A Note on Correct Duration-Convexity Hedges Using Treasury Futures

(January 2005)

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JEL classification: G13; G15; G19

Keywords: Duration and Convexity; Convexity-based Hedge; Treasury Futures; Cheapest-to-deliver Treasury; Cross Hedges; CBOT; Conversion Factoring System; Numerical analysis.

***Corresponding author.** The authors have received valuable comments from Andrew Chen, Andrew Karolyi, Robin Grieves, Joseph Williams, and participants of the 12th Annual PACAP/FMA Finance Conference, the 13th Australasian Conference of Banking and Finance, and the 2002 Eastern Finance Association. The authors acknowledge the research grant of Nanyang Business School and the Best Paper Award at the 13th Australasian Conference of Banking and Finance.

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ABSTRACT

This note is motivated by rarely understood hedging implications of the CBOT Conversion Factoring System that (1) the convexity-based hedge using Treasury futures is essential since the convexity of Treasury futures can be negative and (2) there are arbitrage-free relations between the projected duration and convexity exposures of Treasury futures and those of cheapest-to-deliver Treasuries around the first day of delivery month. This note takes into account the hedging implications in deriving correct equations for the convexity-based hedge using Treasury futures. Our hedge is a forward-looking cross hedge since the hedge ratios are obtained from cross matching the projected exposures of the cash bonds and the cheapest-to-deliver Treasuries as of the first day of delivery month. A numerical analysis indicates that on average, the forward-looking hedge is about 1.8% cheaper than traditional duration-convexity hedges. The analysis also indicates that the hedge cost saving would be substantially larger in circumstances where convexity-based hedges are most needed, namely, when high-duration bonds are hedged, term structure of interest rates is steep, or the cheapest to deliver pays high coupons.

I. INTRODUCTION

This note is motivated by rarely understood hedging implications of the CBOT Conversion Factoring System (CFS): namely, (1) the convexity-based hedge using Treasury futures is essential since the duration-based hedge ignores the possibility of a negative convexity of Treasury futures; and (2) there are arbitrage-free relations between the projected duration and convexity exposures of Treasury futures and those of cheapest-to-deliver (CTD) Treasuries around the first day of delivery month. This note takes into account the hedging implications in deriving correct equations for the convexity-based hedge using Treasury futures.

The rarely understood hedging implications of the CFS have been partially recognized only recently in Rendleman Jr. (1999) who took into account the arbitrage-free relations at the first day of delivery month in developing a forward-looking duration-based hedge. Extant literature on the duration-convexity hedge using Treasury futures[1] did not incorporate the hedging implications mainly because they did not fully recognize the bias in the CFS[2], which not only leads to the arbitrage-free relations around the first day of delivery month but also indicates the necessity of convexity-based hedge.

This note fills in the gap in extant literature by explaining hedging implications of the bias in the CFS, deriving correct equations for duration-convexity hedges, and providing detailed comparisons of the costs for the forward-looking and the traditional convexity-based hedges. Hence, this note attempts to contribute to relevant literature by providing: (1) a non-trivial extension of the duration-based hedge in Rendleman Jr (1999) to a convexity-based hedge; (2) a

[1] Traditional duration-convexity hedges using Treasury futures are explained in, e.g., Schaefer (1984), Bierwag, et al. (1988), Goodman and Vijayaraghavan (1988), Daigler (1993), Daigler and Copper (1998a, 1998b), and Lien and Tse (2000). For other types of hedges, see, e.g., combination hedge (e.g., Leschhorn, 2001), key rate hedge (e.g., Ho, 1992), and scenario hedge (e.g., Hill and Vaysman, 1998).

[2] For early discussions on the CFS bias and its implications on the CTD determination, see Kane and Marcus (1984), Daigler (1993), and Benninga and Wiener (1999).

logical framework for a non-option theoretic approach to convexity-based hedges[3]; and (3) analytical details on how to pair hedge positions in different Treasury futures in the face of, among other things, different maturity-coupon characteristics of the cash bonds being hedges and the CTDs, different shapes of the term structure of interest rates, and different hedge horizons.

The forward-looking duration-convexity hedge is of interests also to market practitioners for several reasons. First, such price-based hedge rules can be superior to regression-based backward-looking hedge rules. For empirical evidence and related arguments, see Sercu and Wu (2000). Second, the hedge-ratio equations can show how each hedging instrument contributes to respective hedges against duration and convexity exposures as well as short-term spot and forward interest-rate exposures. As illustrated in the numerical analysis, the hedge-ratio equations can provide detailed information useful for non-arbitrary pairing of long and short Treasury futures contracts.

The numerical analysis indicates that on average, the forward-looking hedge is about 1.8% cheaper than traditional duration-convexity hedges. The analysis also indicates that the hedge cost saving would be substantially larger when high-duration bonds are hedged, term structure of interest rates is steep, or the cheapest to deliver pays high coupons. These practically important results are robust across various hedge horizons, different shapes of term structure, and alternative pairing of Treasury futures.

The remainder of this note is arranged as follows. The next section explains the hedging implications of the CBOT CFS. In the third section, this note develops the forward-looking convexity-based hedge. The fourth section provides a detailed numerical analysis on hedge costs

[3] See Daigler (1993) for a description of delivery-related options and see Koenigsberg (1990 and 1991), Boyle (1989), and Hemler (1990) for pricing these options. Because the price-volatility linkage between Treasury futures and their CTDs is yet to be quantified in an option-theoretic framework, an option-theoretic approach to convexity-based hedge is yet to be developed. One noteworthy problem to an option-theoretic approach to convexity-based

of the forward-looking and traditional convexity-based hedges as well as practical implications of the main results of our numerical analysis. The final section contains a summary and conclusions.

II. The CBOT Conversion Factoring System and Its Hedging Implications

The Treasury futures market is unique in several aspects. For market liquidity, short positions are allowed to deliver any Treasuries as long as they meet certain eligibility criteria. For example, deliverable Treasuries for the 30-year bond futures must have noncallable remaining maturities of 15 years or longer. Yet, only one price is quoted for each type of Treasury futures contracts. For example, in the case of 30-year bond futures, the futures quote closely follows the present value of a notional 20-year 6% coupon straight Treasury of US$1 par. To make short positions indifferent to delivering different coupon-maturity grades, CBOT computes the conversion factor for each deliverable Treasury and applies it in computing the invoice price: [Futures Settlement Price X Conversion Factor] + Accrued Coupons. The settlement price is the settlement quote times the futures contract size that is $100,000 for most futures contracts, whereas the conversion factor is computed from discounting to the first day of delivery month the future cashflows of the deliverable Treasury at the 6% flat rate and then dividing by its par value.[4] Due to the value of delivery-related options that short positions implicitly bought from long positions, the invoice amount tends to be smaller than the market price of the Treasury: I.e., [Treasury Price – Invoice Price] = Delivery Cost. Since the delivery cost can substantially differ across deliverables, the short position delivers the cheapest to deliver Treasury.

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hedge is the difficulty in quantifying the possible relation between a negative convexity in Treasury futures and a positive convexity in their CTDs. See Koenigsberg (1990) and Koenigsberg (1991).

[4] The flat rate was 8% for many years until 2000 March Treasury futures contracts, whereas the par amount is $100,000 for most Treasuries.

In computing the CF, CBOT rounds down the remaining maturity of the deliverable Treasury as of the first day of delivery month to the nearest quarter in the case of 30-year and 10-year Treasury futures and to the nearest month in the case of 5-year and 2-year Treasury futures: i.e., n years and q quarters in the case of long bond and note futures; n years and z months in the case of short note futures. Due to this feature of maturity rounding down, the CBOT CF formula is stated as: $CF = a \times [0.5C + b + c] - d$. Loosely speaking, "0.5C" is the first semi-annual coupon in decimals, "*c*" captures the value of all other coupons in decimals as of the first coupon date, and "*b*" captures the value of \$1 par Treasury as of the same date. Hence, $[0.5C + b + c]$ reflects the deliverable Treasury's full price as of the first coupon date. Since "*a*" is the discount factor for the period between the first coupon date and the first day of delivery month, $a \times [0.5C + b + c]$ reflects the deliverable Treasury's full price as of the first day of delivery month. Since "*d*" captures the accrued coupons, the $CF = a \times [0.5C + b + c] - d$ can be regarded as the Treasury's relative quote in decimals. [5][6]

In the case of long-maturity futures, the first coupon is assumed to be paid either on the first day of the delivery month if the rounded-down maturity is n years and even quarters or on the first day of the next March-cycle month if the remaining maturity is n years and odd quarters. In the case of short-maturity futures, it is assumed to be paid on the first day of either the delivery month or one of the next 11 months. Since Treasury futures follow March quarterly-cycle

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[5] Since the a, For alternative explanations of CF computation, see Daigler (1993), Burghardt et al. (1994), Hull (1998), and the CBOT Website.

