Taipei Bank; (24) Taishin International Bank; (25) Ta Chong Bank; (26) The Chinese Bank; (27) The Export-Import Bank of the Republic of China; (28) The Farmers Bank of China; (29) The International Commercial Bank of China; (30) The Shanghai Commercial and Saving Bank; (31) United World Chinese Commercial Bank.

Then, we estimate fitted transition probabilities that consider credit cycle by equation (6) and construct fitted transition matrix. Thus, we list average fitted transition matrix from 1998 to 2003 in Table 2. For Table 1 and 2, we find that credit cycle have significant effect on transition matrix. The credit cycle can explain the special phenomenon in table 1. The default probability is higher in rating class 2* than that in 1*, which is more reasonable and reliable. The default probability will be overestimated and underestimated of less risk rating class to higher risk rating class. On the other hand, the default probability will underestimate in observed transition matrix, especially for a particular rating class, 3*. Furthermore, we estimate the time-varying risk premium and incorporate into fitted transition matrix to get default probability under risk-neutral probability measure. On the other hand, the coefficient α is the correlation between Y_t and credit cycle L_t in equation (3). The estimated value of α is 0.0934 in Table 2.

Table 2. Fitted transition matrix, 1998-2003

Initial Rating	Rating at the end of year			
	1*	2*	3*	D
1*	0.781	0.188	0.014	0.016
2*	0.034	0.743	0.187	0.036
3*	0.016	0.124	0.605	0.255
$\hat{\alpha}$	0.0934			

Third, we estimate the time-varying risk premium to transform fitted transition matrix into risk-neutral fitted transition matrix via equation (10) and (12). Therefore, we list average risk premium in Table 3. The fitted transition matrix under risk-neutral probability measure is shown in Table 4.

Table 3. Average risk premium

Rating	Maturity (Years)					
	1998	1999	2000	2001	2002	2003
1*	0.955	0.968	0.984	0.975	0.956	0.973
2*	0.951	0.870	0.723	0.780	0.793	0.885
3*	0.941	0.994	0.999	0.995	0.993	0.999

Table 4. Fitted transition matrix under risk-neutral probability measure, 1998-2003

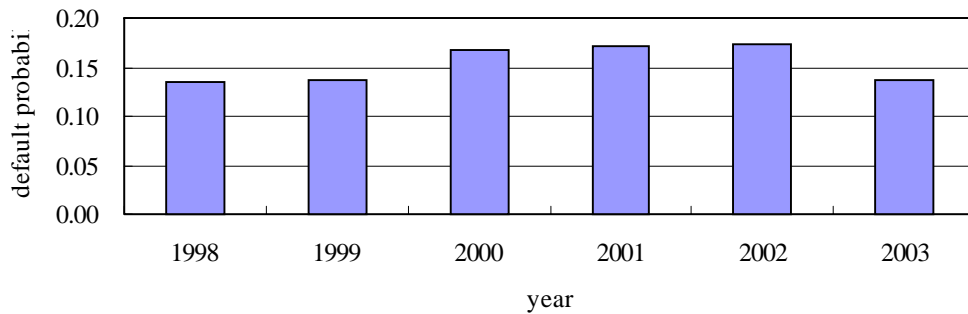
Initial Rating	Rating at the end of year			
	1*	2*	3*	D
1*	0.752	0.187	0.013	0.047
2*	0.028	0.612	0.156	0.205
3*	0.016	0.120	0.599	0.264

Finally, we assess the average default probability of 31 banks by equation (16) and table 5 and figure 1 show the empirical results. In 2000 and 2001, we find that default probabilities are higher than other years, which may be accompanied by a change in the business cycle in Taiwan and explained below.

Table 5. Default probability

Year	1998	1999	2000	2001	2002	2003
Default probability	0.134	0.137	0.167	0.171	0.160	0.136

Figure 1. Average default probability



One contributions of the paper is incorporating the credit cycle and time-varying risk premium into transition matrix, which is never discussed in previous studies. Broadly speaking, the credit cycle means the values of default rates and of end-of-period risk ratings not predicted, using observed transition matrix, by the initial mix of credit grades. In good years, credit cycle will be positive and negative in bad years. Figure 2 represents the historical movement of credit cycle that describes past credit conditions not evident in the observed transition matrices. On the other hand, figure 3 shows the total scores of monitoring indicators of Taiwan⁴ from 1998 to 2003. The cyclical through occurs in 2000-2001 that is the period of economic recession of Taiwan.

According the model setting, the credit cycle will be positive in good years and negative in band years. For figure 2 and 3, we find that credit cycle drops below zero while Taiwan's business cycle from peak to trough in 2000-2001. From Table 5 and figure 1, we find that default probabilities in 2000-2001 are higher than other years. On the whole, the relative high proportion borrowers together with 2000-2001 credit slump accounts for a high number of default that may be due to the business cycle.

⁴ The total scores of monitoring indicators of Taiwan is composed of monetary aggregates M1b, direct and indirect finance, bank clearing, remittance, stock price index, manufacturing new order index, exports, industrial production index, manufacturing inventory to sale ratio, nonagricultural employment, export price index, and manufacturing output price index.