[6] In the CBOT formulae for CF, $a = (1 + 6\% / 2)^{-(v/6)}$, $b = (1 + 6\% / 2)^{-2n}$ if z is less than 7 but, if otherwise, $b = (1 + 6\% / 2)^{-(2n+1)}$, $c = (C / 0.06) \times (1 - b)$, and $d = 0.5C(1 - v/6)$, where *n* is the number of whole years from the first day of delivery month to deliverable Treasury's maturity, *z* is the number of months between *n* years and the Treasury's maturity rounded down to the nearest quarter in the case of Treasuries for 30-year T-bond and 10-year T-note futures (so that *z* can take on the values 0, 3, 6, or 9) and to the nearest month in the case of Treasuries for 5-year and 2-year T-note futures (so that *z* can be any integer between 0 and 11), and *v* = *z* if *z* is less than 7 and, if otherwise, *v* = 3 in the case of T-bond and 10-year T-note futures but *v* = *z* - 6 in the case of 5-year and 2-year T-note futures.

delivery months and Treasury coupons fall on the middle of February quarterly-cycle months, the maturity rounding-down brings forward the future cashflows of Treasuries by about 2.5 months in the case of long-maturity futures and about 15 days in the case of short-maturity futures.[7]

The CBOT's CF measure contains a bias that can arise from rounding-down the remaining maturity of deliverable Treasury. Given Treasury futures' expiration in March quarterly-cycle months and Treasuries' coupon payment in February quarter-cycle months, the maturity rounding-down results in bringing up the future cashflows by 2.5 months in the case of deliverable Treasuries for T-bond and 10-year T-note futures (and 15 days in the case of deliverable Treasuries for 5-year and 2-year T-note futures). Hence, the CF measure for relative quote of deliverable Treasuries should yield a non-negligible upward bias.[8] But, this bias due to the maturity rounding-down is not the main bias in CF.

When the prevailing spot rates are substantially different from the 6% flat term structure, which are quite often, the CF bias due to using the 6% flat discount rate in computing the CBOT's CF can be more substantial. This bias in CF measure varies by the coupon-maturity characteristic of the deliverable Treasury. Note that the forward rates applicable to the delivery day and beyond should be used for computing the expected price of deliverable Treasury as of the delivery day. When the forward rate curve lies above the 6% flat term structure, for example, the use of a flat 6% discount rate will produce a substantially upward-biased CF for the deliverable Treasury with long remaining maturity, low coupon, high yield, or large duration. This is so because the cashflows of such deliverable Treasury receive relatively favorable treatment from the use of a low flat discount rate. All else being equal, short positions will prefer

[7] This upward bias in the CF measure as a relative quote for deliverable Treasury as of the first day of delivery month is alternatively recognized as a "pull-to-par" effect in Grieves and Mann (2001).

[8] In fact, this upward bias in CF is alternatively recognized as a "pull-to-par" effect in Grieves and Mann (2001). It can be shown that the bias is about log $(1.06^{5/24})$ in the case of deliverable Treasuries for T-bond and 10-year T-note

to deliver Treasuries with such characteristics because these Treasuries will command relatively large invoice amounts. In general, the CTD tends to have the lowest (highest) duration when the yield is lower (higher) than 6%, although, given the duration, the Treasury with higher yield tends to be the CTD.[9]

Short position will choose the Treasury with the cheapest delivery cost (i.e., the purchase price of deliverable Treasury minus the invoice amount to be paid by CBOT). Given limited deliverable Treasuries, limited open interest of respective Treasury and the presence of CF bias, the deliver-cost minimizing short positions will be able to identify a unique CTD Treasury. As the term structure of interest rates changes during the life span of Treasury futures, the corresponding forward rate curve will change accordingly. Then, the CF bias will change differently across different coupon-maturity characteristics of the deliverable Treasuries. As a consequence, the CTD will change too. The CTD change triggers cash-futures arbitrages that can affect Treasury futures' quote. The time-varying CF bias contributes to CTD existence, the CTD changes over time, and the cash-futures arbitrages.

The arbitrage forces dictate a price-volatility linkage between Treasury futures and the CTD. This is so because, given the CTD existence, the Treasury futures will trade as if it is a contract written on the CTD. This means that Treasury futures' interest-rate exposure should reflect the CTD's interest-rate exposure. As time approaches the delivery day, the Treasury quote should approach the clean price of CTD divided by its conversion factor because cash-futures arbitrages will force the adjusted basis to shrink to zero.[10] In an equilibrium situation where the

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futures and about log $(1.06^{2/52})$ in the case of deliverable Treasuries for 5-year and 2-year T-note futures. The upward bias is roughly equal to 120 and 24 basis points, respectively.

[9] Alternatively stated, the CTD tends to be the deliverable Treasury with the highest repo rate. For details, see Daigler (1993) and Burghardt et al. (1994). For details on CTD choice, see Benninga and Wiener (1999), Livingston (1984), Castelino and Chatterjee (1988), and Kane and Marcus (1984).

[10] In this paper, the adjusted basis refers to the basis adjusted for the CF: namely, a deliverable Treasury's quote minus the corresponding Treasury futures' quote times the CF of the deliverable Treasury. To have the equilibrium

complex effect of delivery-related options is assumed away, the clean price of CTD divided by its CF should be the same as the futures'quote. The arbitrage-free price relationship between Treasury futures and CTD indicates that the dollar duration (convexity) of Treasury futures should equal the dollar duration (convexity) of CTD Treasury divided by the CF.[11]

III. A MODEL OF CONVEXITY-BASED HEDGES WITH TREASURY FUTURES

The assumptions essential in developing the forward-looking cross-hedge model are as follows:[12] the term structure of interest rates may not be flat but shifts in parallel; the forward price of bond is an unbiased estimator of the expected spot price of bond at maturity of the forward contract; neither the marking to market nor the margin requirement exists for Treasury futures contracts; and both the hedge expiration and the futures' expiration fall on the first day of delivery month.

1. Interest Rate Sensitivity of Bond Price

The first and second assumptions imply that an investment strategy of simultaneous buying and selling the bond and the forward bond should generate a zero profit. Hence, a simple carry-cost representation of an arbitrage-free relationship between a bond price and a forward

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convergence between Treasury futures and CTD prices, the adjusted bases for all non-CTD Treasuries should be negative on the delivery day. Treasury futures' quote can be also affected through the change in the value of delivery-related options. For example, the change in term structure can affect the values of delivery-related options, which in turn will affect Treasury futures' quote. This alternative route is not allowed in this paper, whose relevant details are provided in the following section.

[11] For alternative explanations, see Kishimoto (1998) and Rendleman, Jr. (1999). Namely, $F' D'_{F'} = D'_{CTD} (P'_{CTD} + A'_{CTD}) / f'_{CTD}$ and $F' C'_{F'} = C'_{CTD}(P'_{CTD} + A'_{CTD}) / f'_{CTD}$, where F' is the Treasury futures' quote, $D'_{F'}$ ($C'_{F'}$) is the Treasury futures' duration (convexity), $F' D'_{F'}$ ($F' C'_{F'}$) is the Treasury futures' dollar duration (convexity), $(P'_{CTD} + A'_{CTD})$ is the CTD Treasury's full price (i.e., clean price + accrued coupon), $D'_{CTD}(P'_{CTD} + A'_{CTD})$ is the CTD Treasury's dollar duration, $C'_{CTD}(P'_{CTD} + A'_{CTD})$ is the CTD Treasury's dollar convexity, and f'_{CTD} is the CTD Treasury's conversion factor.

bond price is possible. The arbitrage-free relationship dictates that the forward price should exactly equal the cash bond price plus the net cost of carrying the cash bond position through the expiration of forward contract. The net carry cost is computed as the cost of financing the bond purchase minus the revenue of cash coupon to be received during the life span of the forward contract. With the addition of the fourth assumption, the full (dirty) price of coupon bond can be expressed as a sum of certainty-equivalent values of both a forward bond and a series of zero-coupon bonds:

$$P + A = (P' + A')e^{-rt} + \sum_{k=1}^{m} I_k e^{-r_k t_k} \tag{1}$$

where A is the accrued coupon as of now, P is the clean (quoted) price, $P + A$ is the full (dirty) price of bond, $P' + A'$ is the dirty forward price of bond, P' is the clean forward price of bond, A' is the accrued coupon as of the maturity of forward, e^{-rt} is the discount factor for the t period between now and the expiration of forward bond contract, r is the spot interest rate continuously compounded for the t period, m denotes the number of cash coupon payments prior to the expiration of forward bond contract, I_k denotes the k-th interest cash coupon payment occurring at the t_k-th time, and r_k is the continuously-compounded spot interest rate for the period between now and the t_k-th time.

The equation (1) simply states a mathematical equivalence between the market price of the generic bond (the left-hand side) and the intrinsic value of a synthetic position (the right-hand side). The synthetic position is equivalent to the position of both selling the forward bond for the hedge horizon and lending, at a risk-free rate, the present value of all cash coupons to be received between now and the expiration of forward bond. The value of the synthetic position contains two parts. One is the present price of the forward bond that matures at end of the hedge. The

[12] Detailed discussions of such assumptions are available upon request.

other is a series of short-term zero-coupon bonds, which have an identical maturity value equal to the generic bond coupon and their respective maturity matching the respective coupon payment date of the generic bond. When there is only one coupon to be received from now till the maturity of forward contract, the second part contains only one zero-coupon bond. When there is no coupon from now till the maturity, then the second part in the right-hand side of equation (1) simply disappears. To focus on the interest-rate exposure of clean ex-coupon price, different notations are used for the clean and dirty prices. These prices can be same, however, when a coupon payment has just been made and hence there is no accrued coupon.[13]

Under the first assumption, a Taylor expansion of the change in clean bond price with respect to a change in short-term spot interest rate will produce a simple expression containing only two terms when only the first- and second-order terms of differentiation are considered as in the usual practice.[14] The application of the usual definitions for duration and convexity measure leads to an equation for the change in clean price:

$$dP = [-(P' + A')D'e^{-rt} - (P + A)D'']dr + [(P' + A')e^{-rt}(C' + tD') + (P + A)C''](dr)^2 \qquad (2)^{15}$$

[13] For alternative explanation of the equation (1), see Rendleman, Jr. (1999).

[14] In the current mathematical framework, higher-order measures of bond price volatility are not required. For such higher order measures, see, e.g., Nawalkha and Lacey (1990) and Jeffrey (2000). See, also, Chance and Jordan (1996), where the investment holding horizon is also considered as one of the factors affecting bond returns (and, implicitly, the bond price-volatility).

[15] where $D' \equiv -\frac{d(P' + A')}{dr}\frac{1}{P' + A'}$ is a forward-looking Macaulay duration for the bond as of the maturity of forward bond, $C' \equiv \frac{1}{2}\frac{d^2(P' + A')}{dr^2}\frac{1}{P' + A'}$ is the corresponding convexity measure, $D'' \equiv \frac{t(P' + A')e^{-rt} + \sum_{k=1}^{m} t_k I_k e^{-r_k t_k}}{(P + A)}$ is a forward-looking Macaulay duration of the synthetic bond that pays the forward price of bond at the expiration of forward bond and makes *m* coupon payments prior to the expiration of forward bond, and $C'' \equiv \frac{(P' + A')e^{-rt}t^2 + \sum_{k=1}^{m} t_k^2 I_k e^{-r_k t_k}}{2(P + A)}$ is the corresponding convexity measure of the synthetic bond position.

The equation for clean price change contains the first-order effect (the bracket term in front of *dr*) and the second-order effect (the bracket term in front of $(dr)^2$). These two terms can be shown to be the same as the two terms in the corresponding equation for the generic bond. The first-order effect is nothing but a linear combination of dollar durations of forward bond and synthetic bond, which is equivalent to dollar duration of the generic bond. Similarly, the second order effect is a linear combination of dollar convexities of forward bond and synthetic bond as well as dollar duration of forward bond.

Unlike the traditional measure for dollar duration, the bracket term in front of *dr* decomposes dollar duration into two components: (i) dollar duration with respect to changes in short-term forward rates beyond hedge expiration and (ii) dollar duration with respect to changes in short-term spot rates through hedge expiration. The bracket term in front of $(dr)^2$ similarly decomposes the dollar convexity into two components. Such decomposition of the dollar price changes is essential for the forward-looking hedge ratio equations to contain an explicit process that both the short-term spot and the forward rate exposures determine the cross hedge ratios. This explicit process makes a non-arbitrary mix of short- and long-term Treasury futures possible because additional information on the interdependency between the two instruments' respective hedges is available.

2. Interest Rate Sensitivity of Treasury Futures' Quote

Under the first, third, and fourth assumptions, a Taylor-series expansion of the change in Treasury futures quote (dF') with respect to a spot interest-rate change (ignoring terms of higher than the second order) leads to:

$$dF' = \frac{dF'}{dr}dr + \frac{1}{2}\frac{d^2F'}{dr^2}(dr)^2 \qquad (3)$$

Under the first assumption, the CTD (i.e., the one with the most severe upward bias in CF) will exist and can be known. Under the fourth assumption, it will be delivered on the first business day of delivery month. Then, the Treasury futures will trade as if it is a contract written on the CTD not on the notional Treasury. As time approaches the delivery day, the Treasury quote should approach the clean price of CTD divided by its conversion factor. In such a stable equilibrium situation, the arbitrage-free price relationship between Treasury futures and CTD will indicate that the dollar duration (convexity) of Treasury futures should equal the dollar duration (convexity) of CTD Treasury divided by the CF.[16]

Using the arbitrage-free price-volatility relationship, the equation for the change in Treasury futures' quote can be specified in terms of the CTD price, its price-volatility measures and the CF:

$$dF' = -\frac{(P'_{CTD} + A'_{CTD})}{f'_{CTD}} D'_{CTD} dr + \frac{(P'_{CTD} + A'_{CTD})}{f'_{CTD}} C'_{CTD} (dr)^2 \qquad (4)$$

Equation (4) incorporates the correct arbitrage-free relationship between futures' price volatility (duration and convexity) and corresponding CTD's price volatility (duration and convexity). The first-order effect (the term in front of *dr*) is nothing but an alternative expression for Treasury futures' dollar duration, whereas the second-order effect (the term in front of $(dr)^2$) is nothing but an alternative expression of Treasury futures' dollar convexity.

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[16] Namely, with i = 1 or 2 for either Treasury futures, $F_i' D_{F_i}' = D'_{CTD_i} (P'_{CTD_i} + A'_{CTD_i}) / f'_{CTD_i}$ and $F_i' C_{F_i}' = C'_{CTD_i} (P'_{CTD_i} + A'_{CTD_i}) / f'_{CTD_i}$, where F' is the Treasury futures' quote, $D'_{F'}$ ($C'_{F'}$) is the Treasury futures' duration (convexity), $F' D'_{F'}$ ($F' C'_{F'}$) is the Treasury futures' dollar duration (convexity), $P'_{CTD} + A'_{CTD}$ is the CTD Treasury's full price (i.e., clean price + accrued coupon), D'_{CTD} ($P'_{CTD} + A'_{CTD}$) is the CTD Treasury's dollar duration, C'_{CTD} ($P'_{CTD} + A'_{CTD}$) is the CTD Treasury's dollar convexity, and f'_{CTD} is the CTD Treasury's conversion factor.

In this paper, the CTD exists due to the bias in CF. The CTD can change over time as the change in term structure can affect the CFs of deliverable Treasuries differently. Unlike traditional hedges, the cross hedges in this paper allow the CTD to vary during the hedge horizon. Using the arbitrage-free convergence of Treasury futures and adjusted CTD price at end of the hedge horizon, the cross hedge in this paper matches the projected measures for interest-rate exposures of bond and Treasury futures prices. In this sense, the cross hedge is forward-looking and captures the correct price and price-volatility linkages between Treasury futures and the CTD.[17]

3. Simultaneous Equations for Convexity-based Hedge Ratios

To derive the convexity-based cross hedge ratios, a straightforward matching of the exposures of hedging and hedged securities is considered. The two hedging instruments considered here are the T-bond and T-note futures traded in CBOT. Their hedge ratios are denoted as $h_1 \quad and \quad h_2$ respectively. In the current analysis, a positive value for hedge ratio indicates the number of short positions in Treasury futures relative to a unit position in the cash bond, whereas a negative value indicates the number of long positions in the Treasury futures.