That is, the credit risk in 2000-2001 is higher than other years. On the other hand, credit cycle has stayed positive and credit conditions have remained benign and the default probabilities are low during other periods. Consequently, the model for estimating the default probability is accurate and reliable.

Figure 2. Credit cycle from 1998 to 2003

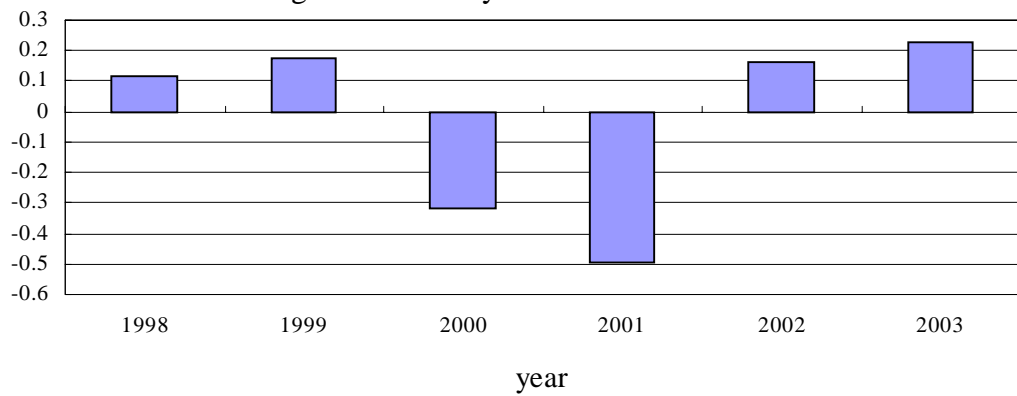
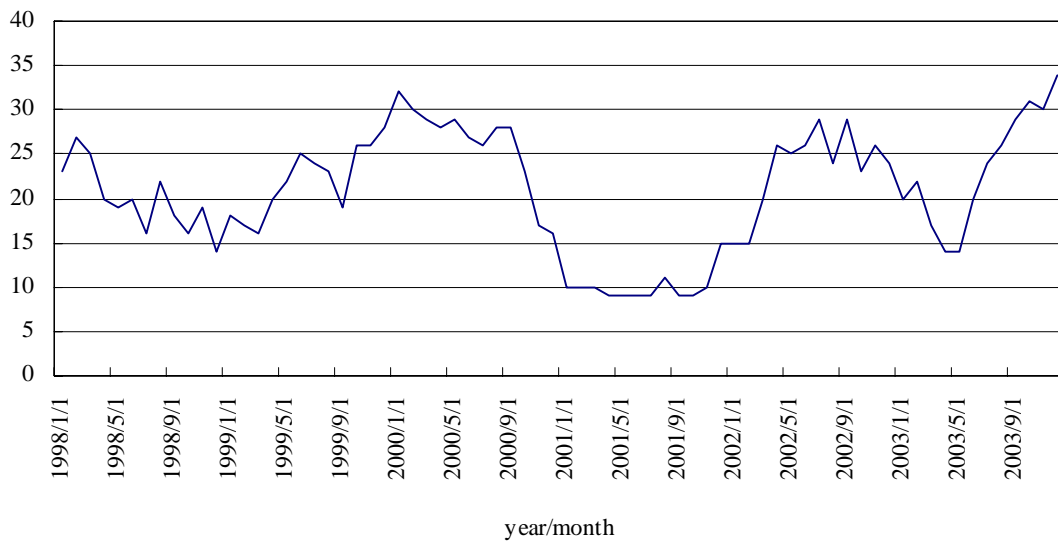


Figure 3. Total scores of monitoring indicators



6. Conclusion

It is important to note that the model depends largely on the borrower's credit ratings and the risk-free term structure of the interest rate, which are forward looking and reflect the current position. They are a timely and reliable measure of credit quality. Therefore, accurate and timely information from the borrower's credit rating data provides a continuous credit monitoring process in this paper. This paper have highlighted some rather great differences between the previous researches such as Jarrow, Lando and Turnbull (1997), Kijima nad Komoribayashi (1998), Wei (2003) and Lu (2006), etc.

This paper focuses on providing a formal model to assess the credit risk of bank loans for thirty-one domestic banks in Taiwan. The proposed model contributes to the literature in following aspects. First, the model incorporate the credit cycle into the transition matrix, called fitted transition matrix. In addition, we compare differences between the observed and fitted transition matrix. The empirical result shows that the observed transition matrix will overestimate and underestimate the default probability of less risk rating class and risky rating class, respectively.

Second, we relax the assumption of risk premium as a time-varying parameter that is assumed as a constant parameter in Jarrow, Lando and Turnbull (1997) and Wei (2003). Finally, the procedure is easy to follow and implement. We recommend that the method also can apply to assess the credit risk of other financial institutions, such as a bills finance company. On the whole, we expect that the paper can help banks estimate their credit risk more carefully and are also an effective tool for any financial institutions' credit review process.

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