Matching the bond's forward-looking duration exposure (the first term in equation (2)) to the two futures' forward-looking duration exposures (the first term in equation (4)) yields equation (5). Equating the bond's forward-looking convexity exposure (the second term in equation (2)) to the two futures' forward-looking convexity exposures (the second term in equation (4)) yields equation (6):

[17] For the case of duration hedges, see the Exhibit 1 in Rendleman Jr. (1999) for a comparison between correct and incorrect hedge-ratio formulas.

$$(P' + A')D'e^{-rt} + (P + A)D'' = h_1 \frac{P'_{CTD_1} + A'_{CTD_1}}{f'_{CTD_1}} D'_{CTD_1} + h_2 \frac{P'_{CTD_2} + A'_{CTD_2}}{f'_{CTD_2}} D'_{CTD_2} \quad (5)$$

$$(P' + A')e^{-rt}(C' + tD') + (P + A)C'' = h_1 \frac{P'_{CTD_1} + A'_{CTD_1}}{f'_{CTD_1}} C'_{CTD_1} + h_2 \frac{P'_{CTD_2} + A'_{CTD_2}}{f'_{CTD_2}} C'_{CTD_2} \quad (6)$$

Using the correct definitions for duration and convexity of Treasury futures, these equations can further simplify as equations (7) and (8) for duration matching and convexity matching, respectively:

$$(P' + A')D'e^{-rt} + (P + A)D'' = h_1 D'_{F_1} F'_1 + h_2 D'_{F_2} F'_2 \quad (7)$$

$$(P' + A')e^{-rt}(C' + tD') + (P + A)C'' = h_1 C'_{F_1} F'_1 + h_2 C'_{F_2} F'_2 \quad (8)$$

The solutions for simultaneous equations (7) and (8) are:

$$h_1 = \frac{[(P' + A')D'e^{-rt} + (P + A)D'']C'_{F_2} - [(P' + A')e^{-rt}(C' + tD') + (P + A)C'']D'_{F_2}}{D'_{F_1} C'_{F_2} F'_1 - C'_{F_1} D'_{F_2} F'_1} \quad (9)$$

$$h_2 = \frac{-[(P' + A')D'e^{-rt} + (P + A)D'']C'_{F_1} + [(P' + A')e^{-rt}(C' + tD') + (P + A)C'']D'_{F_1}}{D'_{F_1} C'_{F_2} F'_2 - C'_{F_1} D'_{F_2} F'_2} \quad (10)$$

The hedge ratio for T-bond futures (h_1) can be decomposed into two parts. The first component refers to the portion of T-bond futures' hedge against long-term interest rate exposure, whereas the second component refers to the portion of T-bond futures' hedge against short-term interest rate exposure. The hedge ratio for T-note futures (h_2) can be similarly decomposed into two parts.

Unlike traditional hedge ratios, the hedge ratios and their decomposition in this paper can provide an additional guidance to the desirable mix of short- and long-term Treasury futures as well as to the performance evaluation of selected hedges. Note that the second hedge ratio (h_2) formula can be further simplified when shorter-term Treasury futures (e.g., 1-year or shorter-term

T-bill or Euro-dollar futures) are employed as the second hedging instrument because, then, the price and price-volatility measures for such futures take simpler expressions. Traditional hedge ratios are equivalent to the current hedge ratios if they are obtained using the correct price-volatility measures of Treasury futures. This mathematical equivalence can be easily shown by recombining the decomposed terms (in the equation (2) and (4)) into the expressions for modified duration and convexity measure of the generic bond.

IV. NUMERICAL ANALYSIS

The magnitude of hedge ratios is mainly determined by the shape of term structure of interest rates, the hedge horizon, the characteristics of the bond or bond portfolio being hedged, the characteristics of cheapest-to-deliver Treasuries, and the characteristics of different types of Treasury futures and their pairing. The numerical analysis in the following highlights the relative difference between the cross hedge ratios of traditional and the current model, which is conditional upon (1) the location and slope parameters in a general term-structure specification, (2) the length of hedge horizon, (3) the coupon-maturity characteristics of the bond being hedged and cheapest-to-deliver Treasuries (CTD), and (4) various pairing of long- and short-term Treasury futures. Descriptive statistics on selected results of these sensitivity analyses are reported in Table I through Table V.

Insert Table I about here

Table I reports descriptive statistics on the difference between the absolute sum of hedge ratios of traditional and the forward-looking approaches for 6 different hedge horizons and 3 different levels of durations for the bond being hedged. The descriptive statistics for each hedge horizon in each Panel are based on 189 numerical observations.

In Panel B of Table I (i.e., the case of bond with the medium-level duration of 7.61), the percentage difference in the mean hedge cost ranges from 1.57% to 1.81% whereas the percentage difference based on the maximum hedge cost ranges from 2.53% to 3.99%. The hedge horizon does not have a statistically significant effect on the variability in the percentage difference in the mean hedge cost. The hedge horizon effect is not statistically significant for the case of bond either of relatively high duration (9.90) or of relatively low duration (3.28). The bond duration does have a statistically significant effect on the percentage difference in the mean hedge cost. For the bond with relatively high duration, the mean percentage difference in mean hedge costs is about 1.9% whereas the same is about 1.3% for the bond with relatively low duration. Although the percentage difference in mean hedge costs for the bond with medium-level duration varies relatively more across hedge horizon, the percentage difference in hedge costs by and large increases with bond duration. [18]

Insert Table II about here

Table II reports descriptive statistics on the difference between the absolute sum of hedge ratios of traditional and the forward-looking approaches for 6 different shapes of term structure and 3 different levels of durations for the bond being hedged. The shape of term structure is captured by the intercept (α) and the slope (β) parameters of a general term-structure specification. For example, the term structure specification with α = 6.5% and β = 0.5% reflects a nearly flat term structure with high level of spot interest rate for short term, whereas the term structure specification with α = 0.5% and β = 6.5% reflects a very-steep term structure with low

[18] It is possible that the hedge-horizon effect does exist but is subdued by the bond duration effect. It is also possible that the hedge-horizon effect may exist in longer horizons of 6 to 12 months. To check this possibility, alternative descriptive statistics are obtained without controlling the duration level and for hedge horizons of 1 to 12 months. These alternative descriptive statistics are based on 1701 (= 189*3*3) observations because 3 bond maturities and 3 bond coupon rates are additionally considered. The detailed analysis not reported here indicates that the hedge-horizon effect does not exist even when the duration level is not controlled for and that the lack of hedge-horizon effect becomes even more distinct when the horizon becomes longer.

level of spot interest rate for short term. The shape of term structure (with α = 3.5% and β = 3.5%) corresponding to the middle of the range in term-structure shape reflect the typical term structure that existed toward the end of 2000. The descriptive statistics for each shape of term structure in each Panel are based on 189 numerical observations.

In Panel B of Table II (i.e., the case of bond with the medium-level duration of 7.90), the percentage difference in the mean hedge cost ranges from 1.75% to 1.92% whereas the percentage difference based on the maximum hedge cost ranges from 3.21% to 3.83%. The effect of term-structure shape does not have a statistically significant effect on the variability in the percentage difference of the mean hedge cost. The effect of term-structure shape is not statistically significant for the case of bond either of relatively high duration (9.92) or of relatively low duration (3.48). The bond duration seems to have a weakly statistically significant effect on the variability in the percentage difference of the mean hedge cost. For the bond with relatively high duration, the mean percentage difference in the mean hedge cost is about 1.9% whereas the same is about 1.7% for the bond with relatively low duration. The percentage difference in the mean hedge cost by and large increases with bond duration.[19]

Insert Table III about here

Table III reports descriptive statistics on the difference between the absolute sum of hedge ratios of traditional and the forward-looking approaches for 6 different coupon rates of the CTD and 3 different levels of durations for the bond being hedged. The descriptive statistics for each CTD coupon rate in each Panel are based on 189 numerical observations.

[19] It is possible that the effect of term-structure shape does exist but is subdued by the bond duration effect. To check this possibility, alternative descriptive statistics are obtained without controlling the duration level. These alternative descriptive statistics are based on 1701 (= 189*3*3) observations because 3 bond maturities and 3 bond coupon rates are additionally considered. The detailed analysis not reported here indicates that the effect of term-structure shape does not exist even when the duration level is not controlled for. A further analysis not reported here indicates, however, a statistically significant positive relationship between the term-structure slope parameter (β) and the mean difference in the mean hedge cost.

In Panel B of Table III (i.e., the case of bond with the medium-level duration of 7.90), the percentage difference in the mean hedge cost ranges from 1.26% to 2.22% whereas the percentage difference based on the maximum hedge cost ranges from 1.96% to 4.00%. The effect of CTD coupon has a statistically significant positive effect on the variability in the percentage difference of the mean hedge cost. This positive effect of CTD coupon carries over to the case of bond either of relatively high duration (9.93) or of relatively low duration (3.47). [20]

Insert Table IV about here

Table IV reports descriptive statistics on the difference between the absolute sum of hedge ratios of traditional and the forward-looking approaches for 6 different bond coupon rates and 3 different bond maturities. Given the strong CTD coupon effect (reported in Table III), Table IV no longer assumes identical coupon rates for the CTD T-bonds and T-notes. The descriptive statistics for each bond coupon rate in each Panel are therefore based on 567 (= 189 * 3) numerical observations.

In Panel B of Table IV (i.e., the case of bond with the medium remaining maturity of 14.25 years), the percentage difference in the mean hedge cost ranges from 1.81% to 1.86% whereas the percentage difference based on the maximum hedge cost ranges from 3.75% to 4.11%. The bond coupon does not have a statistically significant effect on the variability in the percentage difference of the mean hedge cost. The bond coupon effect does not exist for the bonds with relatively short (4.25 years) and long (24.25 years) remaining maturities. For the case of relatively long maturity, the percentage difference in the mean hedge cost ranges from 1.75%

[20] It is possible that the CTD coupon effect may not exist but it appears so because of its interaction with the bond duration effect. To check this possibility, alternative descriptive statistics are obtained without controlling the duration level and for 11 levels of CTD coupon. These alternative descriptive statistics are based on 1701 (= 189*3*3) observations because 3 bond maturities and 3 bond coupon rates are additionally considered. The detailed analysis not reported here indicates that the CTD coupon effect does exist even when the duration level is not controlled for.

to 1.93% whereas the same does not vary across bond coupon rate in the case of relatively short maturity of 4.25 years.[21]

Insert Table V about here

Table V reports descriptive statistics on the difference between the absolute sum of hedge ratios of traditional and the forward-looking approaches for 6 different bond maturities and 3 different bond coupon rates. Although similar to Table IV, Table V intends to ensure the lack of the bond coupon and maturity effects. It also attempts to determine the relative strength between bond coupon and bond maturity effects, if any. The descriptive statistics in each Panel are also based on 567 numerical observations.

In Panel B of Table V (i.e., the case of bond with the medium-level coupon rate of 8%), the percentage difference in the mean hedge cost ranges from 1.79% to 1.85% whereas the percentage difference based on the maximum hedge cost ranges from 3.42% to 4.80%. The bond maturity does not have a statistically significant effect on the variability in the percentage difference of the mean hedge cost. A positive bond maturity effect does exist, however, for the bonds with relatively high (13%) and low (3%) coupon rates. For the case of relatively high coupon bond, the percentage difference in the mean hedge cost ranges from 1.47% to 1.75% whereas the same ranges from 1.66% to 1.87% in the case of relatively low coupon bond. The bond coupon does not have a statistically significant effect on the percentage difference.[22]

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[21] Note that bond duration monotonically increases in bond maturity and monotonically decreases in bond coupon rate. Then, given the positive duration effect on the percentage difference, one should expect a positive bond maturity effect and a negative bond coupon-rate effect. It is possible that the bond coupon effect may exist but it does not appear so because of its interaction with the bond maturity effect. To check this possibility, alternative descriptive statistics are obtained without controlling the bond maturity and for 11 levels of bond maturity. These alternative descriptive statistics are based on 1701 (= 567*3) observations because 3 bond maturities are additionally considered. The detailed analysis not reported here indicates that the bond coupon effect does not exist even when the bond maturity is not controlled for.

[22] It is possible that the bond maturity effect may exist but it does not appear so because of its interaction with the bond coupon effect. To check this possibility, alternative descriptive statistics are obtained without controlling the bond coupon and for 11 levels of bond maturity. These alternative descriptive statistics are based on 1701 (= 567*3)

V. Practical Implications

The series of numerical analysis indicates that the mean hedge-cost saving of forward-looking hedge is about 1.8%. The hedge-cost saving, when high coupon (13%) Treasury is the CTD, becomes around 2.25% in the case of hedging 8% coupon 14.25-year bond.

The larger hedge-cost saving when the high-coupon Treasury is the CTD has an important practical implication. Note, for example, that recent market interest rates for long-term maturity (say, 20 years) Treasuries have been hovering around 6% (the coupon rate for notional Treasury underlying Treasury futures).[23] Note also that, at present, certain seasoned Treasuries with long-term remaining maturities have coupon rates much (at least 6 percentage points) higher than the market interest rates for equivalent maturities. The conversion factor for such high coupon-rate Treasury would be larger than its intrinsic relative value because its future cash flows are discounted at rates lower than the current ones. It is intuitively obvious that such high coupon Treasury with an upward-biased conversion factor would tend to be the CTD. In fact, it has been demonstrated in relevant literature that, for a nearly flat term structure, the CTD would be a high coupon Treasury when (1) the market interest rate for long-term bond is larger than 6% and (2) the coupon rate of eligible Treasury exceeds the market interest rate for comparable maturity.[24] In other words, when actual coupon rates of eligible Treasuries (i.e., those issued previously at times of high interest rates) exceed the market interest rates for equivalent maturities, the Treasury with highest coupon rate tends to be the CTD. Hence, the practical importance of the

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observations because 3 bond coupon rates are additionally considered. The detailed analysis not reported here indicates that the bond maturity effect does not exist even when the bond coupon is not controlled for.

[23] In July 2001, the market rate has been always at least few basis points higher than 6%. In the previous two years, the market rate has been much higher than 6% (e.g., at least 50 basis points higher on average during January 2001).

[24] See, e.g., Benninga and Wiener (1999).

finding applies not only to current times of declining interest rates but also to recent past (and forthcoming future time) of increasing interest rates.

The hedge-cost saving is about 1.95% when hedging high duration bond (say, 3% coupon bond with 24.25-year maturity). This finding that the hedge-cost saving will be more in the case of hedging high duration bond has also an important practical implication. This is so because, under increasing uncertainty on interest-rate changes, convexity-based hedges of such high duration bonds will be the main tasks facing bond portfolio managers.

VI. A SUMMARY AND CONCLUSIONS

This paper addresses the two problems inherent in traditional convexity-hedge models: (i) the neglect of important volatility linkage between Treasury futures and cheapest-to-deliver Treasuries, which leads to incorrect hedge ratios; and (ii) the arbitrary selection of any two non-redundant Treasury futures as hedging instruments.

To address the problem arising from the neglected price-volatility linkage, the current analysis first provides detailed explanations on the nature of conversion factor system and the bias in the CBOT's CF measure for relative quote of deliverable Treasury. It then establishes an arbitrage-free price linkage between Treasury futures and cheapest-to-deliver Treasury and determines an arbitrage-free price volatility (duration and convexity) linkage between them. In so doing, the cross hedge ratio equations incorporate the correct volatility measures for Treasury futures. To address the problem arising from arbitrary selection of any two hedge-instruments, the current analysis considers the interest-rate exposure of a synthetic position rather than the generic bond (or bond portfolio) position. Doing so leads to the forward-looking cross hedge-ratio equations containing an explicit channel through which the cross hedge ratios are affected by both the short-term spot and the forward interest rates beyond the hedge horizon. Hence, hedgers can draw additional information from the hedge-ratio equations and utilize them in choosing a specific mix of hedge instruments.

This paper also provides detailed numerical analysis that documents the main difference between traditional and the forward-looking hedge ratios. The most important finding is that the forward-looking hedge costs less about 1.8% on average, with the maximum hedge-cost saving reaching up to 4.8%. More detailed analysis indicates that the hedge cost saving becomes larger for hedging high duration bond especially when higher coupon Treasury is the cheapest-to-deliver. The hedge-cost saving of forward-looking hedges persists across various maturities of the bond being hedged and the cheapest-to-deliver Treasury. The hedge-cost saving is particularly large when the cheapest-to-deliver is high coupon Treasury. As explained in the main text, high coupon Treasury would tend to be the cheapest-to-deliver Treasury under the current interest rate cycle. The hedge-cost saving will be even larger if term structure becomes steeper as in the recent past.

Given relatively high interest rate environments in the past three decades, the practical importance of the CTD coupon effect is likely to remain strong for many years to come. It is also likely to remain strong even in the next cycles of rising interest rates and subsequent upward revisions of the coupon rate of notional Treasury. This is so because the current interest rates are very low relative to comparable interest rates in the past three decades.

One of the most critical assumptions that facilitated the development of the forward-looking hedge model and numerical analysis is the implicit assumption that delivery-related options do not exist. The issue is of direct relevance to the major numerical findings in this paper. The pricing of these options itself is a very complicated issue that is yet to be resolved. Hence, the relaxation of this unrealistic assumption remains as a promising future research agenda. The relaxation may not, however, change the qualitative direction of the major findings in this paper.

To understand this conjecture, note that the existence of CTD in the current paper is due to the bias in conversion factor and that the CTD choice implies a negative convexity relationship between the interest rate and the Treasury futures' quote. Note also that the main difference between traditional and the current hedge arises from the negative convexity implied by the conversion factor bias. The existence of full-blown delivery-related options would further strengthen the existence of CTD, the importance of CTD choice, and the negative convexity of Treasury futures' quote. An intuition then suggests that the stronger negative convexity resulting from the existence of delivery-related options would rather magnify the relative overhedge in the traditional approach.

The forward-looking duration-convexity hedges using Treasury futures can also apply to other fixed income markets, e.g., UK and Japan markets, since these markets adopt similar conversion-factor systems to deliverable Treasuries. To the extent that the interest rate cycles in these markets closely follow the interest rate cycles in the US, which is quite often, one would find results that are similar to those in this paper.

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TABLE I

Descriptive Statistics of the Difference between Traditional and Forward-looking Cross-Hedge Ratios (by Various Hedge Horizons and Bond Durations)

Panel A: High-Duration Bond (24.25-year maturity; 3% coupon; 9.90 Duration; 83.9 Convexity)							
Hedge Horizon	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
6 months	0.9805	0.9799	0.9727	0.9925	1.9485	2.7346	0.7467
5 months	0.9808	0.9800	0.9733	0.9927	1.9244	2.6740	0.7324
4 months	0.9797	0.9792	0.9639	0.9908	2.0298	3.6106	0.9250
3 months	0.9813	0.9805	0.9742	0.9930	1.8745	2.5786	0.7021
2 months	0.9815	0.9809	0.9744	0.9931	1.8484	2.5645	0.6863
1 months	0.9805	0.9801	0.9643	0.9904	1.9501	3.5675	0.9614
Panel B: Medium-Duration Bond (14.25-year maturity; 8% coupon; 7.61 Duration; 42.3 Convexity)							
Hedge Horizon	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
6 months	0.9843	0.9837	0.9689	0.9989	1.5656	3.1097	0.1100
5 months	0.9842	0.9834	0.9702	0.9986	1.5816	2.9807	0.1385
4 months	0.9825	0.9823	0.9614	0.9932	1.7540	3.8552	0.6818
3 months	0.9840	0.9827	0.9731	0.9980	1.5952	2.6919	0.1971
2 months	0.9840	0.9824	0.9747	0.9977	1.6039	2.5314	0.2270
1 months	0.9819	0.9814	0.9601	0.9916	1.8109	3.9906	0.8356
Panel C: Low-Duration Bond (4.25-year maturity; 13% coupon; 3.28 Duration; 6.40 Convexity)							
Hedge Horizon	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
6 months	0.9880	0.9872	0.9767	0.9973	1.2021	2.3306	0.2693
5 months	0.9874	0.9864	0.9760	0.9964	1.2637	2.3952	0.3573
4 months	0.9839	0.9850	0.9546	0.9955	1.6062	4.5390	0.4471
3 months	0.9861	0.9848	0.9747	0.9950	1.3905	2.5328	0.5001
2 months	0.9854	0.9839	0.9739	0.9940	1.4604	2.6065	0.5955
1 months	0.9816	0.9826	0.9524	0.9931	1.8385	4.7571	0.6928

Note:

(1) Selected descriptive statistics on both the difference and percentage difference are computed for bonds with high-, medium-, and low-duration for six different hedge horizons. The bond maturity is the remaining maturity at end of the hedge horizon. The difference between traditional and forward-looking hedge ratios is computed as "the absolute sum of traditional hedge ratios minus the absolute sum of forward-looking hedge ratios." The %Diff_Mean is computed as the mean difference divided by the absolute sum of traditional hedge ratios, whereas the %Diff_Min (%Diff_Max) is computed as the minimum (maximum) difference divided by the traditional hedge ratios. These percentage differences reflect, therefore, the respective mean and minimum (maximum) percentage differences in hedge costs of traditional and the forward-looking hedges.

(2) As six different hedge horizons, 1-, 2-, 3-, 4-, 5-, and 6-month horizons are considered. Each hedge horizon matches the Treasury futures' expiration.

(3) In using the term structure specification, $r(t) = \alpha + \beta\ [\ln(t+5.0) - \ln(5.0)]$, three pairs of α, β values are considered. For example, the pair "0.005, 0.065" reflects a very-steep term structure with a low spot rate for short term, whereas the pair "0.065, 0.005" reflects a nearly-flat term structure with a high spot rate for short term. The pair "0.035, 0.035" reflects a term structure that is neither too steep nor too flat and indicates a term structure with a short-term spot rate that is neither too high nor too low.

(4) Table I does not assume a unique CTD T-bond for the 30-year T-bond futures and a unique CTD T-notes for each of the 10-year, 5-year, or 2-year T-note futures. It considers, instead, 3 kinds of CTD T-bonds and 7 kinds of CTD T-notes, which differ only by their remaining maturities. As the CTD T-bond's remaining maturities, near-maximum (29.25), near-medium (22.25 years), and near-minimum (15.25 years) eligible maturities are considered. As the remaining maturities of the CTD T-note corresponding to the 10-year T-note futures, near-maximum (9.25 years), near-medium (7.75 years), and near-minimum (6.25 years) eligible maturities are considered. As the remaining maturities of the CTD T-note corresponding to the 5-year T-note futures, near-maximum (4.75 years), near-medium (4.50 years), and near-minimum (4.25 years) eligible maturities are considered. As the remaining maturity of the CTD T-note corresponding to the 2-year T-note futures that seldom trade, only the near-minimum (1.75 years) eligible maturity is considered because the near-minimum is only 3 months away from the near-maximum (2.0 years) eligible maturity. As CTD coupon rates, 3%, 8%, and 13% are considered. Since, given the 3 CTD coupon rates, each combination of 3 CTD T-bonds and 7 CTD T-notes corresponds to each alternative pairing of T-bond futures with T-note futures, the total number of hedge-instrument pairs considered for Table I is 63 (= 3*3*7).

(5) Since there are 63 hedge-instrument pairs and 3 shapes of term structure for each horizon for each type of bond being hedged, the total number of observations used for the descriptive statistics in each row of Table I is 189 (= 63*3).

(6) All the descriptive statistics are statistically significantly positive at the 1% level.

TABLE II
Descriptive Statistics of the Difference between Traditional and Forward-looking Cross-Hedge Ratios
(by Various Shapes of Term Structure and Bond Durations)

Panel A: High-Duration Bond (24.25-year maturity; 3% coupon; 9.92 Duration; 83.9 Convexity)							
Term-Str. (α, β)	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
0.005, 0.065	0.9790	0.9784	0.9685	0.9875	2.1046	3.1524	1.2490
0.015, 0.055	0.9795	0.9789	0.9678	0.9882	2.0464	3.2213	1.1752
0.025, 0.045	0.9801	0.9794	0.9671	0.9891	1.9880	3.2904	1.0885
0.035, 0.035	0.9807	0.9800	0.9664	0.9902	1.9292	3.3594	0.9824
0.045, 0.025	0.9813	0.9804	0.9657	0.9911	1.8698	3.4285	0.8866
0.055, 0.015	0.9816	0.9807	0.9650	0.9920	1.8437	3.4979	0.8000
0.065, 0.005	0.9826	0.9817	0.9643	0.9928	1.7417	3.5675	0.7216
Panel B: Medium-Duration Bond (14.25-year maturity; 8% coupon; 7.90 Duration; 43.4 Convexity)							
Term-Str. (α, β)	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
0.005, 0.065	0.9808	0.9806	0.9677	0.9954	1.9215	3.2288	0.4599
0.015, 0.055	0.9812	0.9809	0.9679	0.9953	1.8783	3.2090	0.4713
0.025, 0.045	0.9816	0.9812	0.9666	0.9952	1.8375	3.3431	0.4773
0.035, 0.035	0.9820	0.9817	0.9650	0.9952	1.7999	3.5022	0.4779
0.045, 0.025	0.9825	0.9822	0.9633	0.9958	1.7469	3.6664	0.4190
0.055, 0.015	0.9823	0.9818	0.9617	0.9970	1.7697	3.8330	0.2974
0.065, 0.005	0.9808	0.9806	0.9677	0.9954	1.9215	3.2288	0.4599
Panel C: Low-Duration Bond (4.25-year; 13% coupon; 3.48 Duration; 7.39 Convexity)							
Term-Str. (α, β)	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
0.005, 0.065	0.9830	0.9830	0.9568	0.9973	1.7011	4.3180	0.2693
0.015, 0.055	0.9830	0.9830	0.9562	0.9972	1.7030	4.3830	0.2774
0.025, 0.045	0.9829	0.9830	0.9555	0.9971	1.7058	4.4486	0.2854
0.035, 0.035	0.9829	0.9831	0.9549	0.9971	1.7098	4.5145	0.2933
0.045, 0.025	0.9829	0.9831	0.9542	0.9970	1.7149	4.5808	0.3010
0.055, 0.015	0.9828	0.9830	0.9535	0.9969	1.7212	4.6473	0.3086
0.065, 0.005	0.9827	0.9830	0.9529	0.9968	1.7287	4.7141	0.3161

Note:

(1) Selected descriptive statistics on both the difference and percentage difference are computed for bonds with high-, medium-, and low-duration for six different hedge horizons. The bond maturity is the remaining maturity at end of the hedge horizon. The difference between traditional and forward-looking hedge ratios is computed as "the absolute sum of traditional hedge ratios minus the absolute sum of forward-looking hedge ratios." The %Diff_Mean is computed as the mean difference divided by the absolute sum of traditional hedge ratios, whereas the %Diff_Min (%Diff_Max) is computed as the minimum (maximum) difference divided by the traditional hedge ratios. These percentage differences reflect, therefore, the respective mean and minimum (maximum) percentage differences in hedge costs of traditional and the forward-looking hedges.

(2) As various shapes of the term structure specification, $r(t) = \alpha + \beta\,[\ln(t+5.0) - \ln(5.0)]$, seven combinations of α's and β's values are considered. For example, the term structure specification with $\alpha = 5.5\%$ and $\beta = 0.5\%$ reflects a nearly-flat term structure with high level of spot interest rate for short term, whereas the term structure specification with $\alpha = 0.5\%$ and $\beta = 5.5\%$ reflects a very-steep term structure with low level of spot interest rate for short term.

(3) As hedge horizons, 1-, 3-, and 6-month horizons are considered. Each Treasury futures' expiration is assumed to match the hedge expiration.

(4) Table II does not assume a unique CTD T-bond for the 30-year T-bond futures and a unique CTD T-notes for each of the 10-year, 5-year, or 2-year T-note futures. It considers, instead, 3 kinds of CTD T-bonds and 7 kinds of CTD T-notes, which differ only by their remaining maturities. As the CTD T-bond's remaining maturities, near-maximum (29.25), near-medium (22.25 years), and near-minimum (15.25 years) eligible maturities are considered. As the remaining maturities of the CTD T-note corresponding to the 10-year T-note futures, near-maximum (9.25 years), near-medium (7.75 years), and near-minimum (6.25 years) eligible maturities are considered. As the remaining maturities of the CTD T-note corresponding to the 5-year T-note futures, near-maximum (4.75 years), near-medium (4.50 years), and near-minimum (4.25 years) eligible maturities are considered. As the remaining maturity of the CTD T-note corresponding to the 2-year T-note futures that seldom trade, only the near-minimum (1.75 years) eligible maturity is considered because the near-minimum is only 3 months away from the near-maximum (2.0 years) eligible maturity. As CTD coupon rates, 3%, 8%, and 13% are considered. Since, given the 3 CTD coupon rates, each combination of 3 CTD T-bonds and 7 CTD T-notes corresponds to each alternative pairing of T-bond futures with T-note futures, the total number of hedge-instrument pairs considered for Table II is 63 (= 3*3*7).

(5) Since there are 63 hedge-instrument pairs and 3 hedge horizons for each type of bond being hedged under each term-structure shape, the total number of observations used for the descriptive statistics in each row of Table II is 189 (= 63*3).

(6) All the descriptive statistics are statistically significantly positive at the 1% level.

TABLE III
Descriptive Statistics of the Difference between Traditional and Forward-looking Cross-Hedge Ratios
(by Various Coupon Rates of CTD and Bond Durations)

Panel A: High-Duration Bond (24.25-year maturity; 3% coupon; 9.93 Duration; 84.0 Convexity)							
CTD Coupon	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
3%	0.9853	0.9849	0.9792	0.9928	1.4737	2.0776	0.7217
5%	0.9818	0.9821	0.9746	0.9913	1.8168	2.5414	0.8685
7%	0.9803	0.9806	0.9748	0.9905	1.9717	2.5232	0.9505
9%	0.9793	0.9792	0.9708	0.9900	2.0703	2.9238	1.0009
11%	0.9781	0.9779	0.9673	0.9897	2.1940	3.2676	1.0337
13%	0.9772	0.9770	0.9643	0.9894	2.2788	3.5675	1.0560
Panel B: Medium-Duration Bond (14.25 years, 8% coupon, 7.90 Duration, 43.4 Convexity)							
CTD Coupon	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
3%	0.9874	0.9878	0.9804	0.9980	1.2625	1.9639	0.1971
5%	0.9836	0.9842	0.9750	0.9971	1.6404	2.4960	0.2906
7%	0.9810	0.9818	0.9712	0.9965	1.8969	2.8836	0.3529
9%	0.9792	0.9801	0.9668	0.9960	2.0798	3.3227	0.3969
11%	0.9779	0.9789	0.9631	0.9957	2.2141	3.6882	0.4293
13%	0.9778	0.9779	0.9600	0.9955	2.2230	3.9992	0.4540
Panel C: Low-Duration Bond (4.25-year maturity; 13% coupon; 3.47 Duration; 7.40 Convexity)							
CTD Coupon	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
3%	0.9905	0.9914	0.9802	0.9973	0.9475	1.9797	0.2693
5%	0.9867	0.9880	0.9756	0.9951	1.3302	2.4381	0.4902
7%	0.9837	0.9845	0.9685	0.9962	1.6306	3.1510	0.3809
9%	0.9812	0.9820	0.9625	0.9960	1.8844	3.7530	0.3955
11%	0.9790	0.9802	0.9573	0.9959	2.0999	4.2682	0.4081
13%	0.9771	0.9780	0.9529	0.9958	2.2862	4.7141	0.4191

Note:

(1) Selected descriptive statistics on both the difference and percentage difference are computed for bonds with high-, medium-, and low-duration for six different hedge horizons. The bond maturity is the remaining maturity at end of the hedge horizon. The difference between traditional and forward-looking hedge ratios is computed as "the absolute sum of traditional hedge ratios minus the absolute sum of forward-looking hedge ratios." The %Diff_Mean is computed as the mean difference divided by the absolute sum of traditional hedge ratios, whereas the %Diff_Min (%Diff_Max) is computed as the minimum (maximum) difference divided by the traditional hedge ratios. These percentage differences reflect, therefore, the respective mean and minimum (maximum) percentage differences in hedge costs of traditional and the forward-looking hedges.

(2) As the six different coupon rates of CTD, 3%, 5%, 7%, 9%, 11%, and 13% are considered.

(3) As hedge horizons, 1-, 3-, and 6-month horizons are considered. Each Treasury futures' expiration is assumed to match the hedge expiration.

(4) In using the term structure specification, $r(t) = \alpha + \beta\,[\ln(t+5.0) - \ln(5.0)]$, three pairs of α, β values are considered. The pair "0.005, 0.065" reflects a very-steep term structure with a low spot rate for short term, whereas the pair "0.065, 0.005" reflects a nearly-flat term structure with a high spot rate for short term. The pair "0.035, 0.035" reflects a term structure that is neither too steep nor too flat. It also reflects a term structure with a short-term spot rate that is neither too high nor too low.

(5) Table III does not assuming a unique CTD T-bond for the 30-year T-bond futures and a unique CTD T-notes for each of the 10-year, 5-year, or 2-year T-note futures. It considers, instead, 3 kinds of CTD T-bonds and 7 kinds of CTD T-notes, which differ only by their remaining maturities. As the CTD T-bond's remaining maturities, near-maximum (29.25), near-medium (22.25 years), and near-minimum (15.25 years) eligible maturities are considered. As the remaining maturities of the CTD T-note corresponding to the 10-year T-note futures, near-maximum (9.25 years), near-medium (7.75 years), and near-minimum (6.25 years) eligible maturities are considered. As the remaining maturities of the CTD T-note corresponding to the 5-year T-note futures, near-maximum (4.75 years), near-medium (4.50 years), and near-minimum (4.25 years) eligible maturities are considered. As the remaining maturity of the CTD T-note corresponding to the 2-year T-note futures that seldom trade, only the near-minimum (1.75 years) eligible maturity is considered because the near-minimum is only 3 months away from the near-maximum (2.0 years) eligible maturity. Since each combination of 3 CTD T-bonds and 7 CTD T-notes corresponds to each alternative pairing of T-bond futures with T-note futures, the total number of hedge-instrument pairs considered for Table III is 27 (= 3*7).

(6) Since there are 27 hedge-instrument pairs, 3 hedge horizons, and 3 shapes of term structure for each type of bond being hedged for each CTD coupon rate considered, the total number of observations used for the descriptive statistics in each row of Table III is 189 (= 27*3*3).

(7) All the descriptive statistics are statistically significantly positive at the 1% level.

TABLE IV

Descriptive Statistics of the Difference between Traditional and Forward-looking Cross-Hedge Ratios

(by Various Coupon Rates and Remaining Maturities of Bond Being Hedged)

Panel A: Long-Maturity Bond (24.25 years)							
Bond Coupon	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
3%	0.9807	0.9801	0.9643	0.9928	1.9251	3.5675	0.7217
5%	0.9811	0.9807	0.9651	0.9943	1.8936	3.4851	0.5725
7%	0.9818	0.9810	0.9658	0.9953	1.8238	3.4188	0.4715
9%	0.9821	0.9814	0.9664	0.9959	1.7886	3.3642	0.4099
11%	0.9823	0.9815	0.9668	0.9963	1.7705	3.3184	0.3684
13%	0.9825	0.9818	0.9672	0.9966	1.7537	3.2796	0.3385
Panel B: Medium-Maturity Bond (14.25 years)							
Bond Coupon	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
3%	0.9818	0.9810	0.9625	0.9968	1.8244	3.7523	0.3194
5%	0.9819	0.9816	0.9612	0.9974	1.8069	3.8802	0.2572
7%	0.9818	0.9814	0.9603	0.9979	1.8190	3.9660	0.2140
9%	0.9814	0.9813	0.9597	0.9982	1.8567	4.0276	0.1823
11%	0.9814	0.9814	0.9593	0.9984	1.8552	4.0740	0.1581
13%	0.9815	0.9814	0.9589	0.9986	1.8538	4.1101	0.1389
Panel C: Short-Maturity Bond (4.25 years)							
Bond Coupon	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
3%	0.9833	0.9825	0.9524	0.9987	1.6728	4.7586	0.1324
5%	0.9833	0.9830	0.9525	0.9981	1.6706	4.7477	0.1917
7%	0.9833	0.9833	0.9526	0.9976	1.6686	4.7380	0.2353
9%	0.9833	0.9834	0.9527	0.9973	1.6666	4.7292	0.2747
11%	0.9833	0.9834	0.9528	0.9967	1.6671	4.7213	0.3285
13%	0.9833	0.9835	0.9529	0.9973	1.6656	4.7141	0.2693

Note:

(1) Selected descriptive statistics on both the difference and percentage difference are computed for bonds with high-, medium-, and low-duration for six different hedge horizons. The bond maturity is the remaining maturity at end of the hedge horizon. The difference between traditional and forward-looking hedge ratios is computed as "the absolute sum of traditional hedge ratios minus the absolute sum of forward-looking hedge ratios." The %Diff_Mean is computed as the mean difference divided by the absolute sum of traditional hedge ratios, whereas the %Diff_Min (%Diff_Max) is computed as the minimum (maximum) difference divided by the traditional hedge ratios. These percentage differences reflect, therefore, the respective mean and minimum (maximum) percentage differences in hedge costs of traditional and the forward-looking hedges. Each Treasury Futures' expiration is also assumed to match the hedge expiration.

(2) As the six different coupon rates of Bond, 3%, 5%, 7%, 9%, 11%, and 13% are considered.

(3) As hedge horizons, 1-, 3-, and 6-month horizons are considered.

(4) In using the term structure specification, $r(t) = \alpha + \beta\ [\ln(t+5.0) - \ln(5.0)]$, three pairs of α, β values are considered. The pair "0.005, 0.065" reflects a very-steep term structure with a low spot rate for short term, whereas the pair "0.065, 0.005" reflects a nearly-flat term structure with a high spot rate for short term. The pair "0.035, 0.035" reflects a term structure that is neither too steep nor too flat. It also reflects a term structure with a short-term spot rate that is neither too high nor too low.

(5) Table IV does not assuming a unique CTD T-bond for the 30-year T-bond futures and a unique CTD T-notes for each of the 10-year, 5-year, or 2-year T-note futures. It considers, instead, 3 kinds of CTD T-bonds and 7 kinds of CTD T-notes, which differ only by their remaining maturities. As the CTD T-bond's remaining maturities, near-maximum (29.25), near-medium (22.25 years), and near-minimum (15.25 years) eligible maturities are considered. As the remaining maturities of the CTD T-note corresponding to the 10-year T-note futures, near-maximum (9.25 years), near-medium (7.75 years), and near-minimum (6.25 years) eligible maturities are considered. As the remaining maturities of the CTD T-note corresponding to the 5-year T-note futures, near-maximum (4.75 years), near-medium (4.50 years), and near-minimum (4.25 years) eligible maturities are considered. As the remaining maturity of the CTD T-note corresponding to the 2-year T-note futures that seldom trade, only the near-minimum (1.75 years) eligible maturity is considered because the near-minimum is only 3 months away from the near-maximum (2.0 years) eligible maturity. As CTD coupon rates, 3%, 8%, and 13% are considered. Since, given the 3 CTD coupon rates, each combination of 3 CTD T-bonds and 7 CTD T-notes corresponds to each alternative pairing of T-bond futures with T-note futures, the total number of hedge-instrument pairs considered for Table IV is 63 (= 3*3*7).

(6) Since there are 63 hedge-instrument pairs, 3 hedge horizons, and 3 shapes of term structure for each coupon rate and remaining maturity of bond, the total number of observations used for the descriptive statistics in each row of Table IV is 567 (= 63*3*3).

(7) All the descriptive statistics are statistically significantly positive at the 1% level.

TABLE V
Descriptive Statistics of the Difference between Traditional and Forward-looking Cross-Hedge Ratios
(by Various Remaining Maturities and Coupon Rates of Bond Being Hedged)

Panel A: High-Coupon (13%) Bond							
Bond Maturity	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
24.25	0.9825	0.9818	0.9672	0.9966	1.7537	3.2796	0.3385
20.25	0.9828	0.9816	0.9645	0.9977	1.7240	3.5501	0.2298
16.25	0.9836	0.9826	0.9608	0.9989	1.6385	3.9235	0.1138
12.25	0.9845	0.9833	0.9575	0.9998	1.5458	4.2528	0.0192
8.25	0.9853	0.9842	0.9547	0.9999	1.4675	4.5342	0.0147
4.25	0.9854	0.9855	0.9524	0.9970	1.4648	4.7571	0.3050
Panel B: Medium-Coupon (8%) Bond							
Bond Maturity	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
24.25	0.9817	0.9811	0.9658	0.9980	1.8314	3.4217	0.1977
20.25	0.9815	0.9811	0.9655	0.9986	1.8463	3.4537	0.1388
16.25	0.9818	0.9813	0.9621	0.9974	1.8158	3.7883	0.2630
12.25	0.9818	0.9816	0.9581	0.9987	1.8223	4.1926	0.1311
8.25	0.9821	0.9820	0.9547	0.9999	1.7887	4.5280	0.0144
4.25	0.9815	0.9821	0.9520	0.9944	1.8528	4.7976	0.5626
Panel C: Low-Coupon (3%) Bond							
Bond Maturity	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
24.25	0.9813	0.9808	0.9642	0.9946	1.8694	3.5805	0.5441
20.25	0.9815	0.9809	0.9654	0.9944	1.8503	3.4605	0.5620
16.25	0.9817	0.9810	0.9666	0.9963	1.8270	3.3352	0.3663
12.25	0.9825	0.9815	0.9602	0.9981	1.7496	3.9849	0.1906
8.25	0.9834	0.9821	0.9556	0.9996	1.6636	4.4394	0.0355
4.25	0.9833	0.9825	0.9524	0.9987	1.6728	4.7586	0.1324

Note:

(1) Selected descriptive statistics on both the difference and percentage difference are computed for bonds with high-, medium-, and low-duration for six different hedge horizons. The bond maturity is the remaining maturity at end of the hedge horizon. The difference between traditional and forward-looking hedge ratios is computed as “the absolute sum of traditional hedge ratios minus the absolute sum of forward-looking hedge ratios.” The %Diff_Mean is computed as the mean difference divided by the absolute sum of traditional hedge ratios, whereas the %Diff_Min (%Diff_Max) is computed as the minimum (maximum) difference divided by the traditional hedge ratios. These percentage differences reflect, therefore, the respective mean and minimum (maximum) percentage differences in hedge costs of traditional and the forward-looking hedges.

(2) As the six different remaining maturities of bond being hedged, 4.25, 8.25, 12.25, 16.25, 20.25, and 24.25 years are considered.

(3) As hedge horizons, 1-, 3-, and 6-month horizons are considered. Each Treasury futures’ expiration is assumed to match the hedge expiration.

(4) In using the term structure specification, $r(t) = \alpha + \beta\ [\ln(t+5.0) - \ln(5.0)]$, three pairs of α, β values are considered. The pair “0.005, 0.065” reflects a very-steep term structure with a low spot rate for short term, whereas the pair “0.065, 0.005” reflects a nearly-flat term structure with a high spot rate for short term. The pair “0.035, 0.035” reflects a term structure that is neither too steep nor too flat. It also reflects a term structure with a short-term spot rate that is neither too high nor too low.

(5) Table V does not assuming a unique CTD T-bond for the 30-year T-bond futures and a unique CTD T-notes for each of the 10-year, 5-year, or 2-year T-note futures. It considers, instead, 3 kinds of CTD T-bonds and 7 kinds of CTD T-notes, which differ only by their remaining maturities. As the CTD T-bond’s remaining maturities, near-maximum (29.25), near-medium (22.25 years), and near-minimum (15.25 years) eligible maturities are considered. As the remaining maturities of the CTD T-note corresponding to the 10-year T-note futures, near-maximum (9.25 years), near-medium (7.75 years), and near-minimum (6.25 years) eligible maturities are considered. As the remaining maturities of the CTD T-note corresponding to the 5-year T-note futures, near-maximum (4.75 years), near-medium (4.50 years), and near-minimum (4.25 years) eligible maturities are considered. As the remaining maturity of the CTD T-note corresponding to the 2-year T-note futures that seldom trade, only the near-minimum (1.75 years) eligible maturity is considered because the near-minimum is only 3 months away from the near-maximum (2.0 years) eligible maturity. As CTD coupon rates, 3%, 8%, and 13% are considered. Since, given the 3 CTD coupon rates, each combination of 3 CTD T-bonds and 7 CTD T-notes corresponds to each alternative pairing of T-bond futures with T-note futures, the total number of hedge-instrument pairs considered for Table V is 63 (= 3*3*7).

(6) Since there are 63 hedge-instrument pairs, 3 hedge horizons, and 3 shapes of term structure for each coupon rate and remaining maturity of bond, the total number of observations used for the descriptive statistics in each row of Table V is 567 (= 63*3*3).

(7) All the descriptive statistics are statistically significantly positive at the 1% level.